Excited states from Lattice QCD

“something about excited states/exotica, which uses techniques which could potentially be useful for finite T as well”

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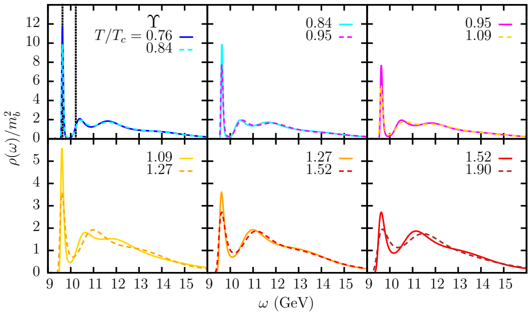
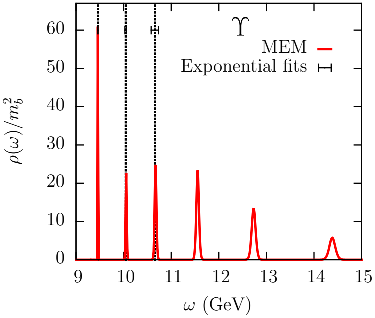
THOR Cost Meeting, Swansea, September 2017

Spectroscopy at zero and finite temperature

MEM is the method of choice at $T > 0$ - where focus is on e.g. patterns of melting/suppression in ground states of S and P waves.





Focus is rather different at $T = 0$ and there has been a lot of rapid progress in the last few years: experimental and in lattice techniques.

Lattice spectroscopy: two strategies for progress

Below thresholds: "gold-plated" quantities characterised by

- Using well-tried and tested and validated methods
- High statistics and improved actions for precise results
- Different actions in agreement
- Most/All systematic errors accounted for e.g. continuum extrapolation, finite volume, physical light quark masses, Nf, EM effects

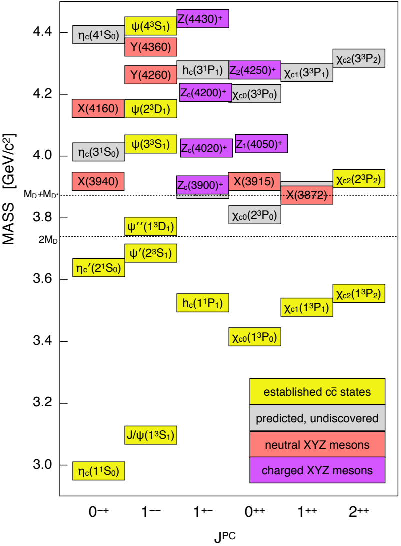
Above thresholds: calculation of high spin & exotic states relatively new

- Physics of molecular/multi hadron states needs relevant operators.
- No continuum extrapolation
- Relatively heavy pions ← already changing
- Improvements underway now that methods are proven

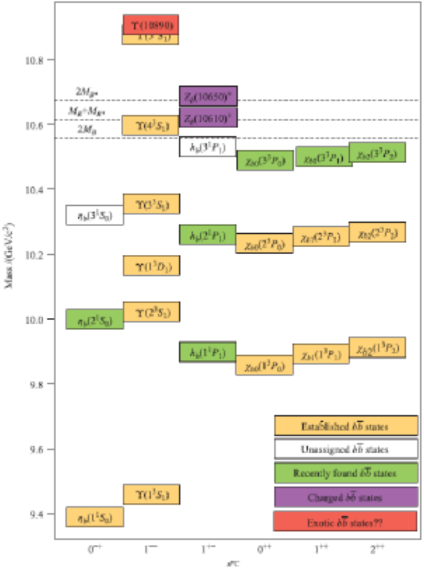
new ideas crucial

- Distillation - quark propagation enabling isoscalars, precision spectroscopy ...
- Framework for scattering & coupled channels

A charming and beautiful renaissance

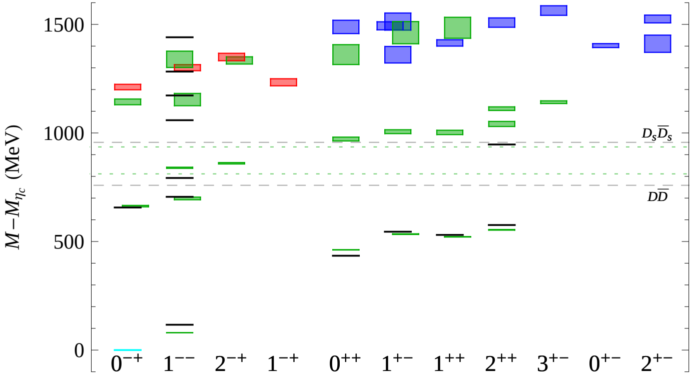


from Ryan Mitchell via Steve Olsen



from X-H Liu, ECT 2017*

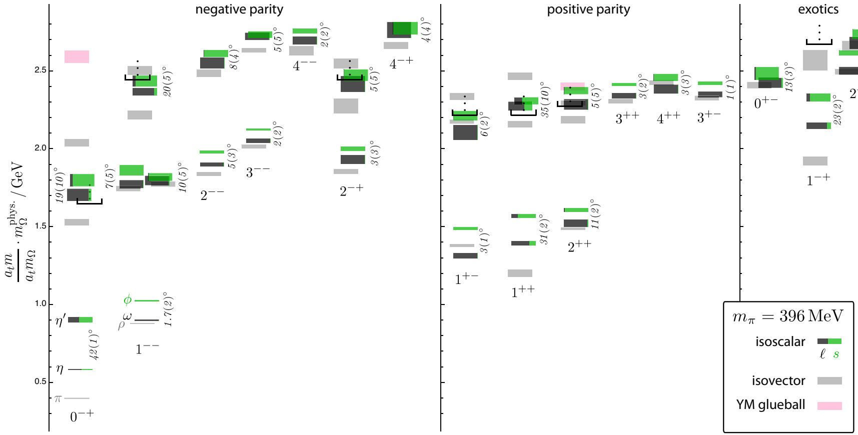
PRECISION SPECTROSCOPY OF HYBRIDS: CHARMONIUM



- Not all fit quark model: spin-exotic (and non-exotic) hybrids determined.
- Same pattern and energy scale in mesons and baryons - heavy and light.
- Very little bottomonium spectroscopy yet ...

[HadSpec: Liu et.al. arXiv:1204.5425]

Precision light spectroscopy



from HadSpec, Dudek et al

Lattice hadron spectroscopy, T=0

In principle, provides a model-independent ab initio description of the QCD spectrum.
Lots of progress and new ideas to tackle spectroscopy

- Algorithms for quark propagation: all-to-all propagators and distillation
- Extracting energies of excited and exotic states
  - Bayesian analysis, χ^2-histogram analysis, variational analysis, ...
- Using symmetries and operator overlaps to identify states at $J \geq 2$
- Extension of Lüscher's finite volume idea for scattering including coupled channels and resonant transitions.
- Despite huge progress the nature of the XYZ states still unclear.

Questions

Does it make sense to study hybrids and/or exotica at finite temperature?
Can we use the techniques from zero temperature to improve spectroscopy?

- smearing plays a key role.

Making Measurements

A recipe for (meson) spectroscopy

Let's look at the basic ideas for extracting energies:

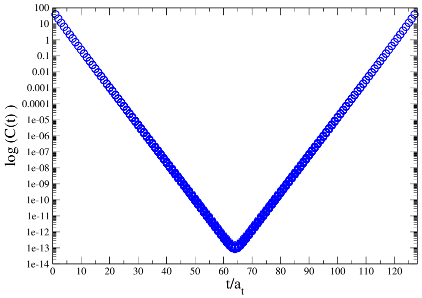
- Ground state energies found from $t \to \infty$ limit of a Euclidean-time correlation function

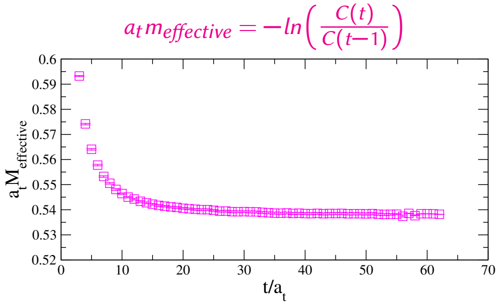
$$\begin{aligned} C(t) &= \langle 0|\phi(t)\phi^\dagger(0)|0\rangle \\ &= \sum k, k' \langle 0|\phi|k\rangle\langle k|e^{-\hat{H}t}|k'\rangle\langle k'|\phi^\dagger|0\rangle \\ &= \sum_k |\langle 0|\phi|k\rangle|^2 e^{-E_k t} \end{aligned}$$

- So, $\lim_{t\to\infty} C(t) = Ze^{-E_0 t}$ and ground states are reliably determined from exponential fits.

From correlators to energies

- In general works well for extracting ground states
- Higher excitation energies hard to extract by just fitting to exponentials.





- The correlator and effective mass of the J/Ψ meson.
- For $\mathcal{O}_i = \mathcal{O}_j$ the correlation function is positive definite and $a_t m_{effective}$ converges monotonically from above.
- Bayesian fitting methods improve multi-exponential fits but not enough to determine/disentangle high-spin states.

Excited states

To go beyond ground state spectroscopy the method of choice is the variational idea: find operator ϕ to maximise $C(t)/C(t_0)$ from sum of basis operators $\phi = \sum_a v_a \phi_a$

[C. Michael & I. Teasdale. NPB215 (1983) 433]; [M. Lüscher & U. Wolff. NPB339 (1990) 222]

Variational method

If we can measure $C_{ab}(t) = \langle 0|\phi_a(t)\phi_b^\dagger(0)|0\rangle$ for all a, b and solve the generalised eigenvalue problem

$$C_{ij}(t)v_j^{(n)} = \lambda^{(n)}(t)C_{ij}(t_0)v_j^{(n)},$$

then

- eigenvalues: $\lambda^{(n)}(t) \sim e^{-E_n t}\left[1 + O(e^{-\Delta E t})\right],\ t \geq t_0/2$ - gives principal correlator.
- eigenvectors: related to overlaps $Z_i^{(n)} = \sqrt{2E_n} e^{E_n t_0/2} v_j^{(n)\dagger} C_{ji}(t_0)$.

For this to be practical, we need

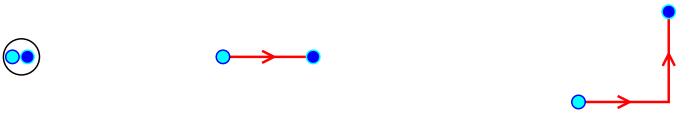
- a “good” basis set that resembles the states of interest
- all elements of this correlation matrix measured

[Blossier et al JHEP 0904 (2009) 094]

Variational method: a method not a magic wand!

Good information in gets good information out!

Lattice operators are bilinears with path-ordered products between the quark and anti-quark field; different offsets, connecting paths and spin contractions give different projections into lattice irreps.



$$\mathcal{O}_{\alpha\beta} = \sum_x \bar{\psi}_\alpha(x)\psi_\beta(x)$$

$$\mathcal{O}^{i}_{\alpha\beta} = \sum_x \bar{\psi}_\alpha(x)U_i(x)\psi_\beta(x+\hat{\imath})$$

$$\mathcal{O}^{ij}_{\alpha\beta} = \sum_x \bar{\psi}_\alpha(x)U_i(x)U_j(x+\hat{\imath})\psi_\beta(x+\hat{\imath}+\hat{\jmath})$$

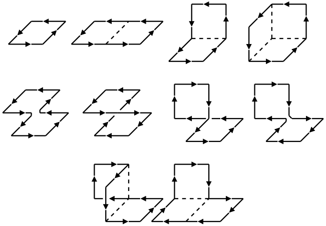
A good operator:

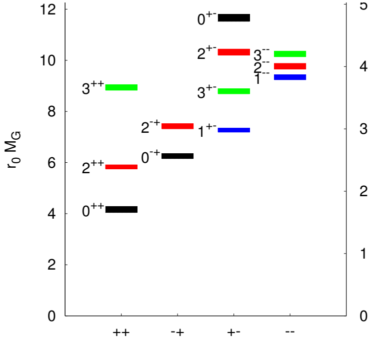
- An operator of definite momentum that transforms under a lattice irrep
- An operator that has strong overlap with the (continuum) state of interest.
- An operator is not noisy ie that produces an acceptable correlator

Glueballs - an early success

- QCD nonAbelian ⇒ allows bound states of glue
- Candidates observed experimentally: $f_0(1370), f_0(1500), f_0(2220)$
- Glueballs can be calculated in lattice QCD
- The interpolating fields are purely gluonic, built from Wilson loops

[Morningstar and Peardon]





Designing good lattice operators - a tale of two symmetries

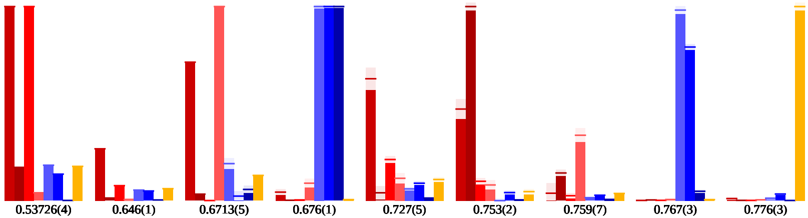
- Lorentz symmetry broken at $a \neq 0$ so $SO(4)$ rotation group broken to discrete rotation group of a hypercube.
- Classify states by irreps of O_h and relate by subduction to J values of $O(3)$.
- 5 irreps of $O(3)$ and an infinite number for J^P so values are distributed across lattice irreps.

- start with continuum operators, built from n derivatives: $\phi = \bar{\psi}\Gamma(D_{i1}D_{i2}D_{i3}\ldots D_{in})\psi$
- Construct irreps of SO(3), then subduce these representations to O_h
- Replace derivative with lattice differences:
- A subduced irrep carries a "memory" of continuum spin J from which it was subduced - it **overlaps** predominantly with states of this J.

J	**0**	**1**	**2**	**3**	**4**
A_1	1	0	0	0	1
A_2	0	0	0	1	0
E	0	0	1	0	1
T_1	0	1	0	1	1
T_2	0	0	1	1	1

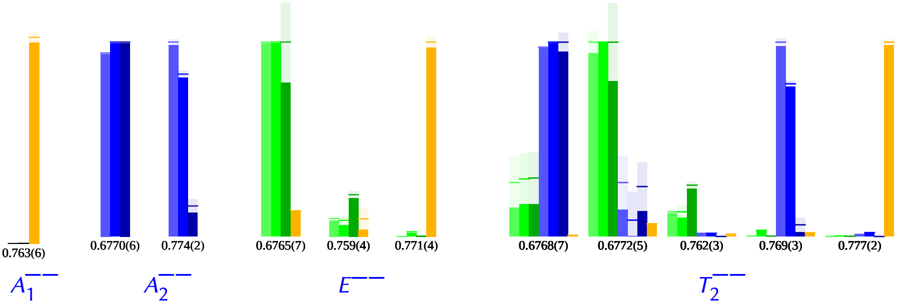
Using GEVP eigenvalues - operator overlaps

- Using $Z = \langle 0|\Phi|k\rangle$, helps to identify continuum spins
- For high spins, can look for agreement between irreps
- Data below for T_1^{--} irrep, colour-coding is **Spin 1**, **Spin 2**, **Spin 3** and **Spin 4**.



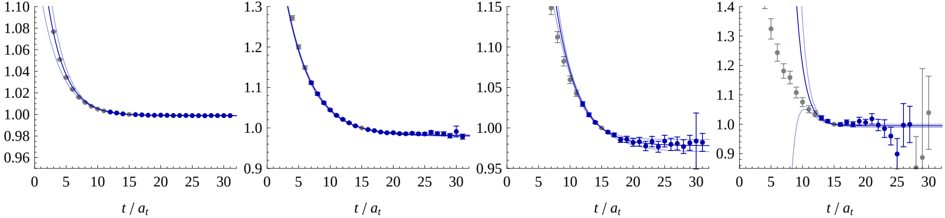
and the rest of the spin 4 state ...

- All polarisations of the spin-4 state are seen
- Spin labelling:



Fitting principal correlators

- Typical fits for a set of excited states in the T_1^{--} irrep in charmonium (26 operators!) are



- plotting $\lambda_n(t) \cdot e^{m_n(t_1 - t_0)}$ with $t_0 = 15$.
- Expect a plateau at 1.0 if single-exp dominates.
- Anisotropy helps the variational method: improving resolution in the temporal direction.
- Smearing crucial.

Variational ideas at finite T

- WHOT-QCD collaboration have explored variational ideas to extract excited states at finite T.
  - 0810.1567, Umeda et al: Ground and first excited states of $J/\Psi, \chi_c$ from a basis of smeared source and sink operators.
    Studied the shape and vol-dependence of BS wavefunctions to distinguish bound and scattering states.
  - 1104.33842, Ejiri et al: Adapted variational method to calculate locations and heights of spectral functions - and compared to MEM.
    Very dependent on "good" operators to resolve a signal in the range of available t at hight temperatures. Using point-smeared operators.
- At T=0, much of the progress has come from looking again at the hadronic building blocks - the propagators.

Making Propagators

Looking back at propagation

Exploiting techniques for excited states requires **efficient** and **precise** determination of propagators.

What's on the market?

- Point propagators - used extensively at finite T, rarely (at least in isolation) at T=0.
- Smeared propagators - via Gaussian, Jacobi etc smearing.
- All to all propagators + smearing.
- Distillation - a "redefinition" of smearing.

How useful can smearing be at finite temperature? Can zero temperature methods be useful?

Point propagators

For better simulations of hadronic quantities look again at the building blocks: the quark propagators

Point (to-all) propagator pros

- doesn't require vast computing resources

Point (to-all) propagator cons

- restricts the accessible physics
  - flavour singlets and condenstates impossible: quark loops need props w sources everywhere in space
- restricts the interpolating basis used
  - a new inversion needed for every operator that is not restricted to a single lattice point
- entangles propagator calculation and operator construction
- not using all information encoded in configurations

Solutions?

- Improve the determination point props to access the physics of interest: smearing. Lots of examples around.
- Compute all elements of the quark propagator: all-to-all propagators. Problem It's expensive - needs and unrealistic number of inversions.
- Work around: Use stochastic estimators (with variance reduction).
- Rethink the problem: combine smearing and propagation ie distillation

I'm just picking a couple of ideas to focus on.

All-to-all propagators

About all-to-all propagators

- Computing all elements of the quark propagator would require full knowledge of the inverse - this is prohibitively expensive
- The lattice representation of the Dirac operator is a large, but very sparse matrix.
- If we are satisfied with an *unbiased estimator* of all elements then sparse matrix methods can be used. Stochastic estimation should be acceptable - we are already using it to generate gauge fields!
- Variance reduction will be crucial

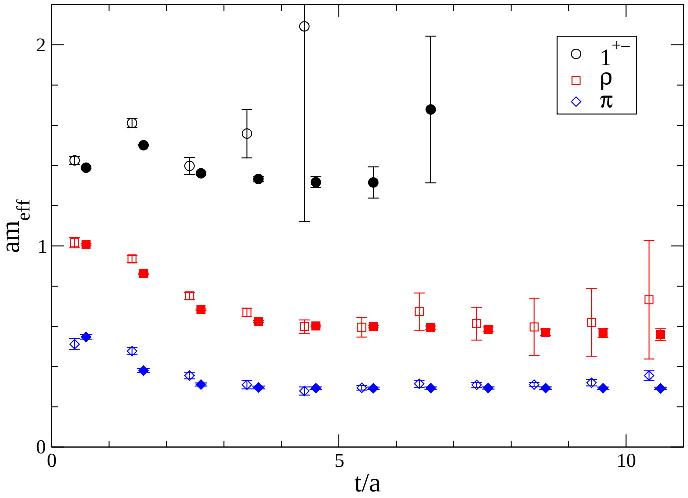
All-to-all quark propagators

- Start with a spectral representation of $Q = \gamma_5 M$ (hermitian so eigenvalues are easier to compute).
- If we can compute all the eigenvectors and eigenvalues, $\{\lambda^{(i)}, v^{(i)}\}$ of

$$Q = \sum_{i=1}^{N} \lambda^{(i)} v^{(i)} \otimes v^{*(i)} \quad \text{then} \quad Q^{-1} = \sum_{i=1}^{N} \frac{1}{\lambda^{(i)}} v^{(i)} \otimes v^{*(i)}$$

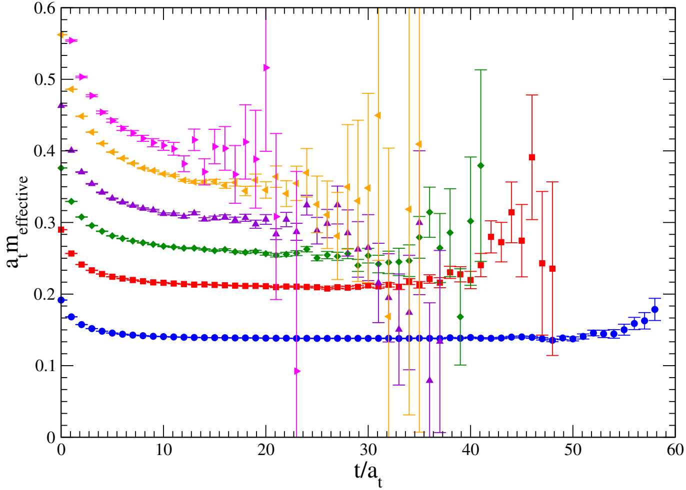
- Even a small sub-set of eigenvectors is computationally expensive, so truncate at $N_{\text{ev}} \ll N$
- Truncated sum violates reflection positivity.
- Use stochastic methods (dilution) to estimate the truncated part.

Comparing methods: point and all-to-all



- $\beta = 5.7$, $12^3 \times 24$, $m_\pi / m_\rho = 0.50$, 75 configs and 100 eigenvectors with time/even-odd/spin dilution.
- Dramatic improvement in precision - especially for higher-lying states.

Extracting excited states energies



$N_f = 2$, $12^3 \times 80$ Wilson fermions, static-light meson energies. Jacobi smearing is used to create a basis of operators for a variational analysis.

Distillation

Distillation distilled...

- A smeared quark field is $\tilde{\psi} = \square\psi$.
- **Define** smearing to be explicitly a very low-rank operator. Rank is $N_{\mathcal{D}}$ ($\ll N_s \times N_c$).

Distillation operator

$$\square(t) = V(t)V^\dagger(t)$$

with $V^a_{\underline{x},c}(t)$ a $N_{\mathcal{D}} \times (N_s \times N_c)$ matrix

- Example (used to date): $\square_\nabla$ the **projection operator into $\mathcal{D}_\nabla$, the space spanned by the lowest eigenmodes of the 3-D laplacian**
- Eigenvectors of ∇^2 not the only choice...

- Consider an isovector meson two-point function:

$$C_M(t_1 - t_0) = \langle\langle \bar{u}(t_1)\Box_{t_1}\Gamma_{t_1}\Box_{t_1} d(t_1) \quad \bar{d}(t_0)\Box_{t_0}\Gamma_{t_0}\Box_{t_0} u(t_0)\rangle\rangle$$

- Substituting the low-rank distillation operator $\Box$ reduces this to a **much smaller** trace:

$$C_M(t_1 - t_0) = \langle \mathrm{Tr}_{\{\sigma,\mathcal{D}\}} \left[\Phi(t_1)\tau(t_1, t_0)\Phi(t_0)\tau(t_0, t_1)\right]\rangle$$

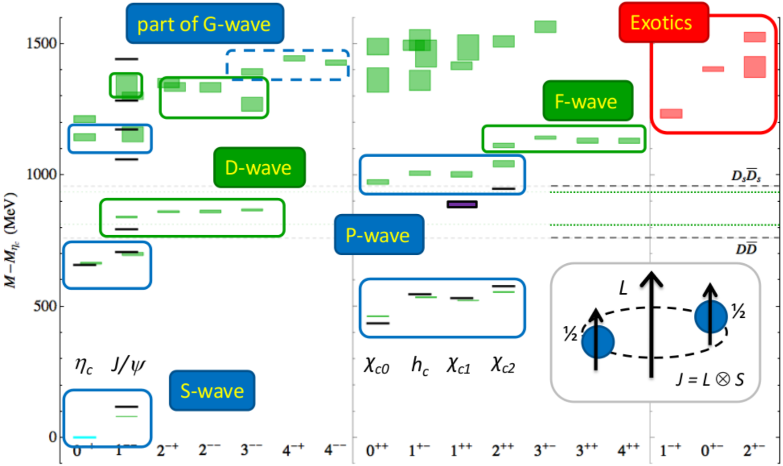
- $\Phi^{\alpha,a}_{\beta,b}$ and $\tau^{\alpha,a}_{\beta,b}$ are $(N_\sigma \times N_\mathcal{D}) \times (N_\sigma \times N_\mathcal{D})$ matrices.

$$\Phi(t) = V^\dagger(t)\Gamma_t V(t) \qquad \tau(t, t') = V^\dagger(t) M^{-1}(t, t') V(t')$$

The "perambulator"

- Note that propagation (via perambulators) and operator construction are separated. Perambulators can be stored and reused for any later operators of interest.

Charmonium with distillation

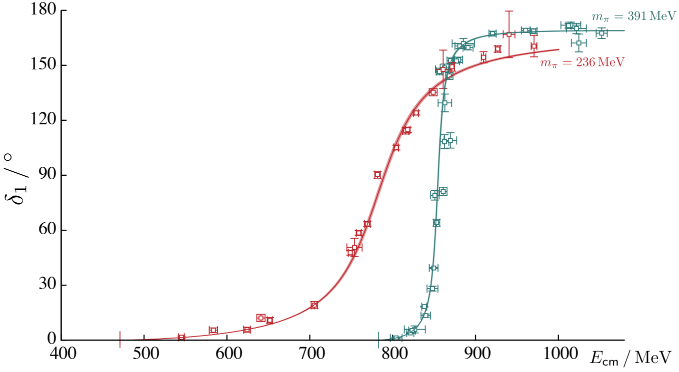


$N_f = 2 + 1$, $m_\pi \sim 400\text{MeV}$. Single hadron operators ie no scattering analysis. Lattice irreps have $\sim$ 10-20 operators in basis.

Extracting scattering information

- Use the finite volume as a tool
- Related lattice energy levels in a finite volume to a decomposition of the scattering amplitude in partial waves in infinite volume

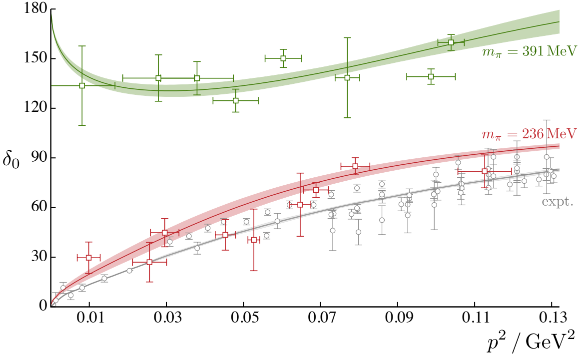
$$\det\left[\cot \delta(E_n^*) + \cot \phi(E_n, \vec{P}, L)\right] = 0$$

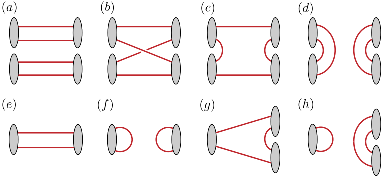


- Includes coupled channels, all disconnected diagrams
- See "HadSpec" for more eg a_0, σ meson etc

SIGMA MESON

Isoscalar $\pi\pi$ scattering including all diagrams.

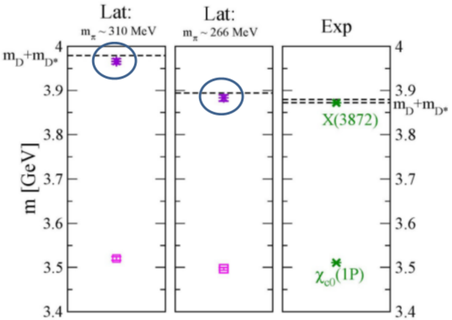




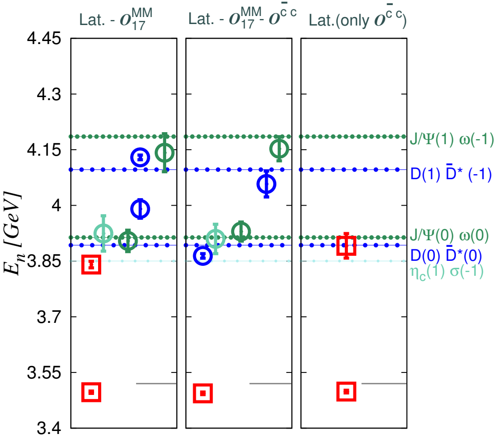
From $m_\pi = 236, 391$MeV resolve a σ that evolves from a bound-state below $\pi\pi$ threshold at heavier mass to a broad resonance at lighter mass.
Note that lattice can also be a tool to study mass-dependence!

The XYZs: X(3872) - a first look (no coupled channels)

Prelovsek & Leskovec 1307.5172



Padmanath, Lang, Prelovsek 1503.03257



Ground state: $\chi_{c1}(1P)$
$D\bar{D}^*$ scattering mx: pole just below thr.
Threshold location? Finite vol effects?

X(3872) not found if $c\bar{c}$ not in basis.

Within 1MeV of $D^0\bar{D}^{0*}$ and 8MeV of D^+D^* thresholds: isospin breaking effects important?

Summary

- Hadronic spectroscopy has changed dramatically in recent years at zero and finite temperature.
- Progress has come from improvements to quite different methods - Bayesian methods at T=0 and variational analysis and quark propagation at T>0.
- At T=0, excited and exotic hadronic states are determined with high precision and the finite volume is used to extract scattering information in increasingly complex scenarios.
- New(ish) methods imply a cost: - all-to-all, eigenvectors for distillation, many operators for variational. But typical improvements in spectroscopic quantities could not otherwise be achieved.
- Multi-hadron and scattering calculations (not discussed) are now quite sophisticated, using these methods - is this useful at finite temperature?
- Can we import some ideas to finite temperature spectroscopy?

Example: $J^{PC} = 2^{++}$ meson creation operator

- Need more information to discriminate spins. Consider continuum operator that creates a 2^{++} meson:

$$\Phi_{ij} = \bar{\psi}\left(\gamma_i D_j + \gamma_j D_i - \frac{2}{3}\delta_{ij}\gamma \cdot D\right)\psi$$

- Lattice: Substitute gauge-covariant lattice finite-difference D_{latt} for D
- A reducible representation:

$$\Phi^{T_2} = \{\Phi_{12}, \Phi_{23}, \Phi_{31}\}$$

$$\Phi^{E} = \left\{\frac{1}{\sqrt{2}}(\Phi_{11} - \Phi_{22}), \frac{1}{\sqrt{6}}(\Phi_{11} + \Phi_{22} - 2\Phi_{33})\right\}$$

- Look for signature of continuum symmetry:

$$\langle 0|\Phi^{(T_2)}|2^{++(T_2)}\rangle = \langle 0|\Phi^{(E)}|2^{++(E)}\rangle$$

Bad news: the bill!

- For constant resolution distillation space scales with N_s
- The cost of a calculation scales with V^2

The problem:

- To maintain constant resolution, need $N_{\mathcal{D}} \propto N_s$
- **Budget:**

Fermion solutions	construct τ	$\mathcal{O}(N_s^2)$
Operator constructions	construct Φ	$\mathcal{O}(N_s^2)$
Meson contractions	$\mathrm{Tr}[\Phi\tau\Phi\tau]$	$\mathcal{O}(N_s^3)$
Baryon contractions	$\bar{B}\tau\tau\tau B$	$\mathcal{O}(N_s^4)$

- Ok for reasonable lattices (eg with $N_s = 16^3$, $N_D = 64$) but scaling this to a 32^3 volume requires $N_D = 512$. Expensive.
- Distillation does not preclude stochastic estimation - use both for large V.