Dense baryons from holography

Florian Preis

in collaboration with
Andreas Schmitt

Vienna University of Technology
Institute for Theoretical Physics

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TECHNISCHE UNIVERSITÄT WIEN
Vienna University of Technology

INSTITUTE for THEORETICAL PHYSICS
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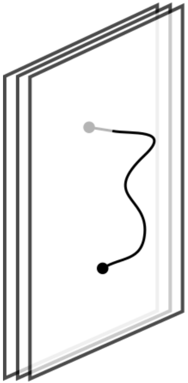
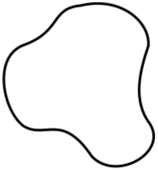
Outline

D-branes in IIA/IIB string theory





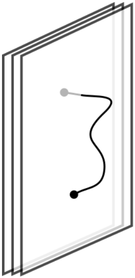
$U(N)$ gauge fields $+\ldots$
$g_{\mathrm{YM}}^2 = \pi g_s$
$M \propto N/g_s$

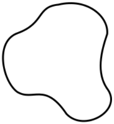
gravitons $+\ldots$
$G \propto g_s^2 (\alpha')^4$
$(R/\sqrt{\alpha'})^{\#} \propto g_s N$

Different perspectives I

Consider low energy limit $\sqrt{\alpha'} \to 0$ and $g_s \ll 1$

1.) $g_s N \ll 1$



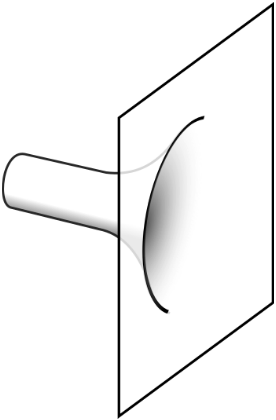


- open strings decouple from closed strings $G \propto g_s^2(\alpha')^4$
- $SU(N)$ Super Yang-Mills (SYM) theory
- supergravity (SUGRA) excitations $G \propto g_s^2(\alpha')^4$ in flat space

Different perspectives II

Consider low energy limit $\sqrt{\alpha'} \to 0$ and $g_s \ll 1$

2.) $g_s N \gg 1$



- D-branes collapse $\Rightarrow$ black brane geometry
- near horizon: SUGRA excitations on a curved background

decoupled from

asymptotic region: SUGRA excitations in flat space

Gauge/gravity duality

Maldacena, Adv.Theor.Math.Phys. 2, 231 (1998)

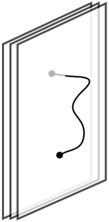
large N, large λ SYM theory in flat space

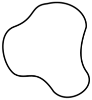
small curvature, classical SUGRA on near horizon geometry

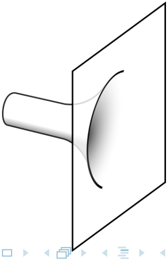
- VEVs of operators
- sources
- global symmetries

$\Leftrightarrow$

- normalizable modes
- non-norm. modes
- local symmetries







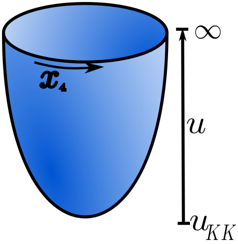
Witten model: breaking super-conformality

Witten, Adv.Theor.Math.Phys. 2: 505,1998

stack of N_c many $D4$-branes wrapped on S^1 (radius M_{KK}^{-1}) with anti periodic boundary conditions for fermions

color branes

geometry

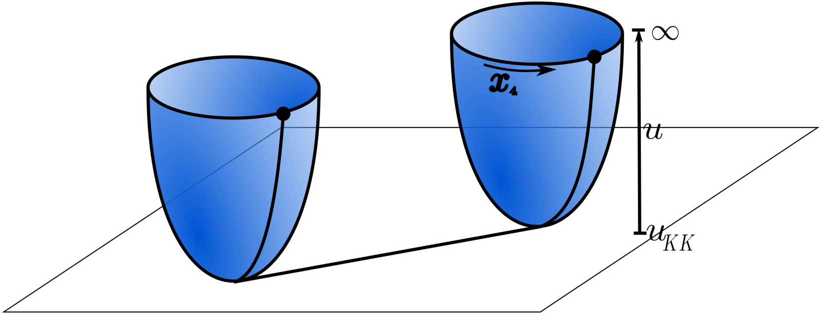


$$\mathrm{d}s^2 = \left(\frac{U}{R}\right)^{3/2} \eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + \left(\frac{R}{U}\right)^{3/2} \frac{\mathrm{d}U^2}{f(U)} + \left(\frac{U}{R}\right)^{3/2} f(U)\mathrm{d}x_4^2 + R^{3/2}U^{1/2}\mathrm{d}\Omega_4^2$$

$$\mathrm{e}^\phi = g_s\left(\frac{U}{R}\right)^{3/4}, \quad F_4 = \frac{N_c}{V_4}\varepsilon_4, \quad f(U) = 1 - \frac{U_{\mathrm{KK}}^3}{U^3}, \quad U_{\mathrm{KK}} = \frac{1}{9}\lambda\alpha' M_{\mathrm{KK}}$$

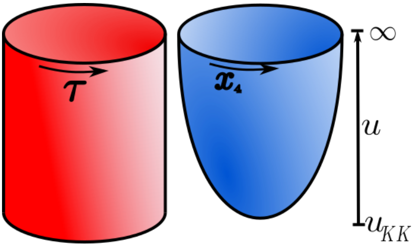
Witten model: confinement

$M_5 \times S^1 \times S^4$ (suppress time, one spatial dimension and internal S^4)



Witten model: low temperature

$S^1 \times \mathbb{R}^3 \times \mathbb{R} \times S^1 \times S^4$ **soft wall**

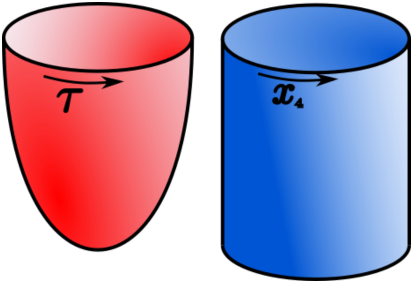


$$ds^2 = \left(\frac{U}{R}\right)^{3/2} \eta_{\mu\nu} dx^\mu dx^\nu + \left(\frac{R}{U}\right)^{3/2} \frac{dU^2}{f(U)} + \left(\frac{U}{R}\right)^{3/2} f(U) dx_4^2 + R^{3/2} U^{1/2} d\Omega_4^2$$

$$f(U) = 1 - \frac{U_{\mathrm{KK}}^3}{U^3}, \quad U_{\mathrm{KK}} = \frac{1}{9}\lambda\alpha' M_{\mathrm{KK}}$$

Witten model: large temperature

$S^1 \times \mathbb{R}^3 \times \mathbb{R} \times S^1 \times S^4$ **black hole**



$$ds^2 = \left(\frac{U}{R}\right)^{3/2} f_T(U)\mathrm{d}\tau^2 + \left(\frac{U}{R}\right)^{3/2} \delta_{ab}\mathrm{d}x^a\mathrm{d}x^b + \left(\frac{R}{U}\right)^{3/2} \frac{\mathrm{d}U^2}{f_T(U)} + R^{3/2}U^{1/2}\mathrm{d}\Omega_4^2$$

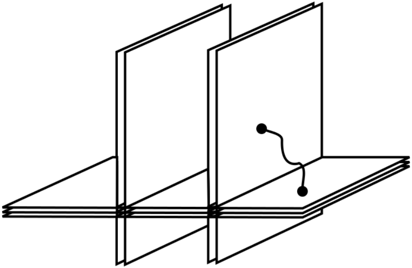
$$f_T(U) = 1 - \frac{U_T^3}{U^3}, \quad U_T = \frac{1}{9}\lambda\alpha' \frac{(2\pi T)^2}{M_{\mathrm{KK}}}$$

Sakai–Sugimoto model: massless flavor

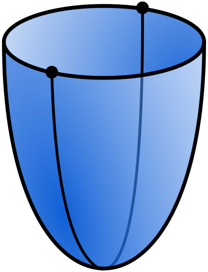
T. Sakai and S. Sugimoto, Prog.Theor.Phys.113:843, 2005

stacks of $N_f \ll N_c$ many $D8$ and $\overline{D8}$-branes

massless quenched quarks



probe branes

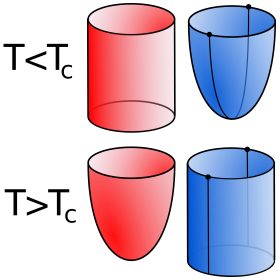


Sakai–Sugimoto model: chiral symmetry (breaking)

boundary-bulk and operator-field correspondence in view of the $D8$-brane action

$$S_{\mathrm{DBI}} \propto \int \mathrm{d}^5 x e^{-\phi} \mathrm{Tr} \sqrt{\det\left(g + 2\pi\alpha' F\right)}$$

$$g_{MN} = \bar{g}_{MN} + \delta g_{MN}, \quad A_M = \bar{A}_M + \delta A_M$$



- chemical potential and quark density:

  $$\text{eg.} \quad \bar{A}_0 = \mu - \frac{2}{3} n U^{-3/2} + \dots$$

- fluctuations describe mesons

Sakai–Sugimoto model: $N_f = 2$ meson spectrum

pions are massless

Table: (Rebhan EPJ Web Conf. 95 (2015) 02005) The masses of isotriplet mesons compared to the experimental value $(m/m_\rho)^{\mathrm{exp.}}$ from [PDG] ($\rho(770), a_1(1260), \rho(1450), a_1(1640)$) and to the large-$N_c$ results from Bali et al. JHEP 06 (2013) 071.

Isotriplet Meson	$\lambda_n = m^2/M^2_{\mathrm{KK}}$	m/m_ρ	$(m/m_\rho)^{\mathrm{exp.}}$	m/m_ρ Bali
0^{-+} (π)	0	0	0.174—0.180	0
1^{--} (ρ)	0.669314	1	1	1
1^{++} (a_1)	1.568766	1.531	1.59(5)	1.86(2)
1^{--} (ρ^*)	2.874323	2.072	1.89(3)	2.40(4)
1^{++} (a_1^*)	4.546104	2.606	2.12(3)	2.98(5)

With Witten–Veneziano formula ($N_f = 3$): $m_{\eta'} = 967\mathrm{MeV}$ (exp: $958\mathrm{MeV}$)

Sakai–Sugimoto model: baryons

recall that we had

$$\begin{aligned} \mathrm{d}s^2 &= \left(\frac{U}{R}\right)^{3/2} \eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + \left(\frac{R}{U}\right)^{3/2} \frac{\mathrm{d}U^2}{f(U)} + \left(\frac{U}{R}\right)^{3/2} f(U)\mathrm{d}x_4^2 + R^{3/2}U^{1/2}\mathrm{d}\Omega_4^2 \\ F_4 &= \frac{N_c}{V_4}\varepsilon_4 \end{aligned}$$

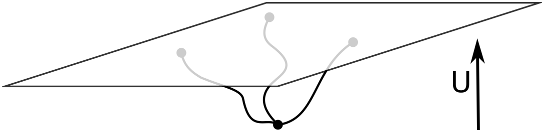
Chern–Simons action couples F_4 to gauge fields on Dp-brane

$$S_{\mathrm{CS}} \propto \int_{\mathcal{M}_{p+1}} F_4 \mathcal{Q}_{p-3}(A)$$

Sakai–Sugimoto model: baryons

Witten, JHEP 9807:006,1998

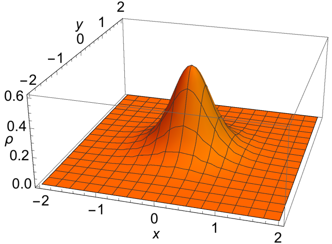
quark vertex ⇔ $D4$-brane on S^4 ⇔ non-trivial gauge bundle

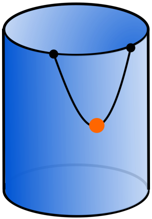


- vertex of quarks at boundary ⇒ strings in the bulk ending on eg. $D4$-brane wrapped on S^4
- $D4$ is charged: $S_{\mathrm{CS}}^{D4} \propto \int_{\mathbb{R}\times S^4} F_4 \wedge \hat{A}_0 \mathrm{d}x^0$
- $\hat{A}_0 \equiv 0$ on compact manifold
  ⇒ need N_c many string endpoints providing charge $-N_c$
- $M_{\mathrm{B}} = \frac{1}{54\pi} M_{\mathrm{KK}} \lambda N_c$, pointlike baryon

Sakai–Sugimoto model: baryons

quark vertex $\Leftrightarrow$ $D4$-brane on S^4 $\Leftrightarrow$ non-trivial gauge bundle





- relevant term $S^{D8}_{\mathrm{CS,B}} \propto \int_{\mathbb{R}\times\mathbb{R}^4\times S^4} F_4 \hat{A}\,\mathrm{Tr}\,F^2$
- $D4$-branes within the $D8$-branes
- $S^{D8}_{\mathrm{CS,B}} = N_c \int_{\mathbb{R}^4\times\mathbb{R}} \frac{1}{8\pi^2}\mathrm{Tr}\,F^2 \hat{A}_0 \mathrm{d}x^0$
- $M_{\mathrm{B}} = \frac{1}{54\pi} M_{\mathrm{KK}} \lambda N_c$, finite size $\sim \lambda^{-1/2}$

Sakai–Sugimoto model: baryon spectrum

Hata et al. Prog.Theor.Phys.117:1157,2007

(n_ρ, n_z)	$(0,0)$	$(1,0)$	$(0,1)$	$(1,1)$	$(2,0)/(0,2)$	$(2,1)/(0,3)$	$(1,2)/(3,0)$
$N\,(I=1)$	940^+	1440^+	1535^-	1655^-	1710^+, ?	2090^-_*, ?	2100^+_*, ?
$\Delta\,(I=3)$	1232^+	1600^+	1700^-	1940^-_*	1920^+, ?	?, ?	?, ?

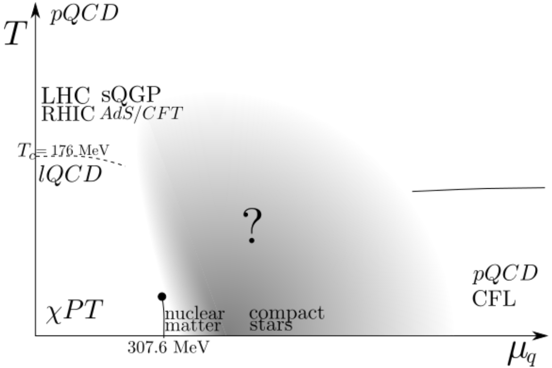
(n_ρ, n_z)	$(0,0)$	$(1,0)$	$(0,1)$	$(1,1)$	$(2,0)/(0,2)$	$(2,1)/(0,3)$	$(1,2)/(3,0)$
$N\,(I=1)$	940^+	1348^+	1348^-	1756^-	$1756^+, 1756^+$	$2164^-, 2164^-$	$2164^+, 2164^+$
$\Delta\,(I=3)$	1240^+	1648^+	1648^-	2056^-	$2056^+, 2056^+$	$2464^-, 2464^-$	$2464^+, 2464^+$

recently Y. Liu and I. Zahed arXiv:1705.01397 [hep-ph]

$\Lambda_c(\frac{1}{2})^+$	$\Xi_c^3(\frac{1}{2})^+$	$\Sigma_c(\frac{1}{2})^+$	$\Lambda_b(\frac{1}{2})^+$	$\Xi_b^3(\frac{1}{2})^+$	$\Sigma_b(\frac{1}{2})^+$
2117 [**2286**]	2320 [**2468**]	2641 [**2453**, **2518**]	[**5619**]	5696 [**5799**]	6022 [**5813**, **5834**]

$$\Xi_{cc}(\frac{1}{2})^+ : \ 3776\,[\mathbf{3519}]\,(PDG),\ [\mathbf{3621}]\,(LHCb)$$

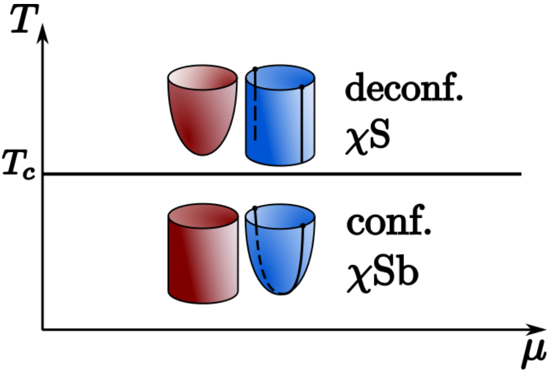
Phase structure of cold dense matter



The Sakai–Sugimoto has all the ingredients and three parameters: λ, M_{KK}, L

Sakai–Sugimoto model: decompactified limit

Nambu–Jona-Lasino (NJL) like model ⇔ limit of small $D8$-brane separation



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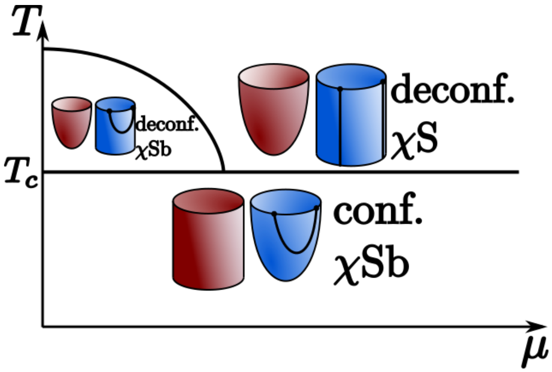

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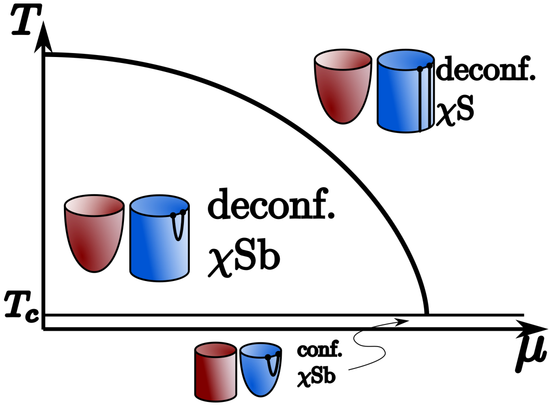
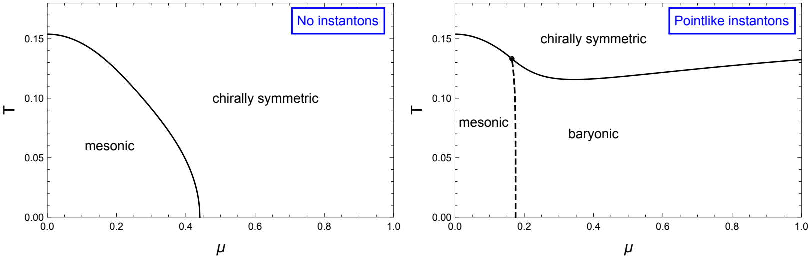


Phase diagram with baryons

Pointlike limit Bergman et al. JHEP0711:056,2007

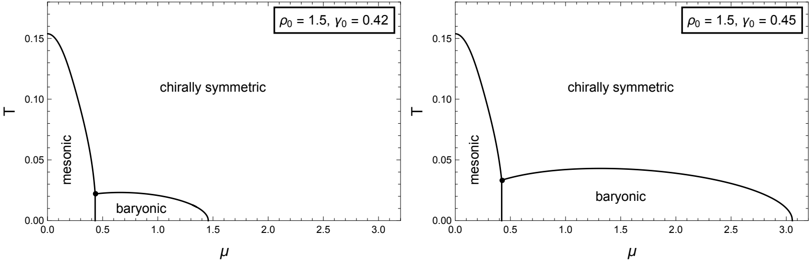


Phase diagram with baryons

Averaging over lattices of *deformed* instantons

FP and A. Schmitt, JHEP 1607 (2016) 001 & EPJ Web Conf. 137 (2017) 09009

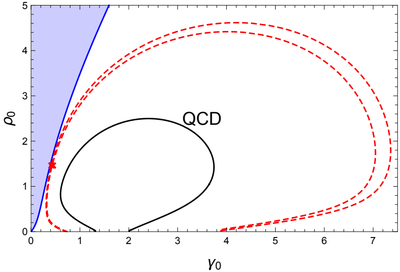
$$\mathrm{Tr}F^2 = \frac{12(\rho/\gamma)^4}{[x^2+(z/\gamma)^2+(\rho/\gamma)^2]^4} \to \frac{1}{V}\sum_{n=0}^{N_z-1}\sum_{i=1}^{i_n}\int d^3X \frac{12(\rho/\gamma)^4}{[(\vec{x}-\vec{x}_{in})^2+(z-z_n)^2/\gamma^2+\rho^2/\gamma^2]^4}$$

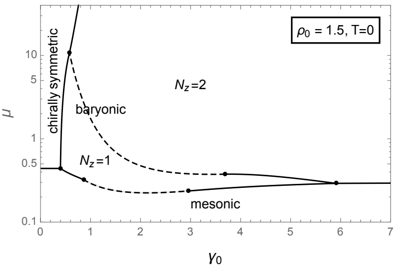


$\gamma = 3/2\gamma_0 u_c^{3/2}$, $\rho = \rho_0 u_c$

Phase diagram with baryons

However, the binding energy is too large $\mu_c/M_0 \sim 0.75$ instead of 0.98





Other approaches

Instanton gas

Ghoroku et al. Phys.Rev. D87, 066006 (2013) & Int.J.Mod.Phys. A29, 1450060 (2014)

Li, Schmitt, Wang, PhysRevD.92.026006

Instanton lattices

Kaplunovsky, Melnikov, Sonnenschein, JHEP11(2012)047 & Mod.Phys.Lett. B29 (2015) no.16, 1540052

Homogeneous solutions

Rozali, Shieh, Van Raamsdonk, Wu, JHEP0801:053,2008

Bolognesi Phys.Rev. D90 (2014) no.10, 105015 & Phys.Rev. D96 (2017) no.3, 034008

Ripley, Sutcliff, Zamaklar, JHEP 1610 (2016) 088

Summary & Conclusions

- The Sakai–Sugimoto model is currently the closest top-down approach to a gravity dual of QCD at asymptotical values of the coupling and N_c. It describes chiral symmetry breaking and confinement.
- It contains glue/glueballs as well as quark matter/hadrons.
- The transition to nuclear matter is first order, but there is no critical endpoint for the liquid-gas phase transition. The binding energy is too large.
  $\Rightarrow$ Need to include the interactions of instantons?
- Our model shows chiral restoration.