Thermodynamics of Parity Doublers in Effective Theory

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Outlines

I. Hadrons in a hot/dense medium
   - Parity doublers: mesons and baryons
   - Survival masses vs. trace anomaly

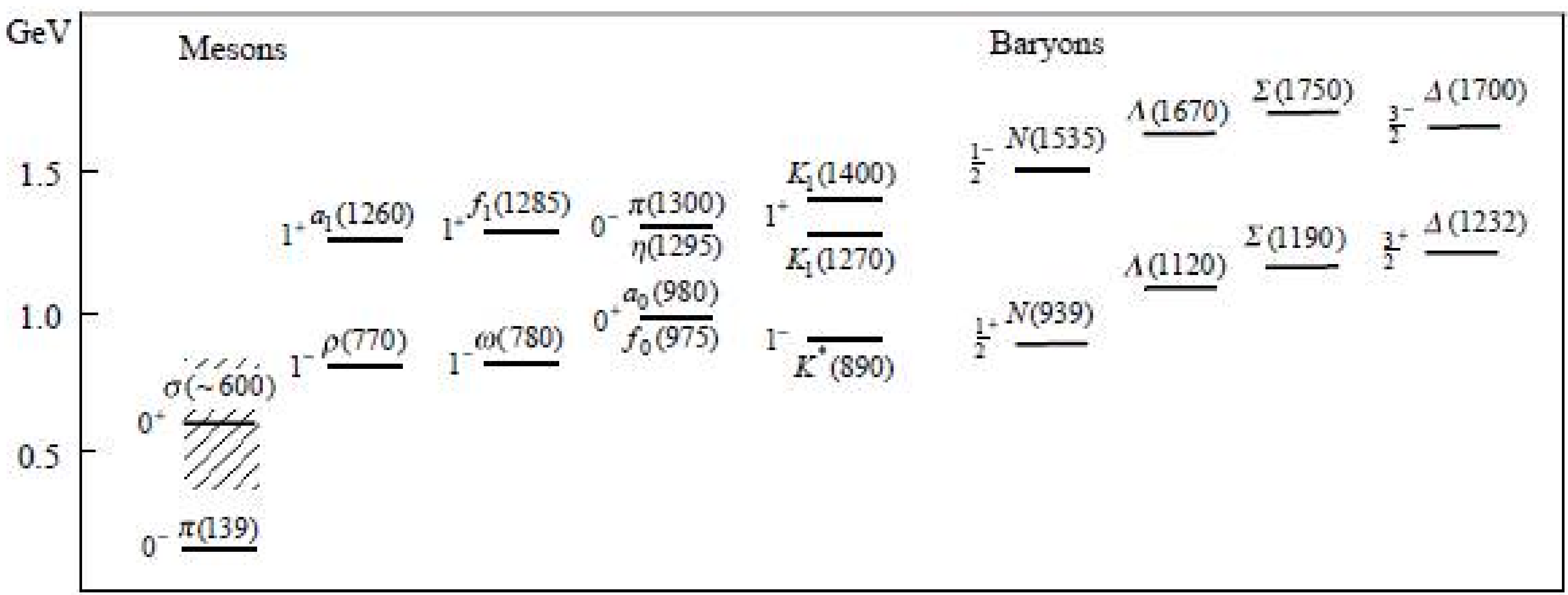
II. Fluctuations and correlations
   - In-medium Hadron Resonance Gas
   - S-matrix approach
   - Chiral-confinement interplay

III. Summary and conclusions

I. Hadrons in a hot/dense medium

Spectra in a chirally broken world

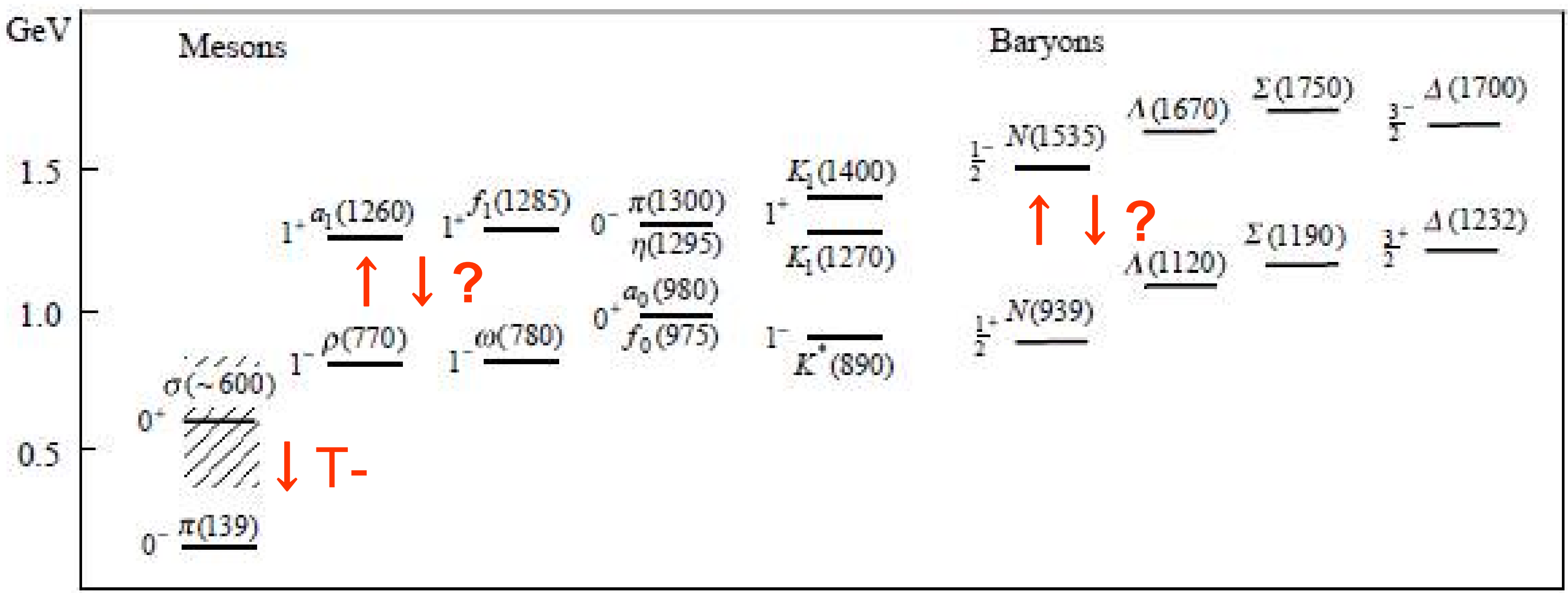
- Lowest pseudo-scalar mesons as NG bosons
- Mass splitting between positive and negative parity hadrons



Spectra in a chirally restored world

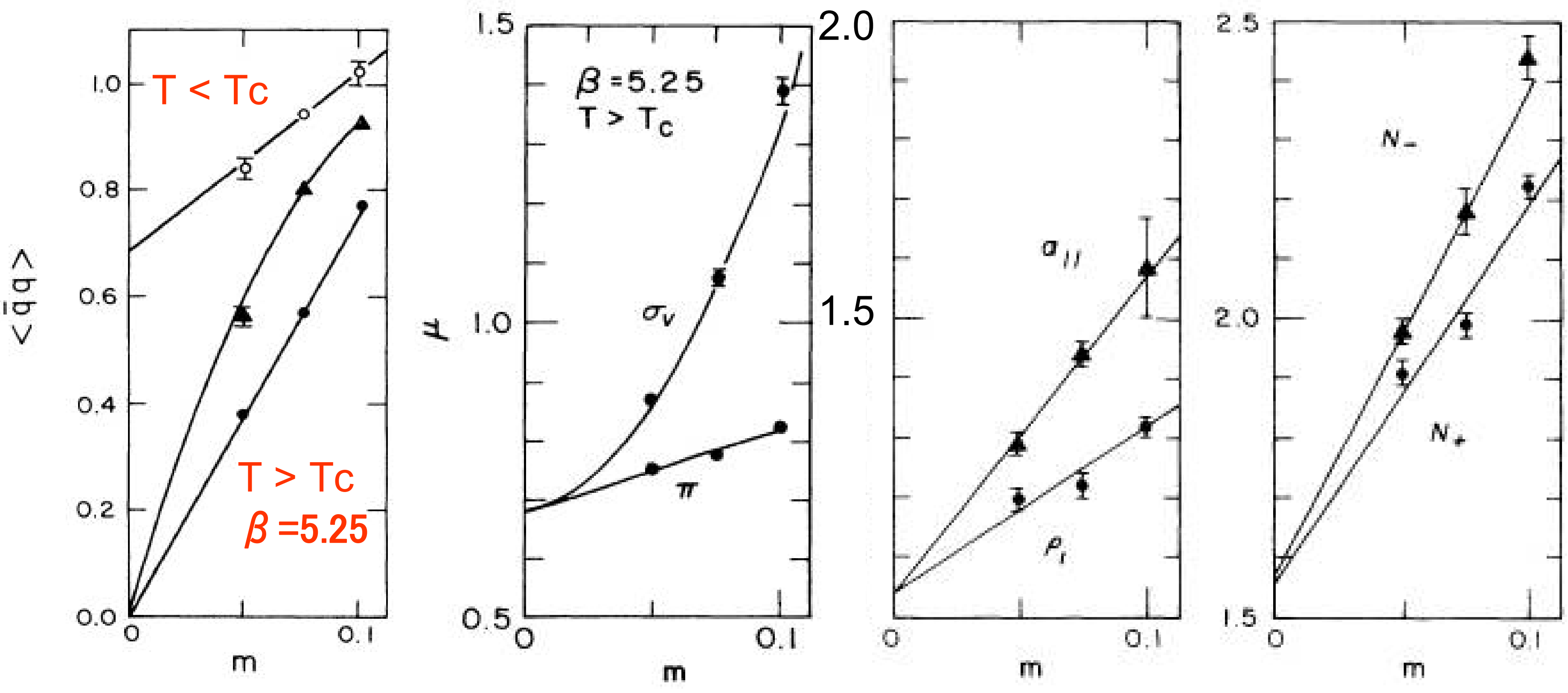
- ❑ Lowest scalar meson → O(4) vector with pion
- ❑ Parity partners degenerate → chiral partners

[mq ≈ 0: helicity eigenstates ≈ parity eigenstates]



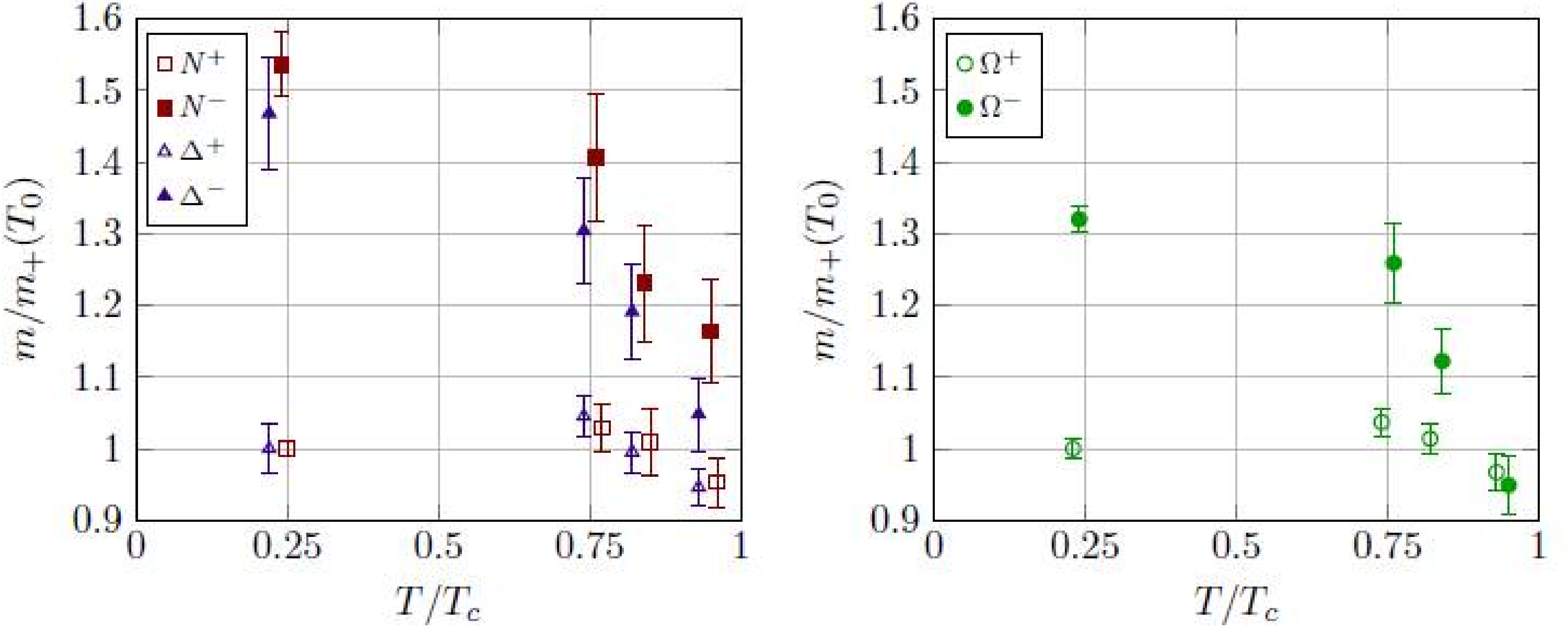
Lattice QCD tells us …

- Screening masses of mesons and nucleons
  [DeTar-Kogut, 1987]



Lattice QCD tells us …

- Temporal correlations in baryonic channels
  [Arts et al. 2015-17: mpi≈400 MeV, mk≈500 MeV, Tch=185 MeV]



Hadron masses vs. CSB

- Gell-Mann—Levy model (1960)

$$\mathcal{L}_{\mathrm{GL}} = i\bar{N}\partial\!\!\!/ N - g\bar{N}(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})N + \mathcal{L}_{\mathrm{meson}}$$

$$\psi_L \to L\,\psi_L\,, \quad \psi_R \to R\,\psi_R \qquad\qquad m_N = g\langle\sigma\rangle$$

- When CS restored → massless nucleon

- Mesons: massless (P), massive (S) by CSB
- Vector mesons (V,A) in LSM can stay massive.

$$R^\mu \to U_R R^\mu U_R^\dagger\,,\; L^\mu \to U_L L^\mu U_L^\dagger \qquad\qquad \frac{m_1^2}{2}\,\mathrm{Tr}\left[(L^\mu)^2 + (R^\mu)^2\right]$$

Non-SCB mass of nucleons

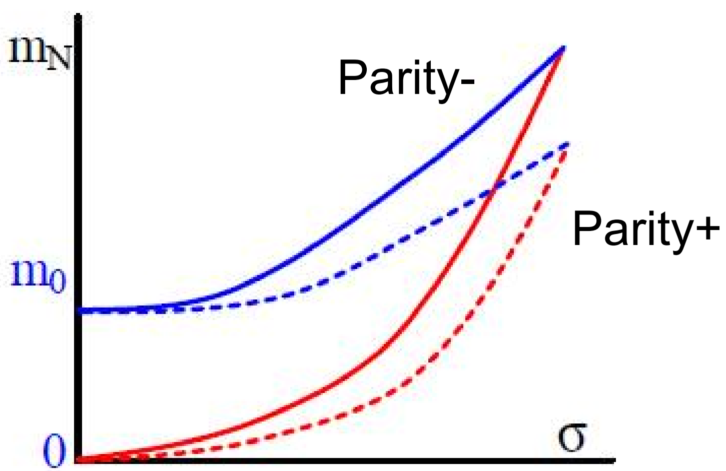
- SU(2) chiral transformation of 2 nucleons

  → how to assign 2 indep. rotation to them?

$$\psi_{1L} \to g_l\, \psi_{1L}\,, \quad \psi_{1R} \to g_r\, \psi_{1R} \;\sim\; \psi_{1L} : (1/2, 0) \quad \psi_{1R} : (0, 1/2)$$

$$\psi_{2L} \to g_r\, \psi_{2L}\,, \quad \psi_{2R} \to g_l\, \psi_{2R} \;\sim\; \psi_{2L} : (0, 1/2) \quad \psi_{2R} : (1/2, 0)$$

$$\mathcal{L}_m = m_0 \left(\bar{\psi}_2 \gamma_5 \psi_1 - \bar{\psi}_1 \gamma_5 \psi_2\right) \;\Rightarrow\; m_{N_\pm} = \frac{1}{2}\left[\sqrt{c_1 \sigma^2 + 4 m_0^2} \mp c_2 \sigma\right]$$



[DeTar-Kunihiro, 1989]

Origin of the survival masses?

- Emergence of a scale in QCD → trace anomaly

$$\partial_\mu J^\mu = T^\mu_\mu \propto \langle H|G^2|H\rangle$$

- in hot matter [Miller, 2007: lattice QCD EoS]

$$\langle G^2\rangle_{T_{\rm chiral}} \simeq \frac{1}{2}\langle G^2\rangle_{T=0}$$

- in nuclear matter [Cohen et al. , 1995: Feynman-Hellmann theorem & low-density approx.]

$$\left\langle \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^{a\mu\nu} \right\rangle_{\rho_N} - \left\langle \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^{a\mu\nu} \right\rangle_{\rm vac}$$

$$= -\frac{8}{9}(M_N - \sigma_N - S)\rho_N + \cdots$$ 5% smaller

How large is m0?

- At Tch: $\langle G^2 \rangle_T$ → m0 = 210 MeV [CS et al.]
- Vacuum: m0 = 270 – 460 MeV [DeTar-Kunihiro, Nemoto et al. , Gallas et al.]
- Nuclear matter
  - Ground state: binding energy, saturation point
  - Preferred m0 ≈ 500-800 MeV (w/ and w/o 4Q)

    [Zschiesche et al. 2007, Gallas et al. 2011]
- Swansea LQCD: near Tch, zero chem.pot.

  m0(octet,decuplet) ≤ m+(T=0)

Let’s take m0 ≈ 900 MeV!

Parity doubling of baryons

- ❑ Baryon octet and decuplet with finite m0
- ❑ Consistent with established phenomenology:
  - ✓ Gell-Mann-Okubo mass formula

$$\frac{3}{4}m_\Lambda + \frac{1}{4}m_\Sigma - \frac{1}{2}(m_N + m_\Xi) = 0$$

  - ✓ Gell-Mann's equal spacing rule

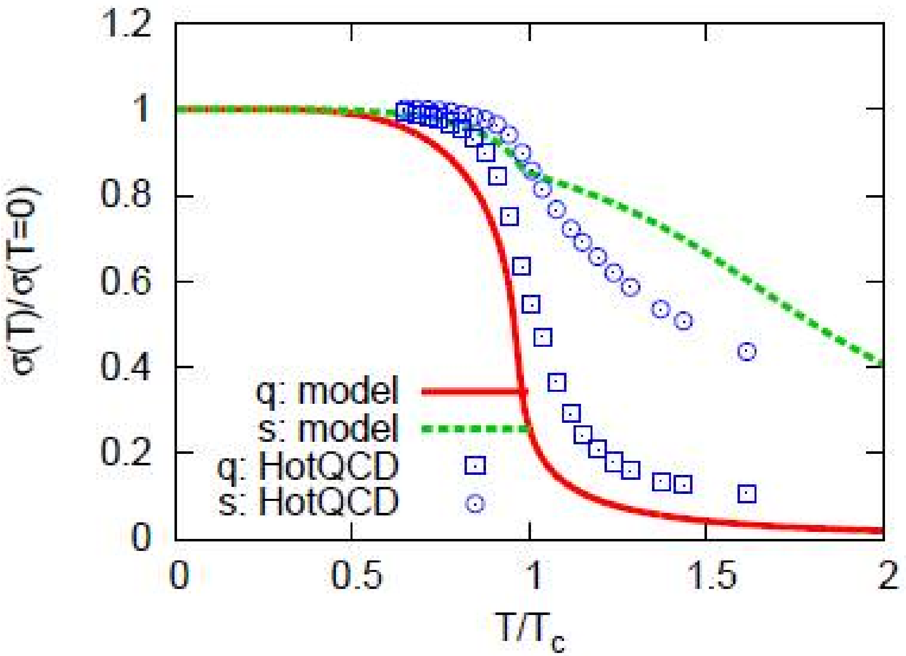
$$m_{\Sigma^*} - m_\Delta = m_{\Xi^*} - m_{\Sigma^*} = m_\Omega - m_{\Xi^*}$$

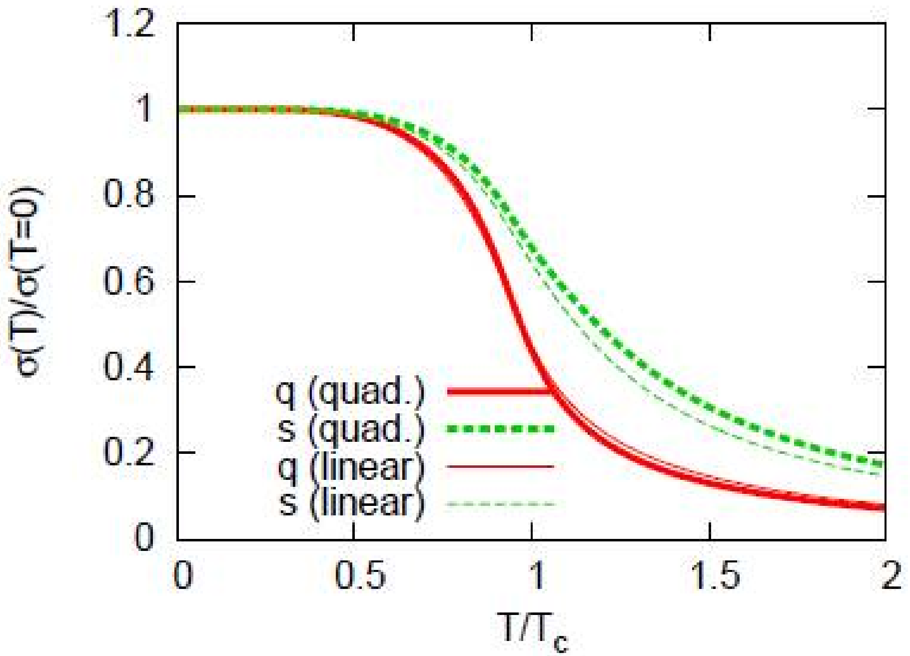
- ❑ Mass relations [CS, arXiv:1707.05081]

$$M_B(\sigma_q, \sigma_s; a, b, m_0)$$

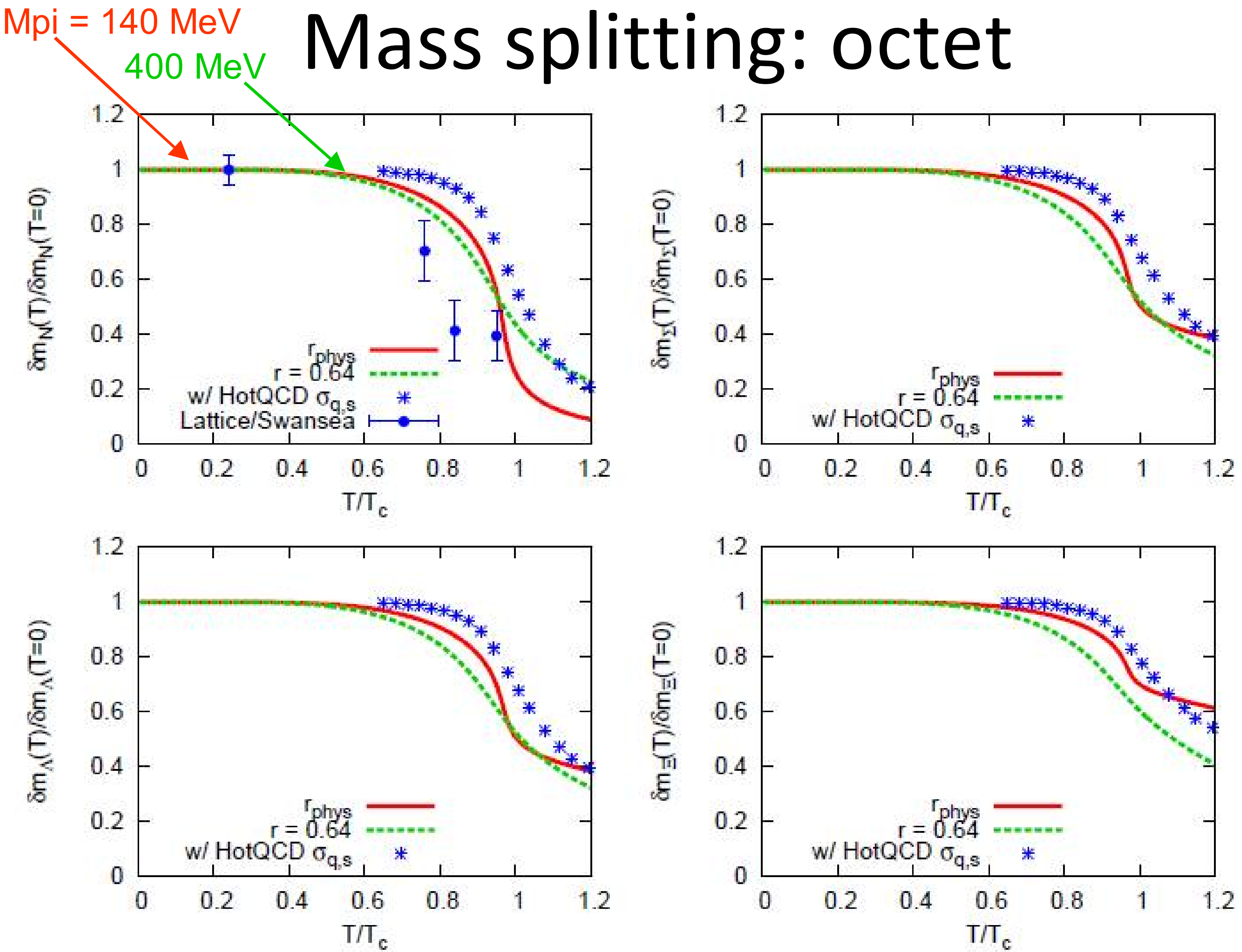
Chiral condensates

- Quark condensates from a model vs. LQCD
- Pion mass dependence: mpi = 140, 400 MeV

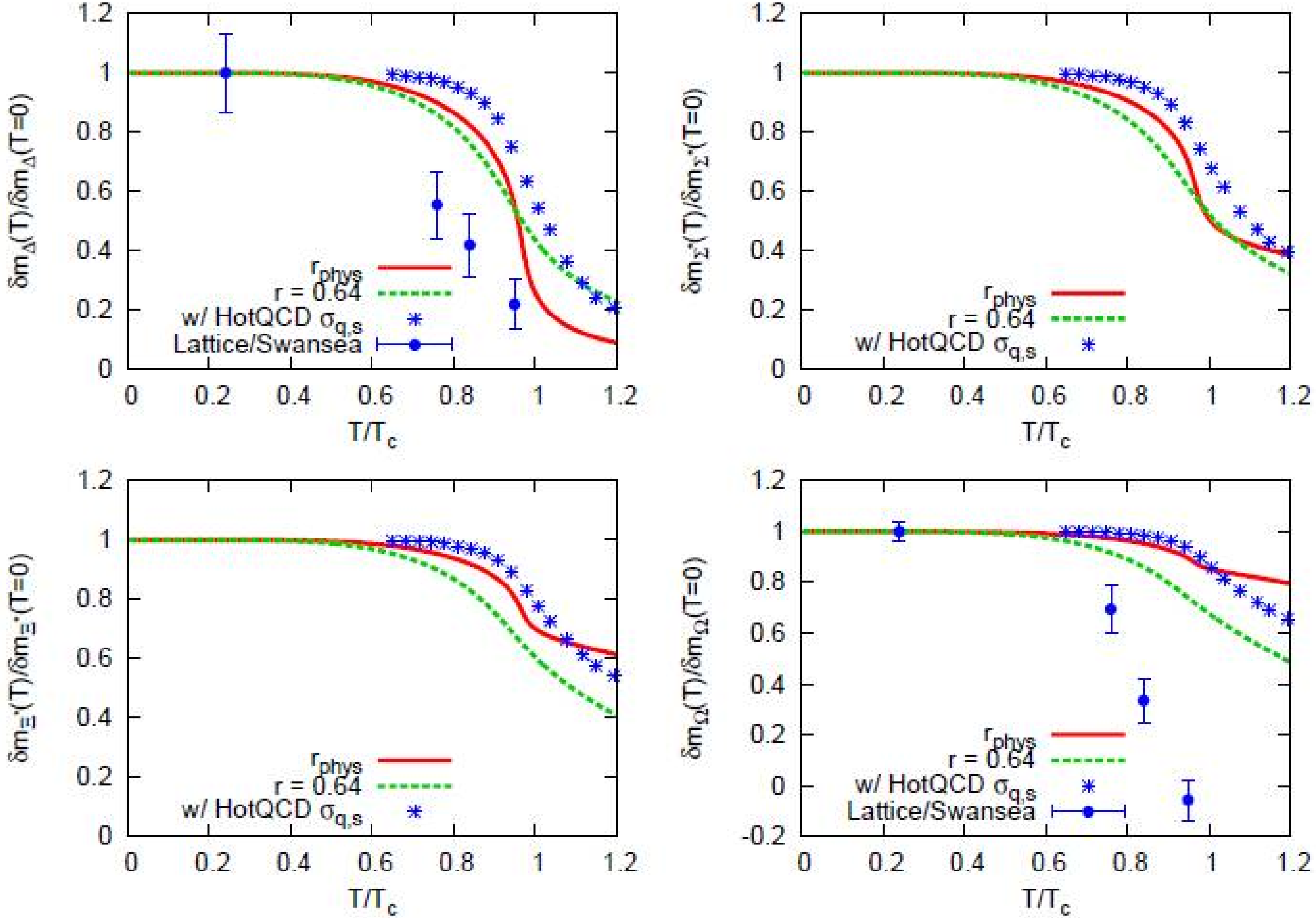




Mass splitting: octet



Mass splitting: decuplet



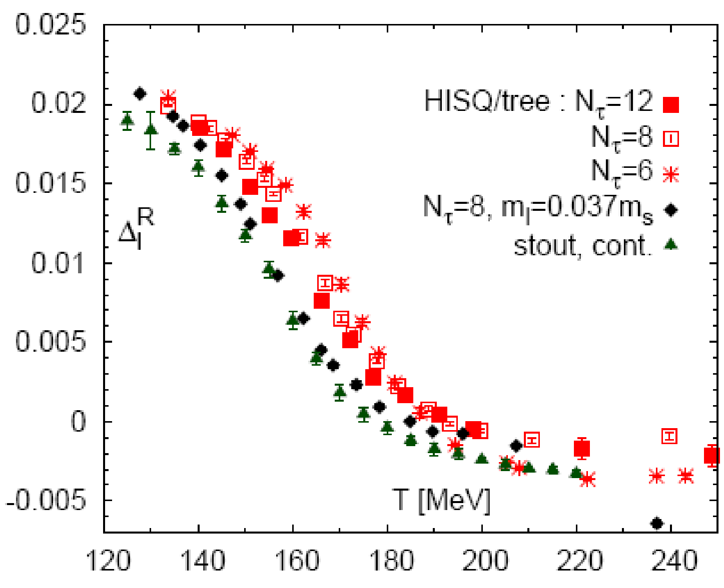
Remarks

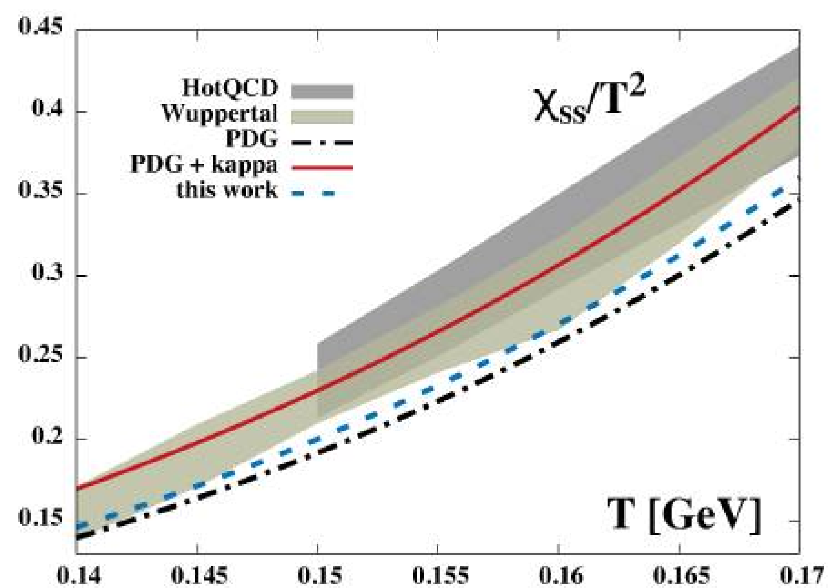
- Nucleons and deltas: δ m drops substantially.
- Much milder trend in hyperons: s-quark effect
- Heavier mpi → mk: δ m(u,d) ≈ δ m(s)
  - more explicit breaking but exact SU(3)
  - Swansea's setup: SU(3) rather than SU(2+1)
  - Still, Omega-baryon mass needs to be understood.
  - Missing piece(s)? --- the onset of deconfinement
- Need simulations with physical mpi!

II. Fluctuations and correlations

Signal of chiral symmetry restoration

- ❑Lattice QCD shows clearly <qqbar> dropping!
- ❑More deviation from HRG in higher-order fluctuations → Missing states? Interactions? and/or in-medium effects?





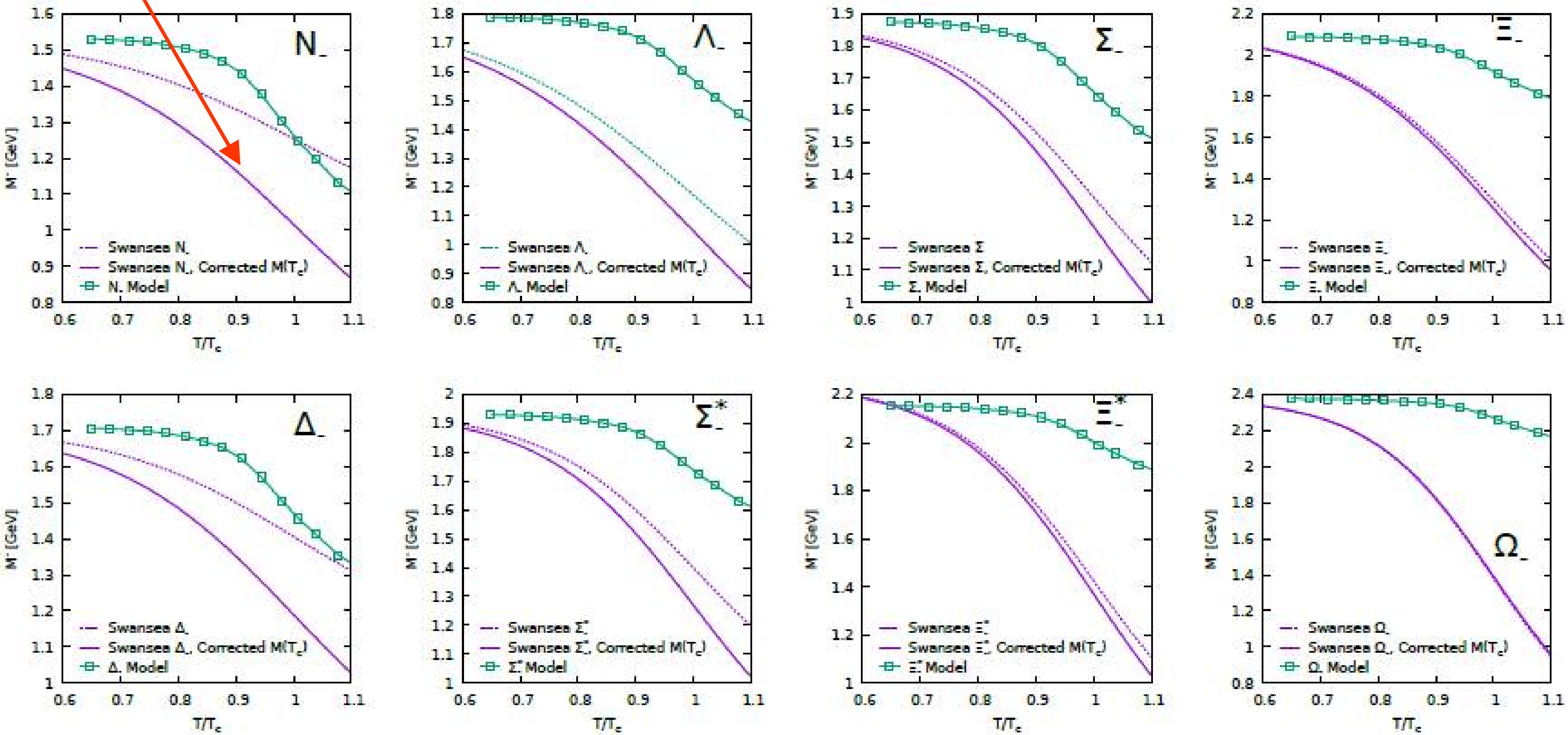
$$M_-^{\mathrm{lat}}(T_c) \times \frac{M_+^{\mathrm{PDG}}}{M_+^{\mathrm{lat}}}$$

In-medium HRG

- T-dep. motivated by Lattice findings [Aarts et al.]

$$M^-(T) = M^-(T=0)\omega(T,b) + M^-(T_c)(1-\omega(T,b))$$

$$\omega(T,b) = \tanh[(1-T/T_c)/b]/\tanh(1/b)$$

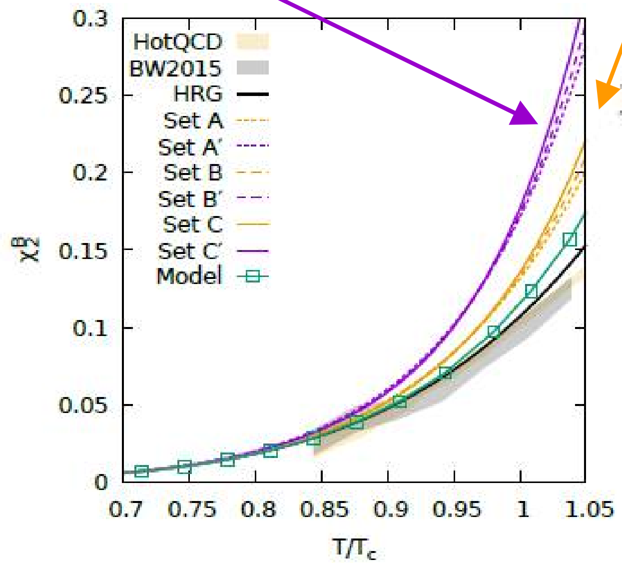


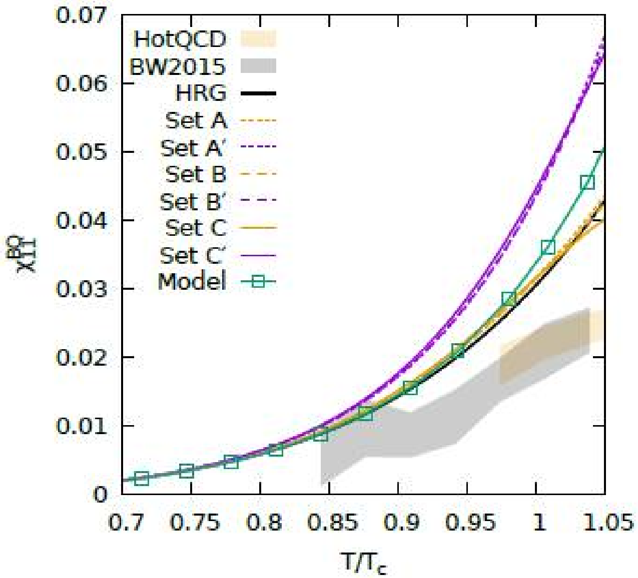
[Morita, Redlich and Sasaki, to appear]

corrected $M_{-}^{\mathrm{lat}}(T_c)$

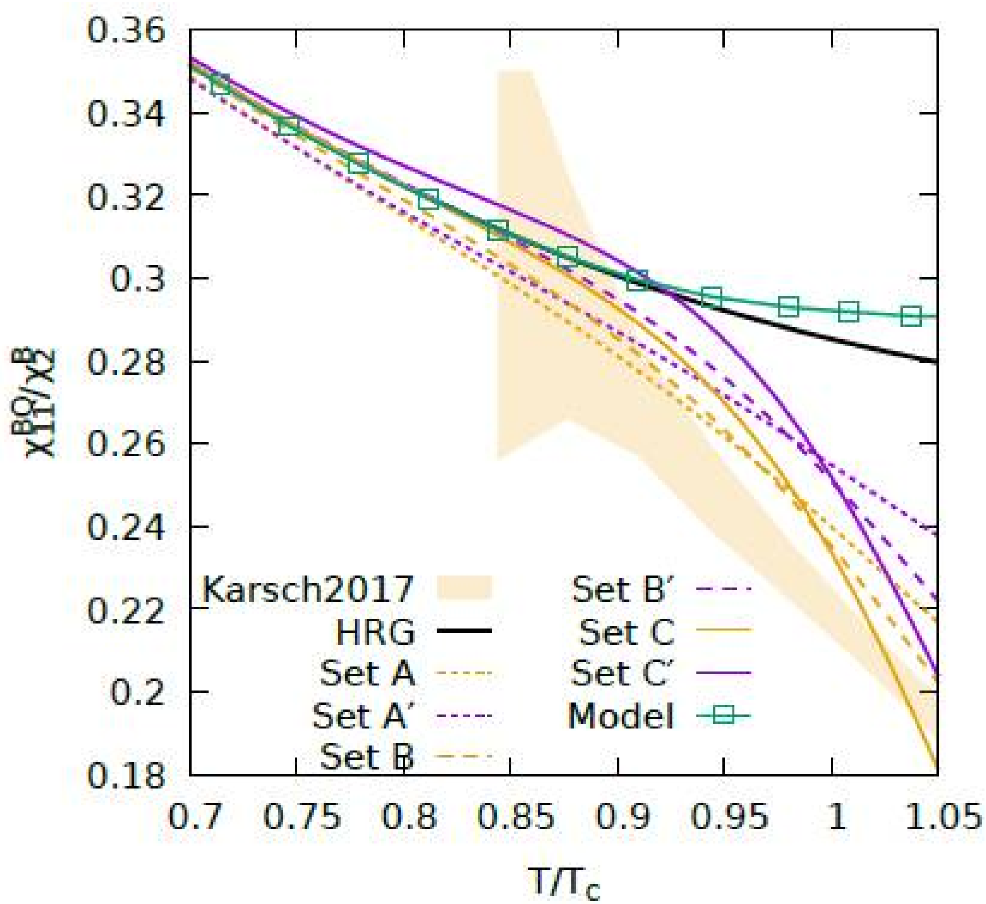
uncorrected

Fluctuations of net-baryon number





$$\chi_{ijk}^{BQS} \equiv \frac{\partial^{i+j+k}}{\partial^i(\mu_B/T)\partial^j(\mu_Q/T)\partial^k(\mu_S/T)} p(T,\mu_B,\mu_Q,\mu_S)/T^4$$



[Morita, Redlich and Sasaki, to appear]

What is missing? --- finite width

❑Thermodynamics of broad resonances

→S matrix approach [Dashen, Ma and Bernstein, 1969]

- Grand canonical potential

$$\Omega = \Omega_0 + \Omega_{\rm int}$$

$$\Delta \ln Z = \int dE\, e^{-\beta E} \frac{1}{4\pi i} {\rm tr} \left[S^{-1} \frac{\overleftrightarrow{\partial}}{\partial E} S \right]_c$$

- Leading contribution: 2-body [Beth-Uhlenbech, 1937]

$$\Delta \ln Z = \int dE\, e^{-\beta E} \times \frac{1}{\pi} \frac{\partial}{\partial E} {\rm tr} \left(\delta_E \right)$$

Phase shift

Dynamical information

What is missing? --- finite width

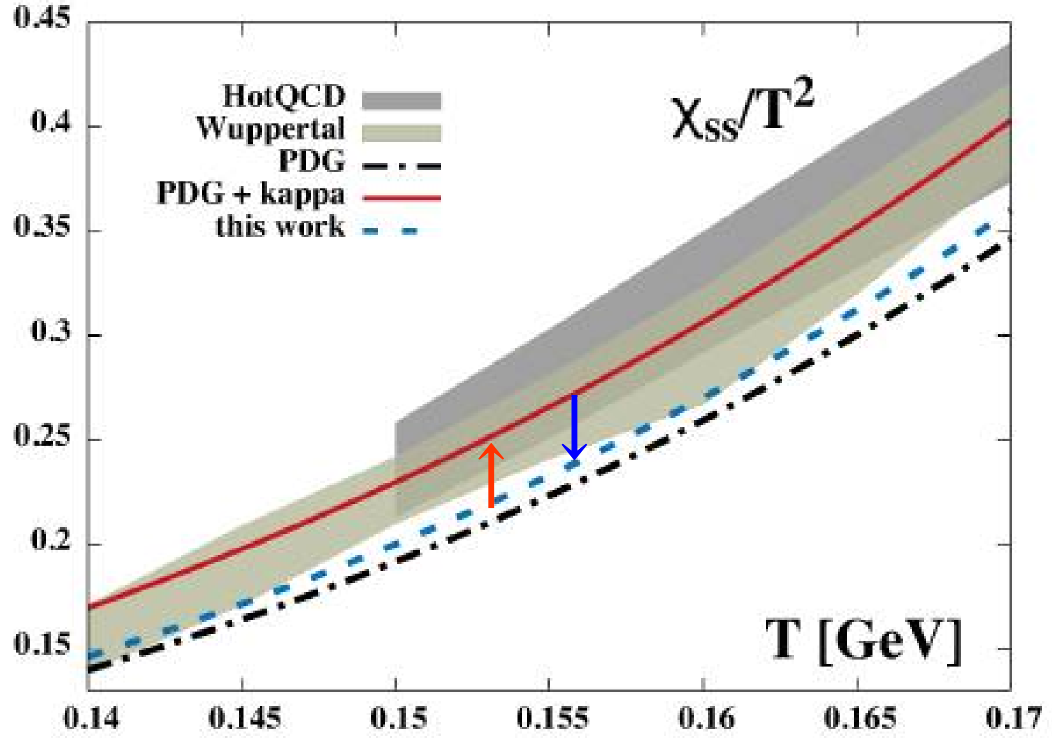
- ❑K0*/ κ (800) meson: chiral partner of kaon

  NOTE: omitted from PDG summary table

- ❑S matrix approach [Friman et al. 2015]

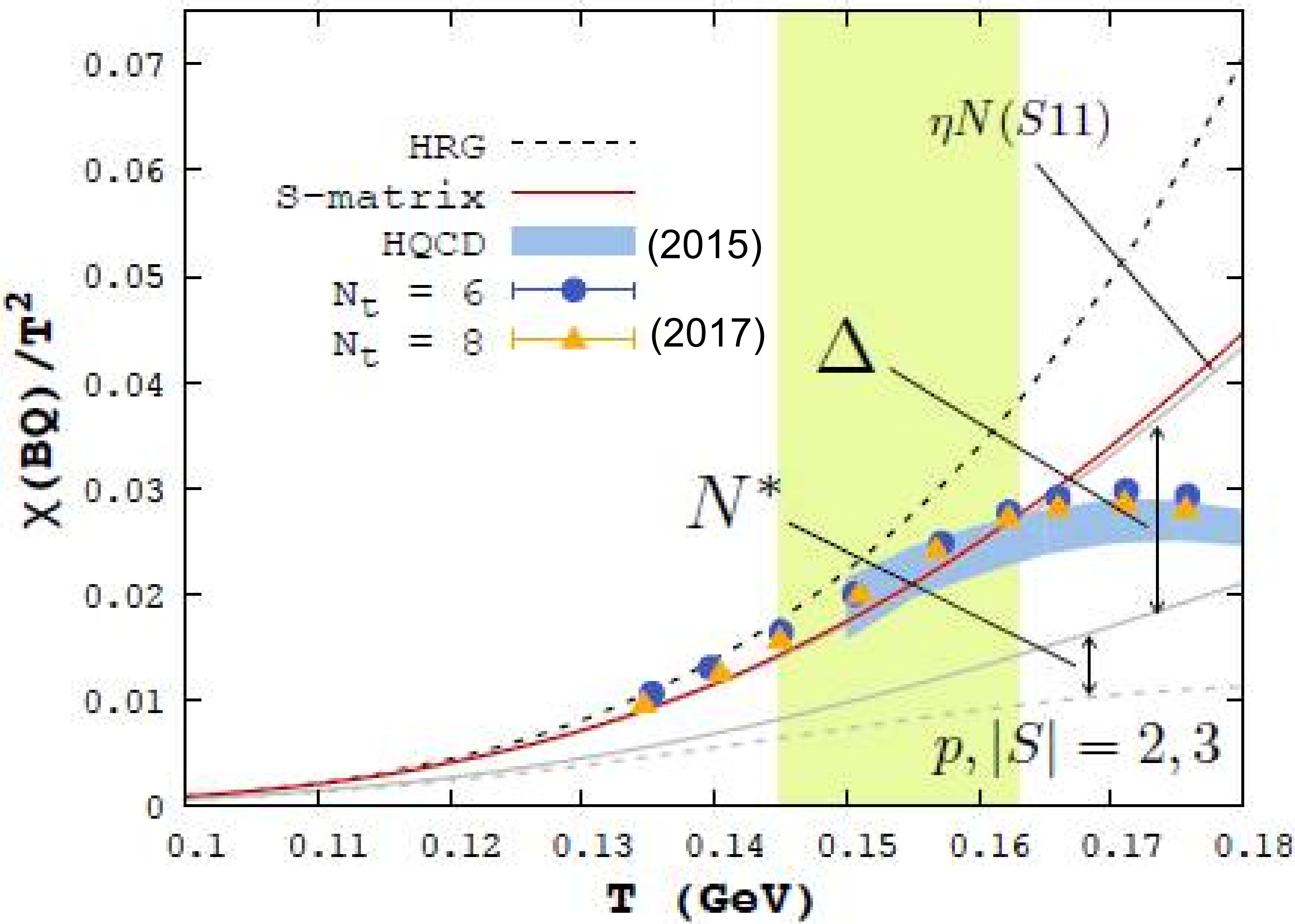
  ✓Empirical π -K phase shift from experiment

$$\Omega = \Omega_{\pi} + \Omega_{K} + \Omega_{\rm int}$$



[Lo et al., to appear]

Pi-Nucleon system



Chiral-confinement interplay

A missing piece: (de)confinement

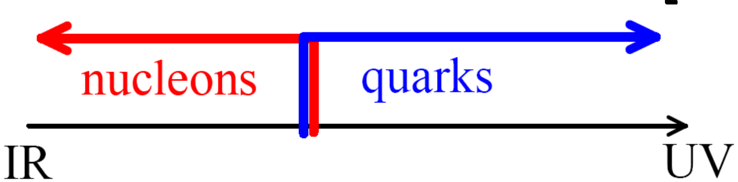
- Modeling conf. of QCD thermodynamics
  - Confined phase = Z(Nc) unbroken phase in YM

$$\hat{L}(\vec{x}) = \mathcal{P}\exp\left[i\int_0^{1/T} d\tau A_4(\vec{x},\tau)\right] \qquad \langle\Phi(\vec{x})\rangle \sim e^{-F_q(\vec{x})/T} \begin{cases} = 0 \\ \neq 0 \end{cases}$$

  - Full QCD: PNJL/PQM ≈ GL theory of σ and Φ
  - At finite density? --- Z(Nc) strongly violated!
- How to suppress quarks at low density?

→IR/UV cutoff in fermion dist. functions

T=mu=0 [Ebert et al. 1996; NJL, SD, AdS/QCD]



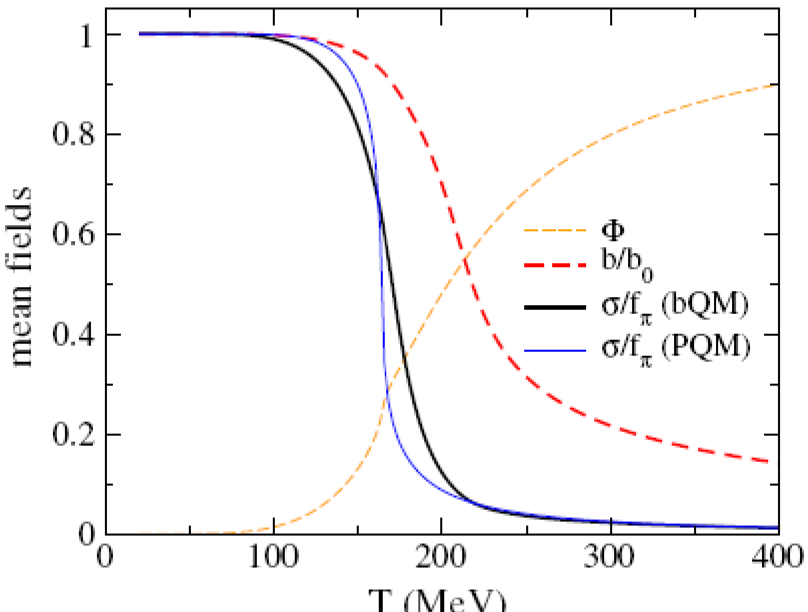
$$n_q = \theta(|\vec{p}| - b) f_q\,, \quad n_{N_\pm} = \theta(b - |\vec{p}|) f_{N_\pm}$$

Polyakov loop vs. IR cutoff

- ❑ Hadron size ≈ 1/b → depends on T and mu!
  - ■ Otherwise SB limit not achieved. [Benic et al. 2015]

$$\Omega = \sum_{X=N_\pm,q} \Omega_X + V_\sigma + V_\omega + V_b\,, \quad V_b = -\frac{\kappa_b^2}{2}b^2 + \frac{\lambda_b}{4}b^4$$

  - ■ Const. b → condensation of a scalar field b: <b>
- ❑ T≠0, μ =0 thermodynamics vs. PQM



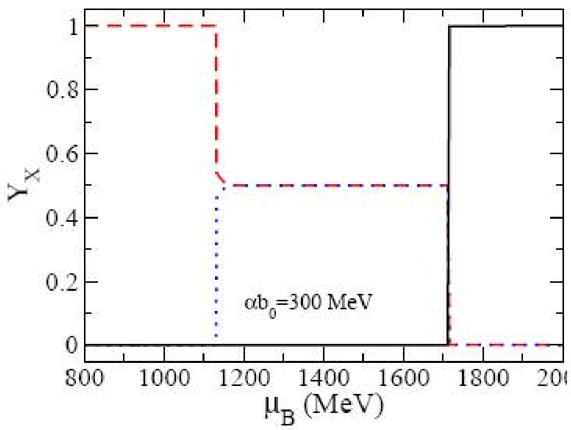
$$T_c^{\mathrm{bQM}} \simeq \left(\frac{12\lambda_\chi\chi_0^2}{\gamma_q g^2} + \frac{3}{\pi^2}b^2\right)^{1/2}$$

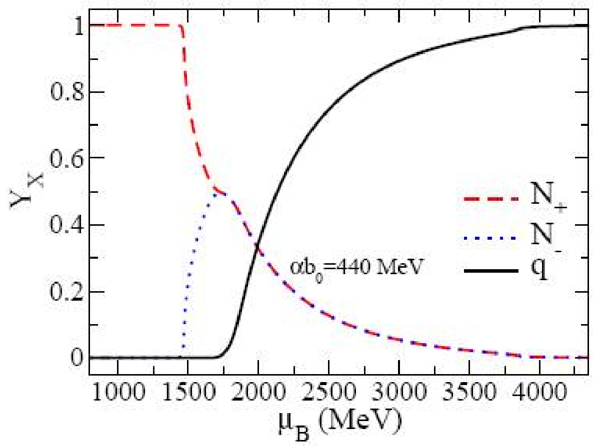
$$T_c^{\mathrm{PQM}} = \left(\frac{12\lambda_\chi\chi_0^2}{\gamma_q g^2} + \frac{2}{\pi^2}\phi^2\right)^{1/2}$$

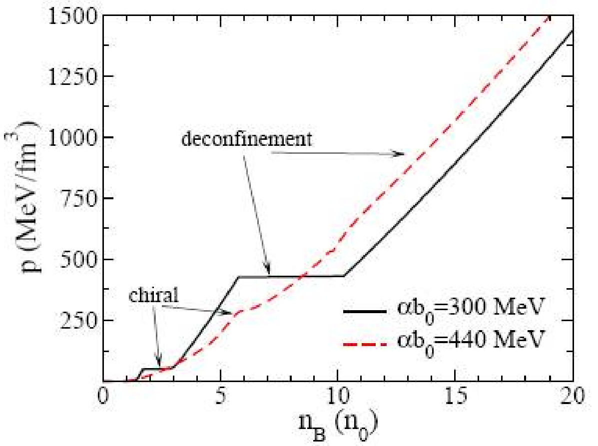
$$\langle\Phi\rangle \approx 1 - \langle b\rangle / \langle b_0\rangle$$

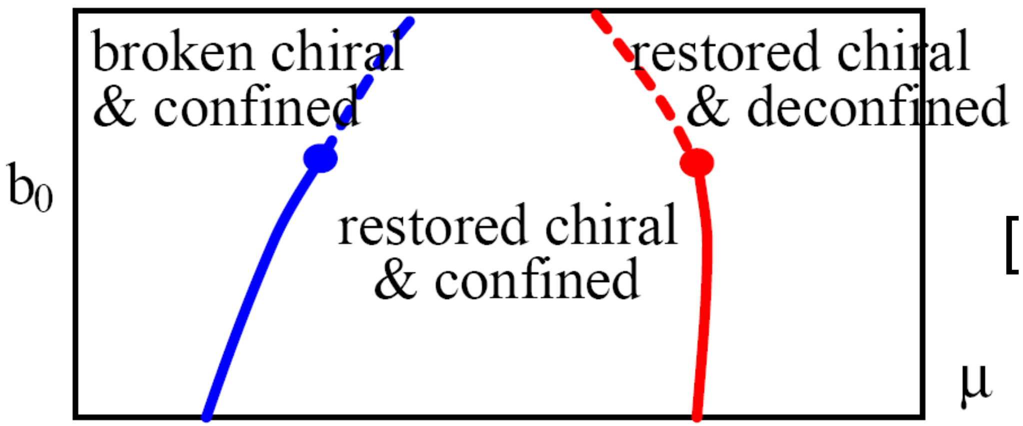
Onset of different fermions

- Fractions of particle num. density







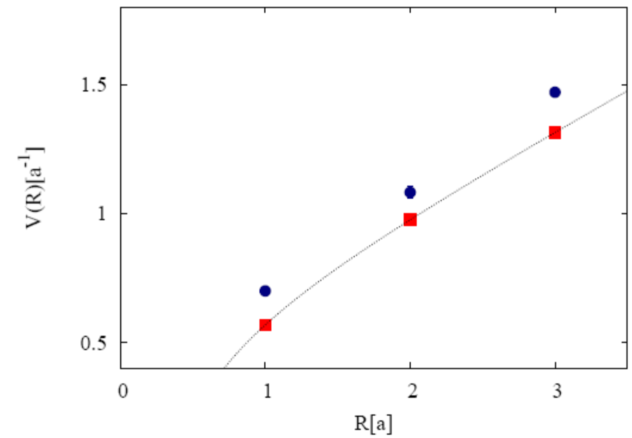


[Marczenko-Sasaki, to appear]

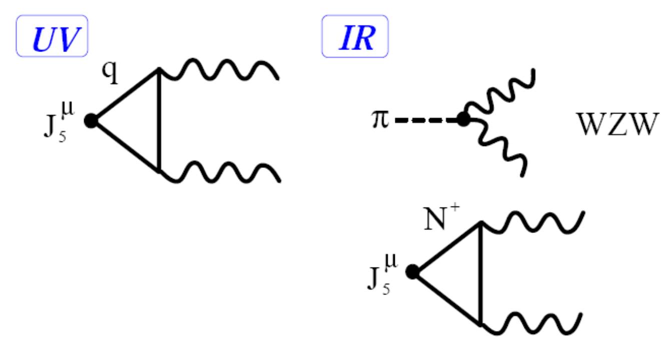
Chiral-confinement interplay

- Dirac zero mode = non-vanishing <qqbar>
  - → when CS is unbroken, then no conf.?
    - String tension unchanged!
    - Chiral restoration ≠ deconf.

    [Gattringer; Bruckmann et al.; Gongyo et al.]



- Anomaly matching: e.g. $\pi^0 \to \gamma\gamma$
  - No WZW w/ unbroken chiral
  - Anomaly can match only if deconf.
  - New collective modes?



- The best we can do: systematic case study

III. Summary and conclusions

- ❑ Emergent parity-doubling structure as a manifestation of restored chiral symmetry
- ❑ Lattice QCD, effective theories
- ❑ In dense QCD: rich phase structure
- ❑ Interplay between CSB and confinement

- ◆ Naive “in-medium HRG” does not work.
- ◆ Effect of resonance widths – beyond HRG
- ◆ Toward more realistic EoS