

(Color-)magnetic flux tubes in dense matter

nuclear matter: A. Haber, A. Schmitt, PRD 95, 116016 (2017)

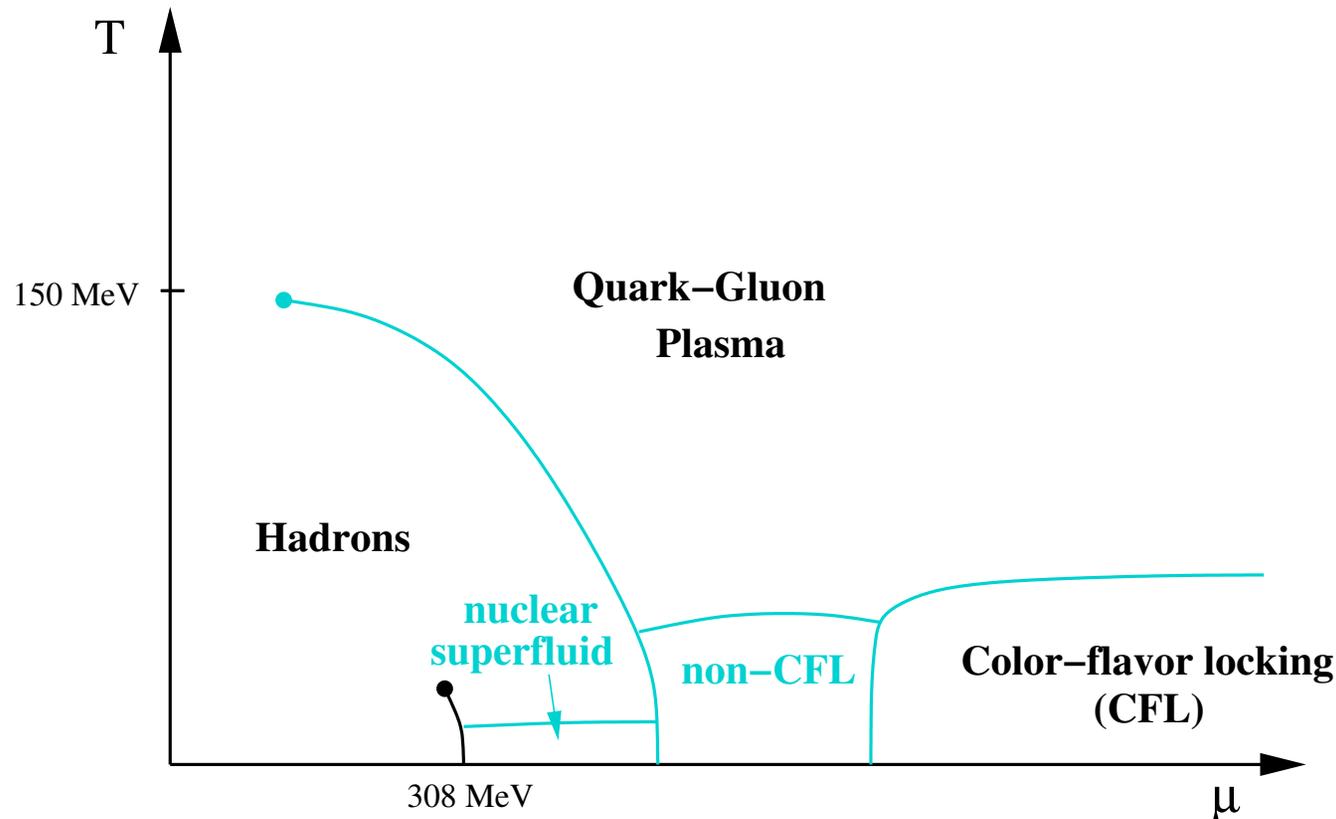
A. Haber, A. Schmitt, EPJ Web Conf. 137, 09003 (2017)

quark matter: A. Haber, A. Schmitt, in preparation (most of this talk)

- dense quark matter:
multi-component superconductor
- phase structure including type-II
color superconductivity
- flux tubes in neutron star cores
("color-magnetic mountains")



● Motivation



- (ultra-)dense QCD at nonzero magnetic field:
type-I/type-II color superconductivity? color-magnetic flux tubes?
- implications for neutron stars: gravitational waves? evolution of magnetic field? (mis-)alignment of rotational and magnetic axis?

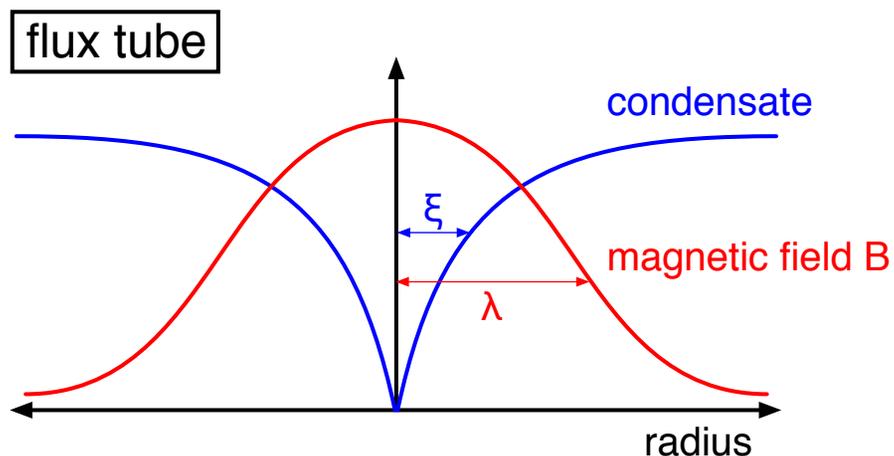
- **Single-component superconductor**

- Ginzburg-Landau potential U for complex field ϕ with charge q coupled to gauge field A^μ

$$U = \frac{\mathbf{B}^2}{2} + |(\nabla + iq\mathbf{A})\phi|^2 - \mu^2|\phi|^2 + \lambda|\phi|^4$$

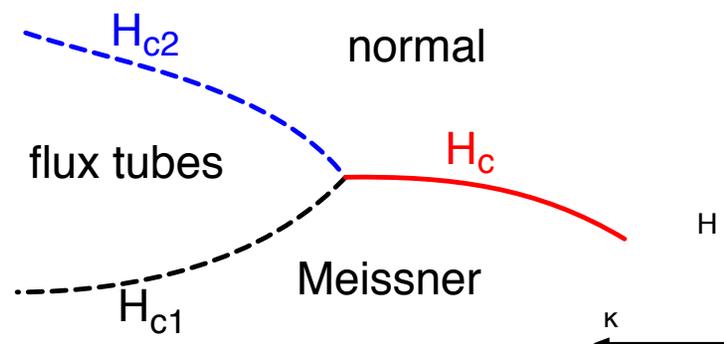
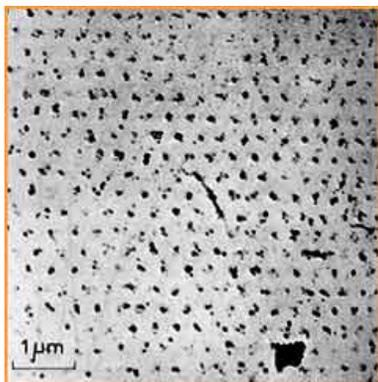
→ textbook scenario: flux tube lattice for $H_{c1} < H < H_{c2}$,
type-I/type-II transition at $\kappa = 1/\sqrt{2}$, etc

- Reminder: type-I/type-II superconductivity



- Ginzburg-Landau parameter

$$\kappa = \frac{\lambda}{\xi}$$



first image of flux tube lattice:
 U. Essmann and H. Träuble
 Phys. Lett. A 24, 526 (1967)

- type-II superconductivity for $\kappa > 1/\sqrt{2}$:
 flux tube lattice for $H_{c1} < H < H_{c2}$
 A.A. Abrikosov, Soviet Physics JETP 5, 1174 (1957)

- **Multi-component superconductor (page 1/2)**

- two fields with charges q_1, q_2 (neutron/proton: $q_1 = 2e, q_2 = 0$)

$$U = \frac{\mathbf{B}^2}{2} + \sum_{i=1,2} [|(\nabla + iq_i \mathbf{A})\phi_i|^2 - \mu_i^2 |\phi_i|^2 + \lambda_i |\phi_i|^4] + 2h |\phi_1|^2 |\phi_2|^2$$

- fields are coupled indirectly via gauge field (if both $q_1, q_2 \neq 0$) and directly with coupling h (neutron/proton system: additional derivative coupling [A. Haber, A. Schmitt, PRD 95, 116016 \(2017\)](#))

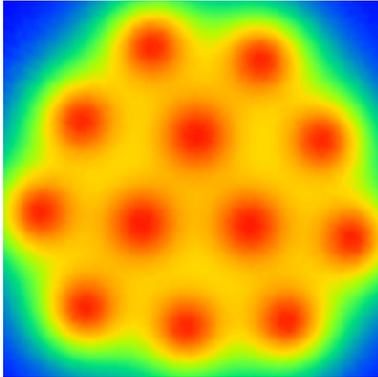
- **Multi-component superconductors (page 2/2)**

- more fields and multiple (color-)gauge fields

$$U = \frac{\mathbf{B}_1^2}{2} + \frac{\mathbf{B}_2^2}{2} + \sum_{i=1}^3 \left[|(\nabla + iq_{i1}\mathbf{A}_1 + iq_{i2}\mathbf{A}_2)\phi_i|^2 - \mu^2|\phi_i|^2 + \lambda|\phi_i|^4 \right]$$
$$- 2h(|\phi_1|^2|\phi_2|^2 + |\phi_1|^2|\phi_3|^2 + |\phi_2|^2|\phi_3|^2)$$

- color superconductor: 3 scalar components
and 3 gauge fields: 1 electromagnetic and 2 color fields
(all commuting, i.e., no non-abelian effects)

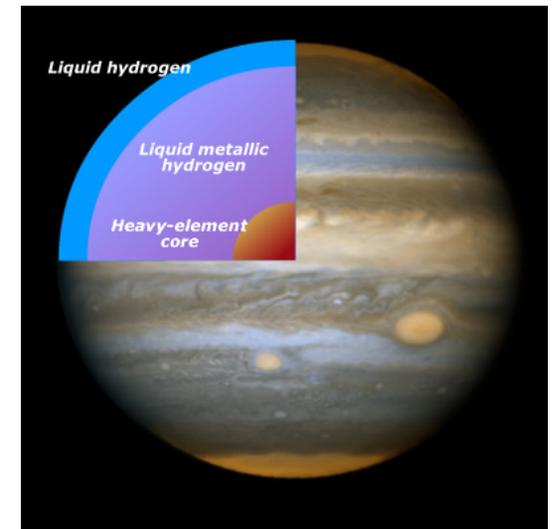
- **Examples (page 1/2)**



- (1) two-band superconductors
("type-1.5" superconductivity → vortex clusters)
J. Carlström, J. Garaud, E. Babaev, PRB 84, 134515 (2011)

- (2) liquid metallic hydrogen (in Jupiter)

E. Babaev, A. Sudbø, N. Ashcroft, Nature 431, 666 (2004)



- (3) ultracold atoms (possibly?)

two-component superfluid I. Ferrier-Barbut, *et al.*, Science 345, 1035 (2014)

single-component "charged" system Y.J. Lin, *et al.*, Nature 462, 628 (2009)

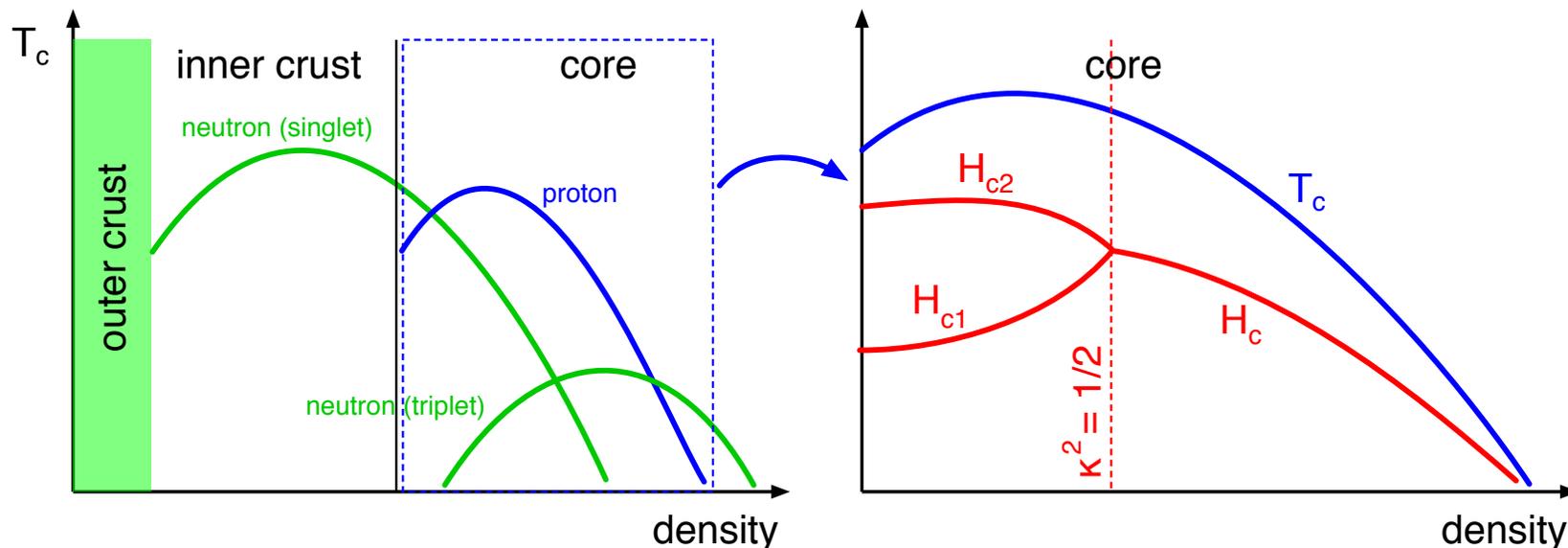
• Examples (page 2/2)

(4) dense neutron/proton matter

M. G. Alford and G. Good, PRB 78, 024510 (2008)

A. Haber, A. Schmitt, PRD 95, 116016 (2017)

→ possible type-I/type-II transition in the interior of neutron stars



(5) color-superconducting quark matter

K. Iida and G. Baym, PRD 65, 014022 (2002)

K. Iida, PRD 71, 054011 (2005)

new – energetically favored – flux tube solutions: A. Haber, A. Schmitt, in preparation

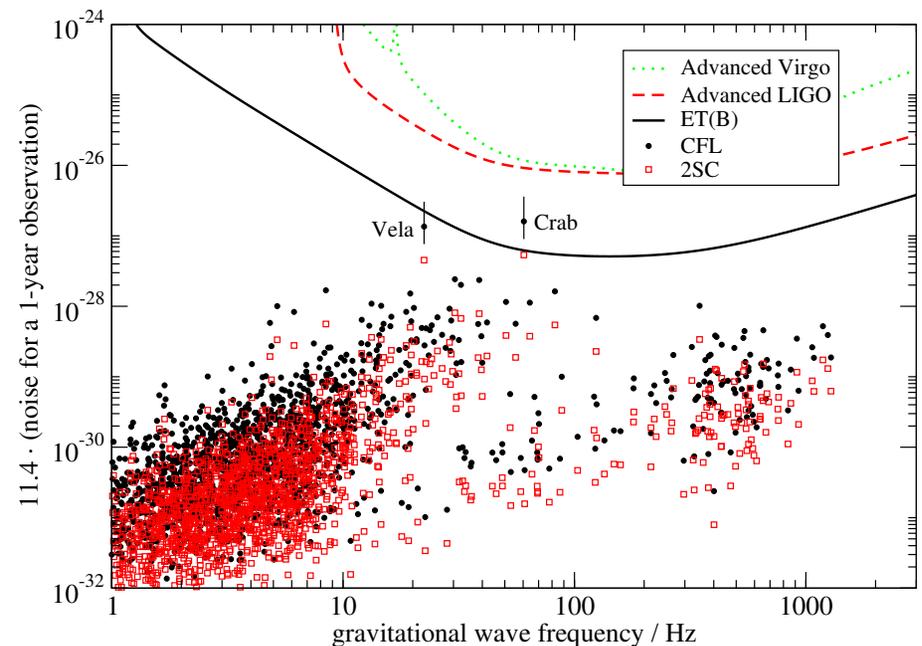
● Astrophysical motivation

- suppose core of neutron star is made of quark matter
- magnetic field induces array of color-magnetic flux tubes
M. G. Alford and A. Sedrakian, *JPG* 37, 075202 (2010)

→ enhanced ellipticity of compact stars with quark matter core

→ gravitational waves

K. Glampedakis, D. I. Jones and L. Samuelsson, *PRL* 109, 081103 (2012)



- CFL and 2SC phases (page 1/2)

- quark Cooper pairs

$$SU(3)_c : \quad [\mathbf{3}]_c \otimes [\mathbf{3}]_c = [\bar{\mathbf{3}}]_c^a \oplus [\mathbf{6}]_c^s \quad (\text{attractive channel})$$

$$SU(3)_f : \quad [\mathbf{3}]_f \otimes [\mathbf{3}]_f = [\bar{\mathbf{3}}]_f^a \oplus [\mathbf{6}]_f^s \quad (\text{overall antisymmetry})$$

\Rightarrow order parameter

$$\Psi \in [\bar{\mathbf{3}}]_c^a \otimes [\bar{\mathbf{3}}]_f^a, \quad \Psi_{ij}^{\alpha\beta} = \Phi_A^B \epsilon^{\alpha\beta A} \epsilon_{ijB}$$

- 3×3 matrix Φ determines pairing pattern

$$\Phi = \begin{pmatrix} \phi & 0 & 0 \\ 0 & \phi & 0 \\ 0 & 0 & \phi \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \phi \end{pmatrix} \quad \begin{pmatrix} \phi_1(\vec{r}) & 0 & 0 \\ 0 & \phi_2(\vec{r}) & 0 \\ 0 & 0 & \phi_3(\vec{r}) \end{pmatrix}$$

color-flavor locking (CFL)

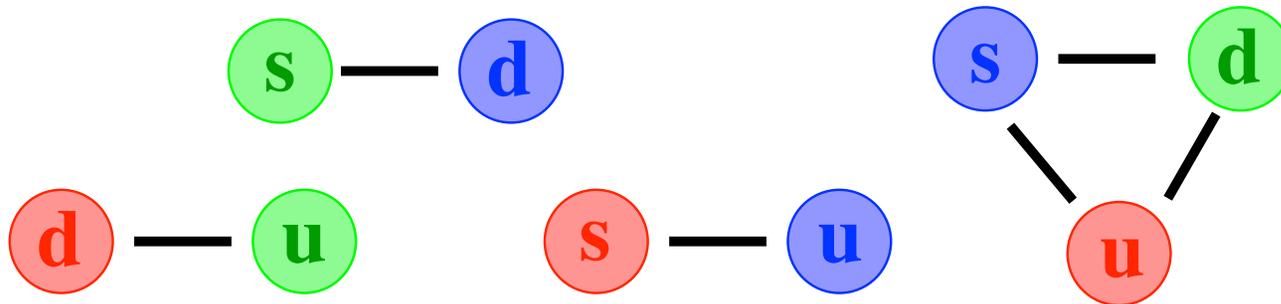
2SC

ansatz for flux tubes

- CFL and 2SC phases (page 2/2)

- color-flavor locking: all quarks pair

M. Alford, K. Rajagopal, F. Wilczek, NPB 537, 443 (1999)



- 2SC phase: possibly preferred due to $m_s \gg m_u, m_d$

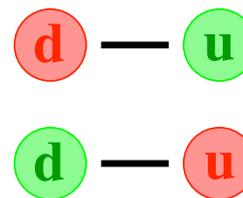
R. Rapp, T. Schäfer, E.V. Shuryak,

M. Velkovsky, PRL 81, 53 (1998)

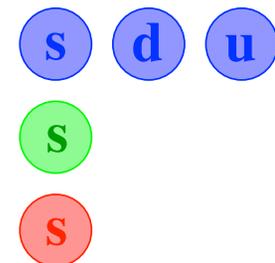
M.G. Alford, K. Rajagopal, F. Wilczek,

PLB 422, 247 (1998)

paired:



unpaired:



- **Mixing of gluons and photons in CFL**

- symmetry breaking pattern of CFL

$$[SU(3)]_c \times \underbrace{SU(3)_L \times SU(3)_R}_{\supset [U(1)]_Q} \times U(1)_B \rightarrow \underbrace{SU(3)_{c+L+R}}_{\supset [U(1)]_{\tilde{Q}}} \times \mathbb{Z}_2$$

- CFL is a superfluid \rightarrow rotational vortices
- Meissner effect for gluons T_1, \dots, T_7 ("color superconductor")
- all Cooper pairs neutral under $\tilde{Q} = Q + \frac{2}{\sqrt{3}}T_8$
and (differently) charged under orthogonal combination \tilde{T}_8
 - \tilde{Q} -magnetic field penetrates CFL
 - Meissner effect for \tilde{T}_8 -magnetic field

(Analogous to gauge field mixing in standard model, $[SU(2)] \times [U(1)] \rightarrow [U(1)]_Q$)

- **Ginzburg-Landau potential**

$$\begin{aligned}
 U = & \frac{\tilde{\mathbf{B}}^2}{2} + \frac{\mathbf{B}_3^2}{2} + \frac{\tilde{\mathbf{B}}_8^2}{2} + \left| \left(\nabla + i\frac{g}{2}\mathbf{A}_3 + i\tilde{g}_8\tilde{\mathbf{A}}_8 \right) \phi_1 \right|^2 + \left| \left(\nabla - i\frac{g}{2}\mathbf{A}_3 + i\tilde{g}_8\tilde{\mathbf{A}}_8 \right) \phi_2 \right|^2 \\
 & + \left| \left(\nabla - 2i\tilde{g}_8\tilde{\mathbf{A}}_8 \right) \phi_3 \right|^2 - \mu^2(|\phi_1|^2 + |\phi_2|^2 + |\phi_3|^2) + \lambda(|\phi_1|^4 + |\phi_2|^4 + |\phi_3|^4) \\
 & - 2h(|\phi_1|^2|\phi_2|^2 + |\phi_1|^2|\phi_3|^2 + |\phi_2|^2|\phi_3|^2)
 \end{aligned}$$

- approximations

- massless quarks, $m_s = m_d = m_u = 0$
- purely bosonic approach \rightarrow neglect effects of magnetic field on Cooper pair constituents
- Ginzburg-Landau parameters μ, λ, h : (mostly) use perturbative results and extrapolate down in density (= to large couplings)

- Ginzburg-Landau potential

$$\begin{aligned}
 U = & \frac{\tilde{\mathbf{B}}^2}{2} + \frac{\mathbf{B}_3^2}{2} + \frac{\tilde{\mathbf{B}}_8^2}{2} + \left| \left(\nabla + i\tilde{g}_8\tilde{\mathbf{A}}_8 + i\frac{g}{2}\mathbf{A}_3 \right) \phi_1 \right|^2 + \left| \left(\nabla + i\tilde{g}_8\tilde{\mathbf{A}}_8 - i\frac{g}{2}\mathbf{A}_3 \right) \phi_2 \right|^2 \\
 & + \left| \left(\nabla - 2i\tilde{g}_8\tilde{\mathbf{A}}_8 \right) \phi_3 \right|^2 - \mu^2(|\phi_1|^2 + |\phi_2|^2 + |\phi_3|^2) + \lambda(|\phi_1|^4 + |\phi_2|^4 + |\phi_3|^4) \\
 & - 2h(|\phi_1|^2|\phi_2|^2 + |\phi_1|^2|\phi_3|^2 + |\phi_2|^2|\phi_3|^2)
 \end{aligned}$$

- ansatz for flux tube solutions

$$\phi_i(r, \theta) = \rho_i(r) e^{in_i\theta}$$

with winding numbers n_1, n_2, n_3

- solve equations of motion for $\rho_1, \rho_2, \rho_3, \mathbf{A}_3, \tilde{\mathbf{A}}_8$
- boundary conditions: homogenous CFL far away from flux tube, $\rho_i = 0$ in center if winding n_i is nonzero

- usually: vortex \rightarrow baryon circulation $\Gamma = \oint d\ell \cdot \mathbf{v}$
- flux tube \rightarrow magnetic flux $\Phi = \oint d\ell \cdot \mathbf{A}$

- CFL line defects can have both!

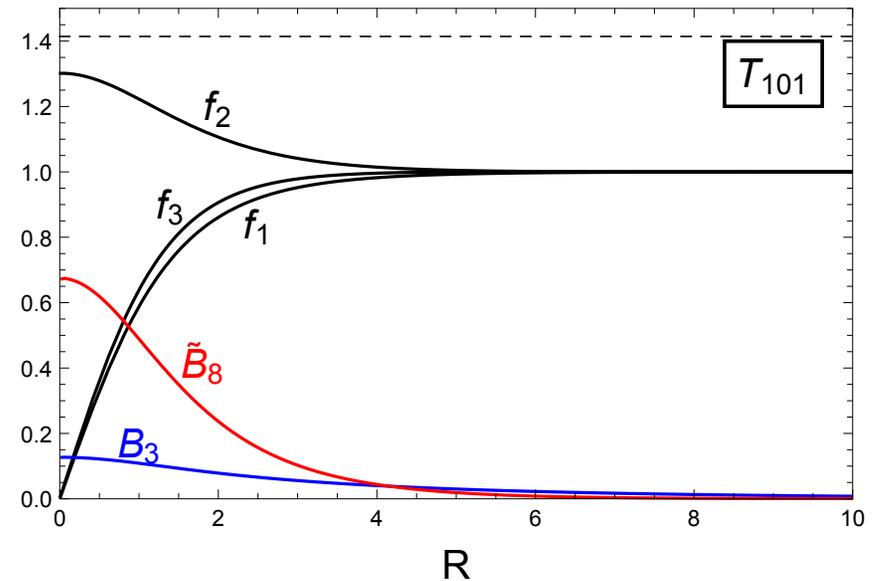
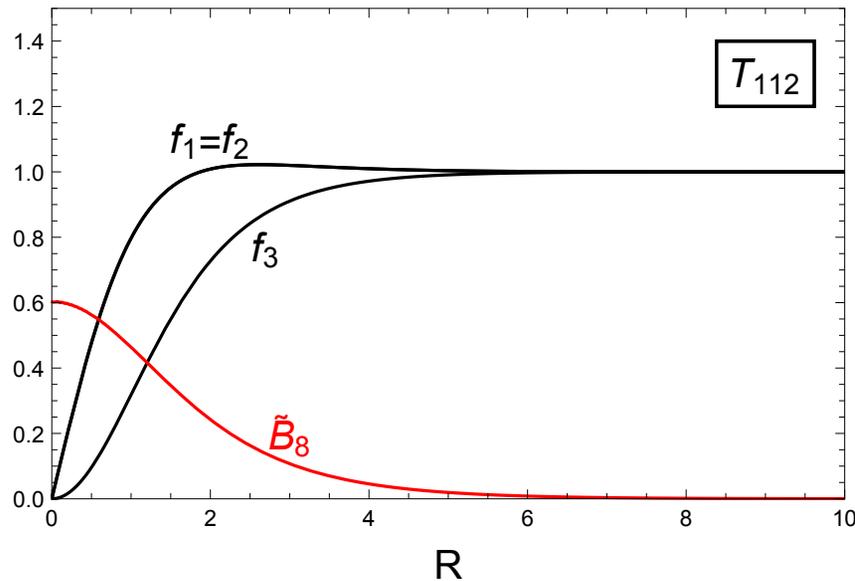
$$\Gamma \propto n_1 + n_2 + n_3, \quad \Phi_3 \propto n_1 - n_2, \quad \tilde{\Phi}_8 \propto n_1 + n_2 - 2n_3$$

CFL line defects	(n_1, n_2, n_3)	$\Gamma [\pi/3\mu_q]$	$\Phi_3 [\pi/g]$	$\tilde{\Phi}_8 [\pi/\tilde{g}_8]$
Global vortex Forbes, Zhitnitsky (2002)	(n, n, n)	$-n$	0	0
"Semi-superfluid" vortex Balachandran, Digal, Matsuura (2006)	$(0, 0, n)$	$-\frac{n}{3}$	0	$\frac{2n}{3}$
Magnetic flux tube T_{112} Iida (2005)	$(n, n, -2n)$	0	0	$-2n$
Magnetic flux tube T_{101} Haber, Schmitt, in prep.	$(n, 0, -n)$	0	$-n$	$-n$

CFL line defects	(n_1, n_2, n_3)	Γ [$\pi/3\mu_q$]	Φ_3 [π/g]	$\tilde{\Phi}_8$ [π/\tilde{g}_8]
Global vortex Forbes, Zhitnitsky (2002)	(n, n, n)	$-n$	0	0
”Semi-superfluid” vortex Balachandran, Digal, Matsuura (2006)	$(0, 0, n)$	$-\frac{n}{3}$	0	$\frac{2n}{3}$
Magnetic flux tube T_{112} Iida (2005)	$(n, n, -2n)$	0	0	$-2n$
Magnetic flux tube T_{101} Haber, Schmitt, in prep.	$(n, 0, -n)$	0	$-n$	$-n$

- vortices ($\Gamma \neq 0$): ”topological” since $\pi_1[U(1)] = \mathbb{Z}$
 - global vortex decays into 3 semi-superfluid vortices
M. G. Alford, S. K. Mallavarapu, T. Vachaspati and A. Windisch, PRC 93, 045801 (2016)
- flux tubes ($\Gamma = 0$): ”non-topological” since $\pi_1[SU(3)] = 0$
 - stabilized through external magnetic field → see rest of the talk

● Flux tube profiles



- flux tube with "unpaired core"

K. Iida, PRD 71, 054011 (2005)

- flux tube with "2SC core"

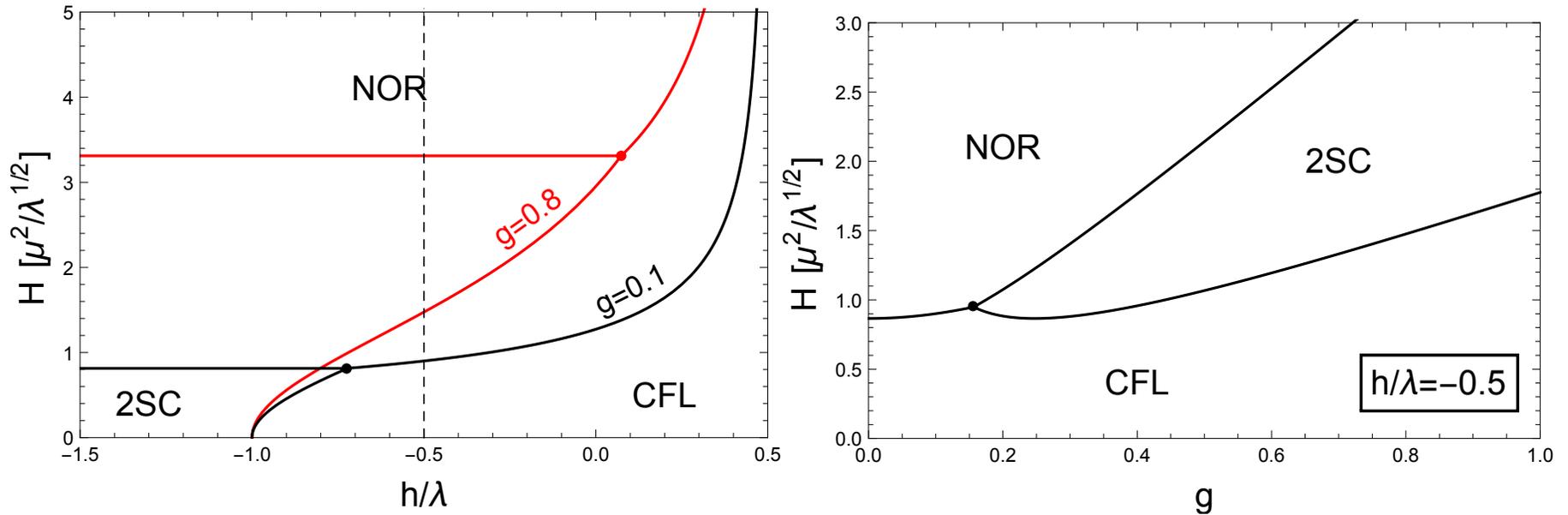
- additional B_3 field
(cost in free energy)

- non-vanishing condensate in
core (gain in free energy)

- which configuration is preferred?

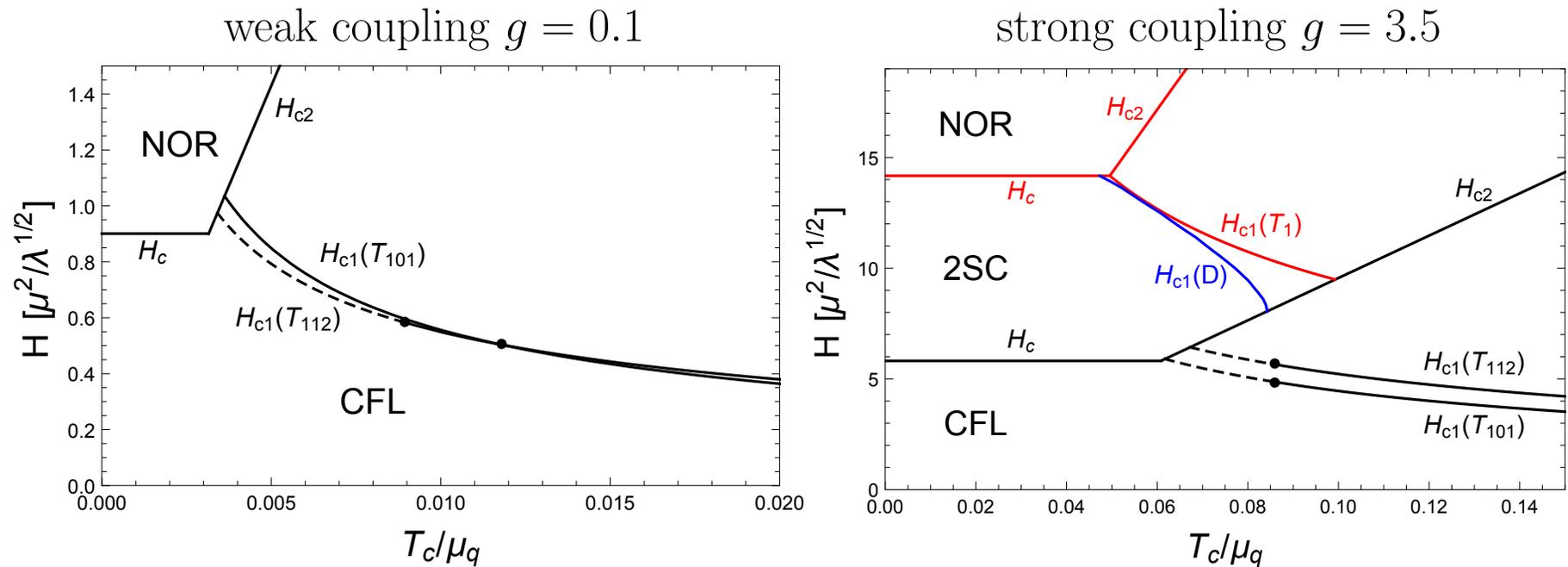
→ compute critical magnetic fields in parameter space

● Phase structure with homogeneous phases



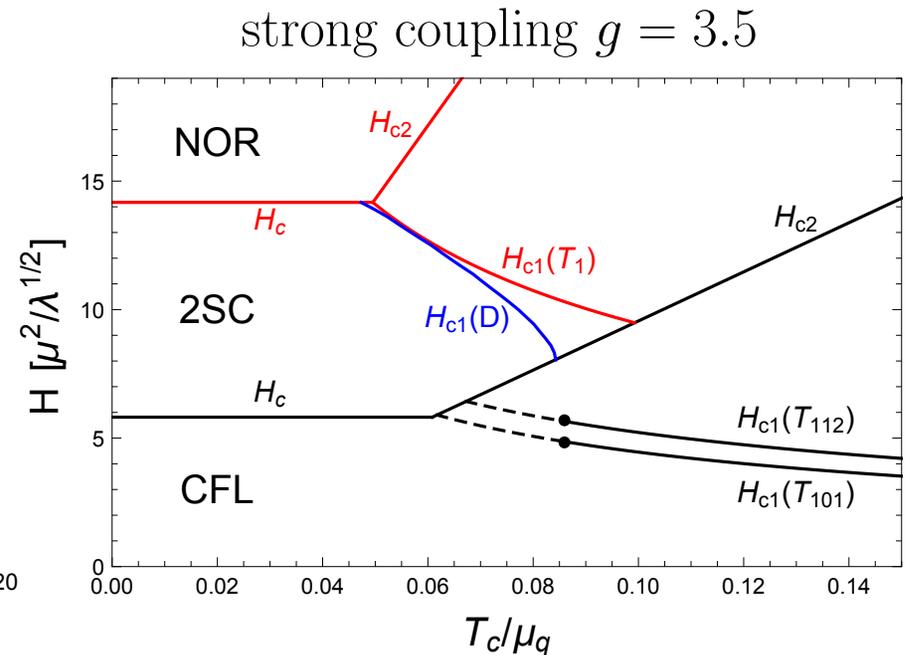
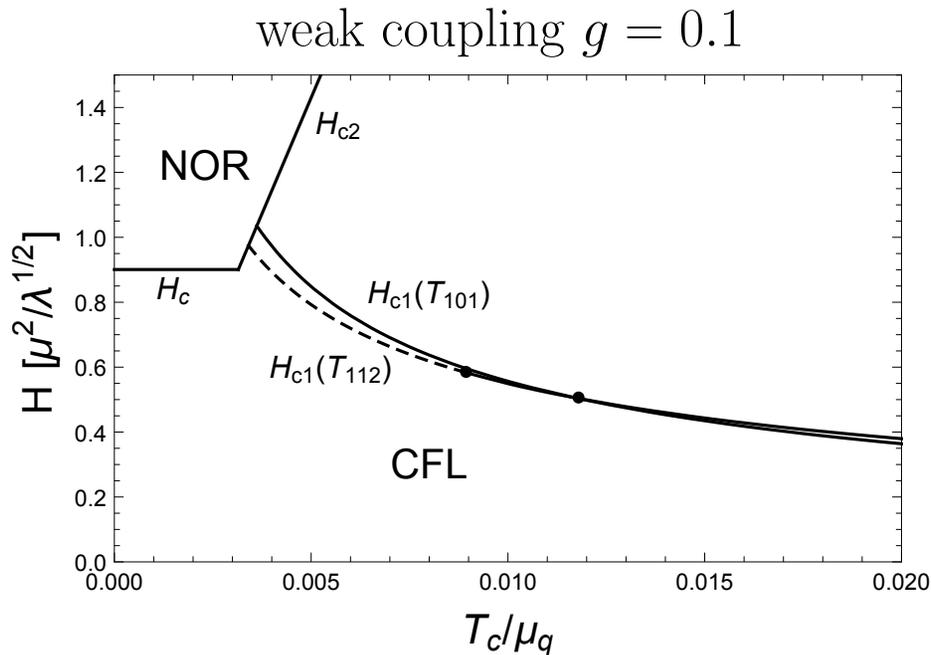
- in weak coupling: $h/\lambda = -0.5$
- CFL superseded by 2SC except for small values of strong coupling constant g
- in neutron stars $\mu_q \simeq 400 \text{ MeV} \Rightarrow g \simeq 3.5$

• Critical magnetic fields



- type-II regime for sufficiently large T_c/μ_q
- type-I/type-II transition complicated (multi-component structure!)
 more details: [A. Haber, A. Schmitt, PRD 95, 116016 \(2017\)](#)

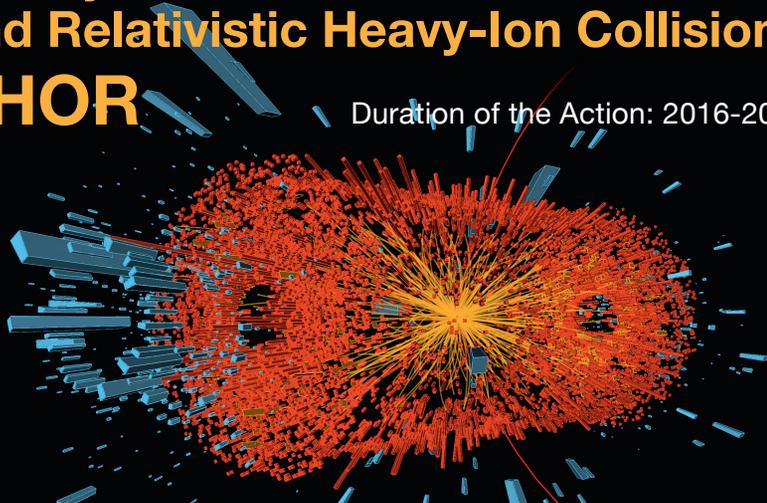
● Critical magnetic fields



- CFL flux tubes with 2SC core (T_{101}) preferred, except for small g
- 2SC domain walls (D) preferred over ordinary 2SC flux tubes (T_1)
- critical fields $H \sim 10^{19}$ G,
creation of flux tubes through cooling into superconducting phase?

- **Summary**

- **dense quark matter** is a **multi-component** superconductor and has various possible line defects
- multi-component superconductors have a **nontrivial type-I/type-II transition**
- CFL flux tubes (without baryon circulation) are **not protected by topology**, but can be **stabilized by a magnetic field**
- we have found solutions with minimized winding:
"CFL tubes with 2SC core" and
"2SC domain walls" (not discussed in this talk)
- defects in superconducting/superfluid nuclear and quark matter are relevant for **neutron star observables**, e.g., **gravitational waves**



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