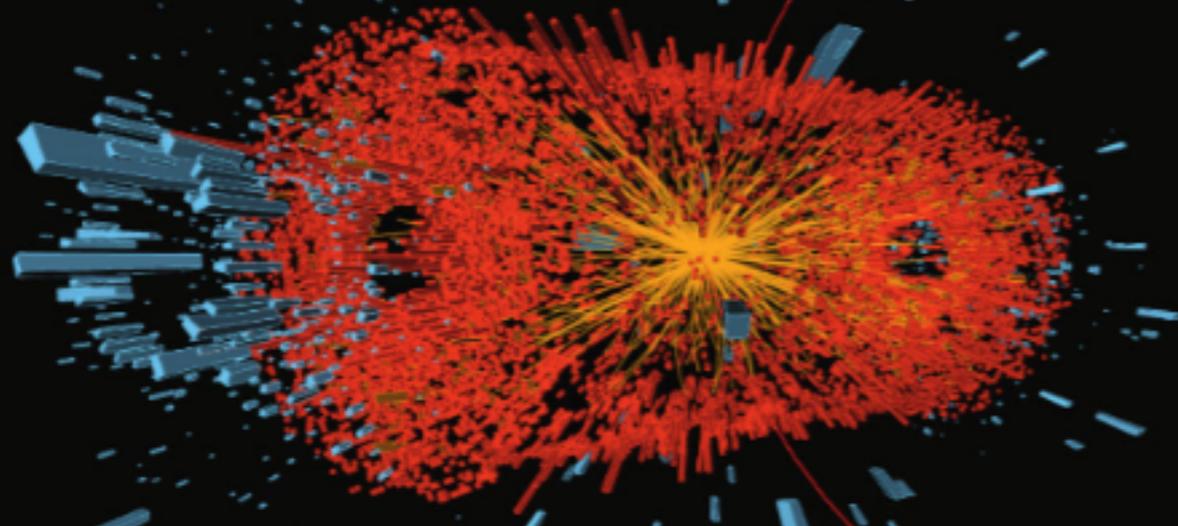


**Theory of Hot Matter
and Relativistic Heavy-Ion Collisions**
THOR

Duration of the Action: 2016-2020



WG1 meeting
Swansea, 12-14 September 2017

Maria Paola Lombardo
INFN

Topology and axions' dark matter from lattice QCD at high T

QCD topology, long standing focus of strong interaction:

-learning about:

the structure of the (s)QGP

fundamental symmetries, η' mass, strong CP problem \rightarrow axions

-hampered by technical difficulties

Recent developments:

-methodological progress: gradient flow, chiral fermions

-first results for dynamical fermions at high temperature:

Trunin *et al.* **J.Phys.Conf.Ser. 668 (2016) no.1, 012123**

Bonati *et al.* **JHEP 1603 (2016) 155**

Borsany *et al.* **Nature 539 (2016) no.7627, 69-71**

Petreczky *et al.* **Phys.Lett. B762 (2016) 498-505**

Burger *et al.* **Nucl. Phys. A, in press**

Taniguchi *et al.* **Phys.Rev. D95 (2017) no.5, 054502**

TmFT

+work in progress

Pseudoscalar spectrum, and $\eta' / U_A(1)$ puzzle

Particle name	Particle symbol \blacklozenge	Antiparticle symbol \blacklozenge	Quark content	Rest mass (MeV/c ²) \blacklozenge
Pion ^[6]	π^+	π^-	$u\bar{d}$	139.570 18 \pm 0.000 35
Pion ^[7]	π^0	Self	$\frac{u\bar{u}-d\bar{d}}{\sqrt{2}}$ [a]	134.9766 \pm 0.0006
Eta meson ^[8]	η	Self	$\frac{u\bar{u}+d\bar{d}-2s\bar{s}}{\sqrt{6}}$ [a]	547.862 \pm 0.018
Eta prime meson ^[9]	$\eta'(958)$	Self	$\frac{u\bar{u}+d\bar{d}+s\bar{s}}{\sqrt{3}}$ [a]	957.78 \pm 0.06
Kaon ^[12]	K^+	K^-	$u\bar{s}$	493.677 \pm 0.016
Kaon ^[13]	K^0	\bar{K}^0	$d\bar{s}$	497.614 \pm 0.024



η' and the $U_A(1)$ problem:

$$U_A(1) \text{ symmetry } q \rightarrow e^{i\alpha\gamma_5} q$$

Would be broken by the (spontaneously generated) $\bar{q}q$:

the candidate Goldstone is the η'
Heavy!! (900 MeV)

BUT:

the divergence of the current

$$j_5^\mu = \bar{q}\gamma_5\gamma_\mu q,$$

$$\partial_\mu j_5^\mu = m\bar{q}\gamma_5 q + \frac{1}{32\pi^2} F \tilde{F}.$$

↑
Contains another term

The $U_A(1)$ symmetry is explicit broken

Particle name	Particle symbol	Antiparticle symbol	Quark content	Rest mass (MeV/c ²)
Pion ^[6]	π^+	π^-	$u\bar{d}$	139.57018 ± 0.00035
Pion ^[7]	π^0	Self	$\frac{u\bar{u}-d\bar{d}}{\sqrt{2}}$ ^[a]	134.9766 ± 0.0006
Eta meson ^[8]	η	Self	$\frac{u\bar{u}+d\bar{d}-2s\bar{s}}{\sqrt{6}}$ ^[a]	547.862 ± 0.018
Eta prime meson ^[9]	$\eta'(958)$	Self	$\frac{u\bar{u}+d\bar{d}+s\bar{s}}{\sqrt{3}}$ ^[a]	957.78 ± 0.06
Kaon ^[12]	K^+	K^-	$u\bar{s}$	493.677 ± 0.016
Kaon ^[13]	K^0	\bar{K}^0	$d\bar{s}$	497.614 ± 0.024

.....

The θ dependence solves the $U_A(1)$ problem:provided that

$$\frac{1}{32\pi^2} \int d^4x F \tilde{F} \quad \text{is different from zero.}$$

It can be proven that $\frac{1}{32\pi^2} \int d^4x F \tilde{F} = Q$ (topological charge)

and

$$Q = n_+ - n_-$$

.....provided that

$$\frac{1}{32\pi^2} \int d^4x F \tilde{F}$$

is different from zero.

It can be proven that

$$\frac{1}{32\pi^2} \int d^4x F \tilde{F} = Q \quad \text{Gluonic definition}$$

and

$$Q = n_+ - n_- \quad \text{Fermionic definition}$$

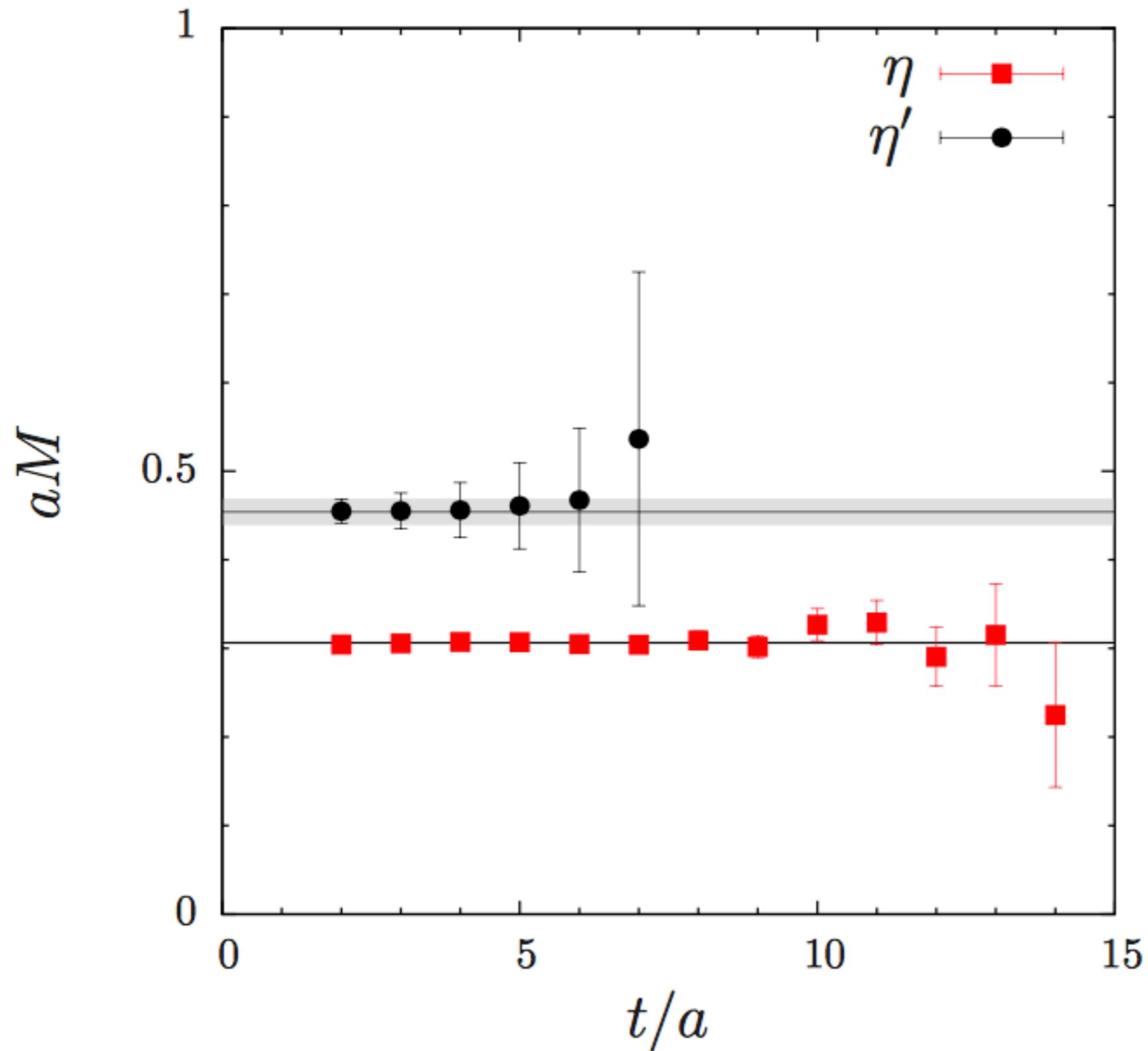
The η' mass may now be computed from the decay of the correlation

$$\langle \partial_\mu j_5^\mu(x) \partial_\mu j_5^\mu(y) \rangle \propto \frac{1}{N^2} \langle F(x) \tilde{F}(x) F(y) \tilde{F}(y) \rangle$$

which at leading order gives the Witten-Veneziano formula

$$m_{\eta'}^2 = \frac{2N_f}{F_\pi^2} \chi_t^{\text{qu}}$$

Lattice studies of η, η'



Otnnad, Urbach, Michael (ETMC)
2013

$$M_\eta = 551(8)_{\text{stat}}(6)_{\text{sys}}$$

$$M_{\eta'} = 1006(54)_{\text{stat}}(38)_{\text{sys}}(+61)_{\text{ex}}$$

OK!

A θ term solves the $U_A(1)$ problem

$$\mathcal{L}_{QCD}(\theta) = \mathcal{L}_{QCD} + \frac{g^2\theta}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}^a F_{\rho\sigma}^a.$$

$$Q = \int d^4x \frac{g^2}{32\pi^2} \text{tr} F \tilde{F}$$

θ term solves $U_A(1)$ problem

$$\mathcal{L}_{QCD}(\theta) = \mathcal{L}_{QCD} + \frac{g^2\theta}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}^a F_{\rho\sigma}^a.$$

Admitted but $\theta < 10^{-9}$

$$Q = \int d^4x \frac{g^2}{32\pi^2} \text{tr} F \tilde{F}$$

but: it poses a naturalness issue ... (more later)...

θ term solves $U_A(1)$ problem

$$\mathcal{L}_{QCD}(\theta) = \mathcal{L}_{QCD} + \frac{g^2\theta}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}^a F_{\rho\sigma}^a.$$

Admitted but $\theta < 10^{-9}$

$$Q = \int d^4x \frac{g^2}{32\pi^2} \text{tr} F \tilde{F}$$

We then consider:

$$Z_{QCD}(\theta, T) = \int [dA][d\psi][d\bar{\psi}] \exp \left(-T \sum_t d^3x \mathcal{L}_{QCD}(\theta) \right) = \exp[-VF(\theta, T)]$$

A note on computation:

$$\mathcal{L}_{QCD}(\theta) = \mathcal{L}_{QCD} + \frac{g^2 \theta}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}^a F_{\rho\sigma}^a.$$

Admitted but $\theta < 10^{-9}$

$$Q = \int d^4x \frac{g^2}{32\pi^2} \text{tr} F \tilde{F}$$

Sign problem

Approach similar in spirit to Taylor expansion for chempot

$$Z_{QCD}(\theta, T) = \int [dA][d\psi][d\bar{\psi}] \exp \left(-T \sum_t d^3x \mathcal{L}_{QCD}(\theta) \right) = \exp[-V F(\theta, T)]$$

Simulations at $\theta = 0$

$$\frac{\partial^2 F(\theta, T)}{\partial \theta^2} \Big|_{\theta=0} \equiv \chi(T), = (\langle Q^2 \rangle - \langle Q \rangle^2) / V$$

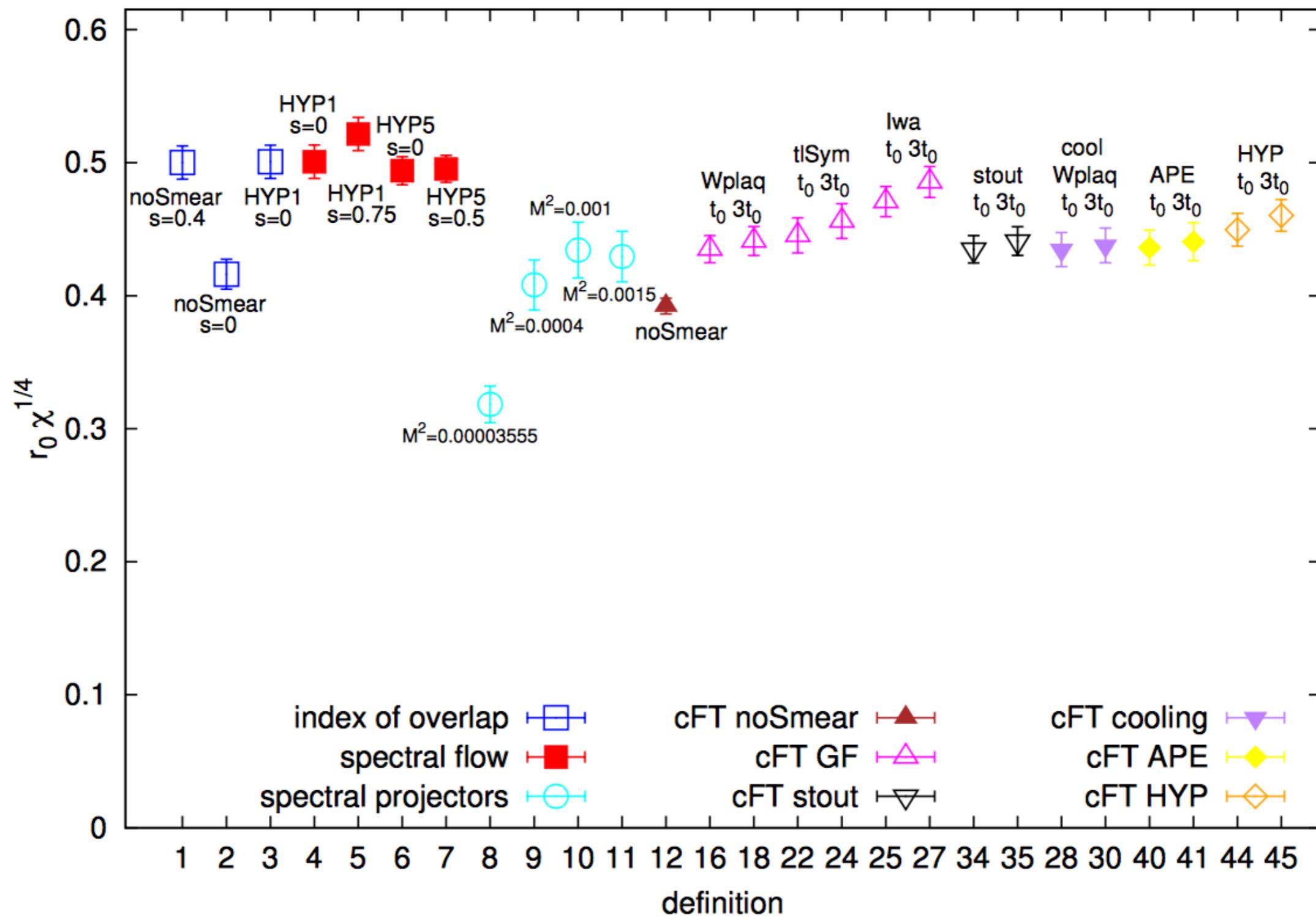
Comparison of topological charge definitions in Lattice QCD

Constantia Alexandrou^{a,b}, Andreas Athenodorou^{b,a}, Krzysztof Cichy^{c,d},
Arthur Dromard^e, Elena Garcia-Ramos^{f,g}, Karl Jansen^f,
Urs Wenger^h, Falk Zimmermannⁱ

2017

Comparison of topological charge definitions

nr	full name	smearing type	short name	type
1	index of overlap Dirac operator $s = 0.4$	–	index nonSmear $s = 0.4$	F
2	index of overlap Dirac operator $s = 0.0$	–	index nonSmear $s = 0$	F
3	index of overlap Dirac operator $s = 0.0$	HYP1	index HYP1 $s = 0$	F
4	Wilson-Dirac op. spectral flow $s = 0.0$	HYP1	SF HYP1 $s = 0.0$	F
5	Wilson-Dirac op. spectral flow $s = 0.75$	HYP1	SF HYP1 $s = 0.75$	F
6	Wilson-Dirac op. spectral flow $s = 0.0$	HYP5	SF HYP5 $s = 0.0$	F
7	Wilson-Dirac op. spectral flow $s = 0.5$	HYP5	SF HYP5 $s = 0.5$	F
8	spectral projectors $M^2 = 0.00003555$	–	spec. proj. $M^2 = 0.00003555$	F
9	spectral projectors $M^2 = 0.0004$	–	spec. proj. $M^2 = 0.0004$	F
10	spectral projectors $M^2 = 0.0010$	–	spec. proj. $M^2 = 0.0010$	F
11	spectral projectors $M^2 = 0.0015$	–	spec. proj. $M^2 = 0.0015$	F
12	field theoretic (clover)	–	cFT nonSmear	G
13	field theoretic (plaquette)	GF (Wplaq, t_0)	pFT GF Wplaq t_0	G
14	field theoretic (plaquette)	GF (Wplaq, $2t_0$)	pFT GF Wplaq $2t_0$	G
15	field theoretic (plaquette)	GF (Wplaq, $3t_0$)	pFT GF Wplaq $3t_0$	G
16	field theoretic (clover)	GF (Wplaq, t_0)	cFT GF Wplaq t_0	G
17	field theoretic (clover)	GF (Wplaq, $2t_0$)	cFT GF Wplaq $2t_0$	G
18	field theoretic (clover)	GF (Wplaq, $3t_0$)	cFT GF Wplaq $3t_0$	G
19	field theoretic (improved)	GF (Wplaq, t_0)	iFT GF Wplaq t_0	G
20	field theoretic (improved)	GF (Wplaq, $2t_0$)	iFT GF Wplaq $2t_0$	G
21	field theoretic (improved)	GF (Wplaq, $3t_0$)	iFT GF Wplaq $3t_0$	G
22	field theoretic (clover)	GF (tlSym, t_0)	cFT GF tlSym t_0	G
23	field theoretic (clover)	GF (tlSym, $2t_0$)	cFT GF tlSym $2t_0$	G
24	field theoretic (clover)	GF (tlSym, $3t_0$)	cFT GF tlSym $3t_0$	G
25	field theoretic (clover)	GF (Iwa, t_0)	cFT GF Iwa t_0	G
26	field theoretic (clover)	GF (Iwa, $2t_0$)	cFT GF Iwa $2t_0$	G
27	field theoretic (clover)	GF (Iwa, $3t_0$)	cFT GF Iwa $3t_0$	G
28	field theoretic (clover)	cool (Wplaq, t_0)	cFT cool (GF Wplaq t_0)	G
29	field theoretic (clover)	cool (Wplaq, $3t_0$)	cFT cool (GF Wplaq $3t_0$)	G
30	field theoretic (clover)	cool (tlSym, t_0)	cFT cool (GF tlSym t_0)	G
31	field theoretic (clover)	cool (tlSym, $3t_0$)	cFT cool (GF tlSym $3t_0$)	G
32	field theoretic (clover)	cool (Iwa, t_0)	cFT cool (GF Iwa t_0)	G
33	field theoretic (clover)	cool (Iwa, $3t_0$)	cFT cool (GF Iwa $3t_0$)	G
34	field theoretic (clover)	stout (0.01, t_0)	cFT stout 0.01 (GF Wplaq t_0)	G
35	field theoretic (clover)	stout (0.01, $3t_0$)	cFT stout 0.01 (GF Wplaq $3t_0$)	G
36	field theoretic (clover)	stout (0.1, t_0)	cFT stout 0.1 (GF Wplaq t_0)	G
37	field theoretic (clover)	stout (0.1, $3t_0$)	cFT stout 0.1 (GF Wplaq $3t_0$)	G
38	field theoretic (clover)	APE (0.4, t_0)	cFT APE 0.4 (GF Wplaq t_0)	G
39	field theoretic (clover)	APE (0.4, $3t_0$)	cFT APE 0.4 (GF Wplaq $3t_0$)	G
40	field theoretic (clover)	APE (0.5, t_0)	cFT APE 0.5 (GF Wplaq t_0)	G
41	field theoretic (clover)	APE (0.5, $3t_0$)	cFT APE 0.5 (GF Wplaq $3t_0$)	G
42	field theoretic (clover)	APE (0.6, t_0)	cFT APE 0.6 (GF Wplaq t_0)	G
43	field theoretic (clover)	APE (0.6, $3t_0$)	cFT APE 0.6 (GF Wplaq $3t_0$)	G
44	field theoretic (clover)	HYP (t_0)	cFT HYP (GF Wplaq t_0)	G
45	field theoretic (clover)	HYP ($3t_0$)	cFT HYP (GF Wplaq $3t_0$)	G



T=0 results satisfactory

(of course there is room for improvement)

From now on:

High Temperature

The two faces of Hot QCD topology



Window to Axions

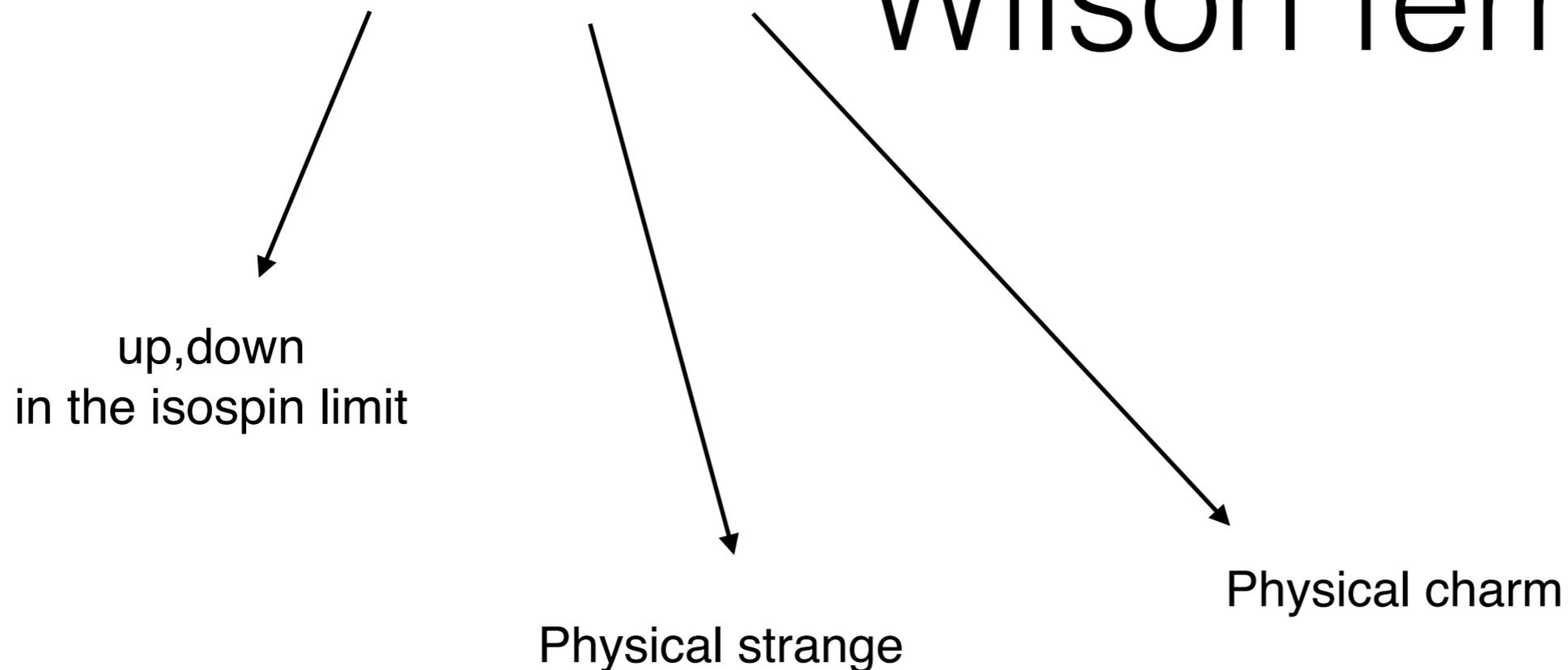
Property of Quark Gluon Plasma

Our setup at a glance

Hot QCD and

$N_f = 2 + 1 + 1$ twisted mass

Wilson fermions



Why $N_f = 2 + 1 + 1$?

T_c

340–380 MeV
RHIC AuAu
200 GeV

420–480 MeV
LHC
2.76 TeV

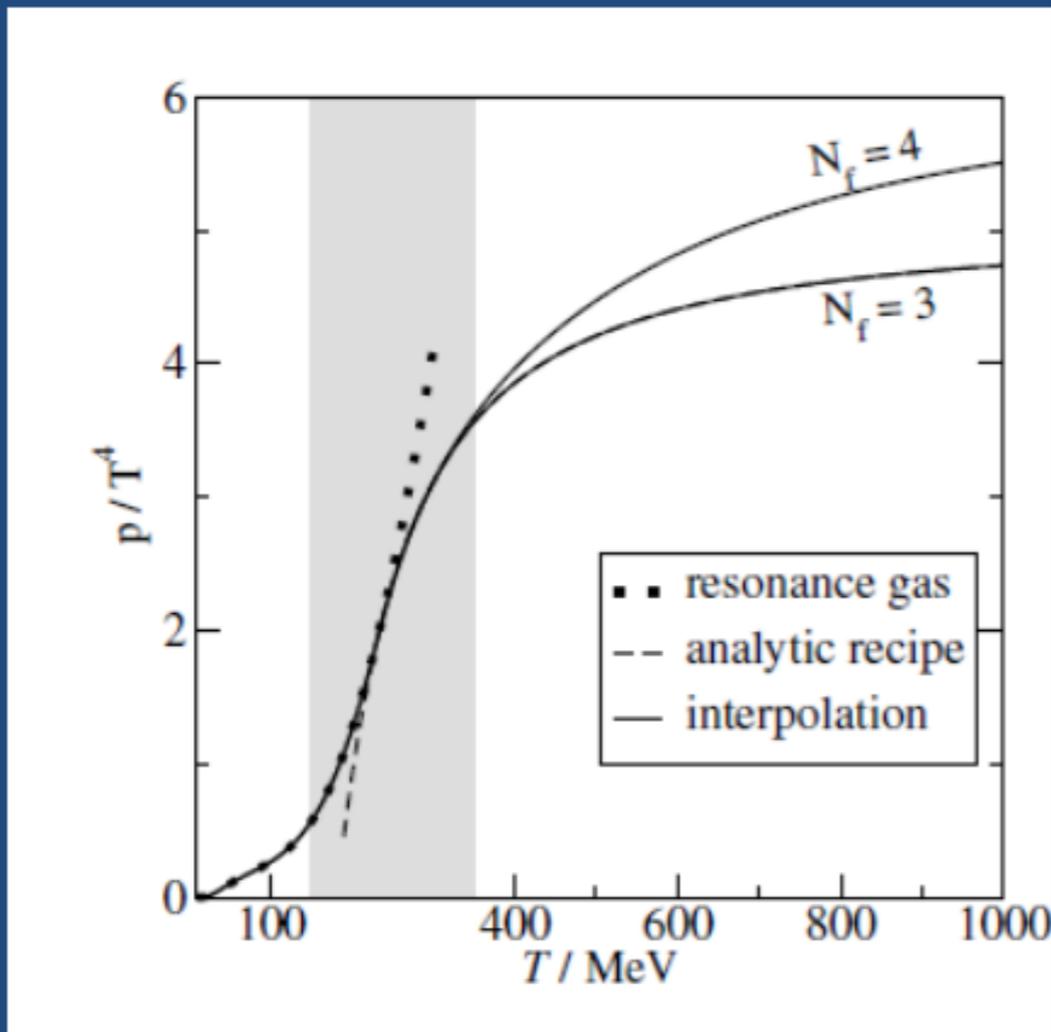
500–600 MeV
LHC hot spots
2.76 TeV

1 GeV
LHC
7 TeV



≈ 200 MeV

Quark Gluon Plasma @ Colliders



Analytic studies suggest that a dynamical charm becomes relevant above 400 MeV, well within the reach of LHC

Laine Schroeder 2006

Fixed
varying
scale

For each lattice spacing we explore a range of temperatures 150MeV — 500 MeV by varying N_t

We repeat this for three different lattice spacings following ETMC T=0 simulations.

Four pion masses

Advantages: we rely on the setup of ETMC T=0 simulations. Scale is set once for all.

Disadvantages: mismatch of temperatures - need interpolation before taking the continuum limit

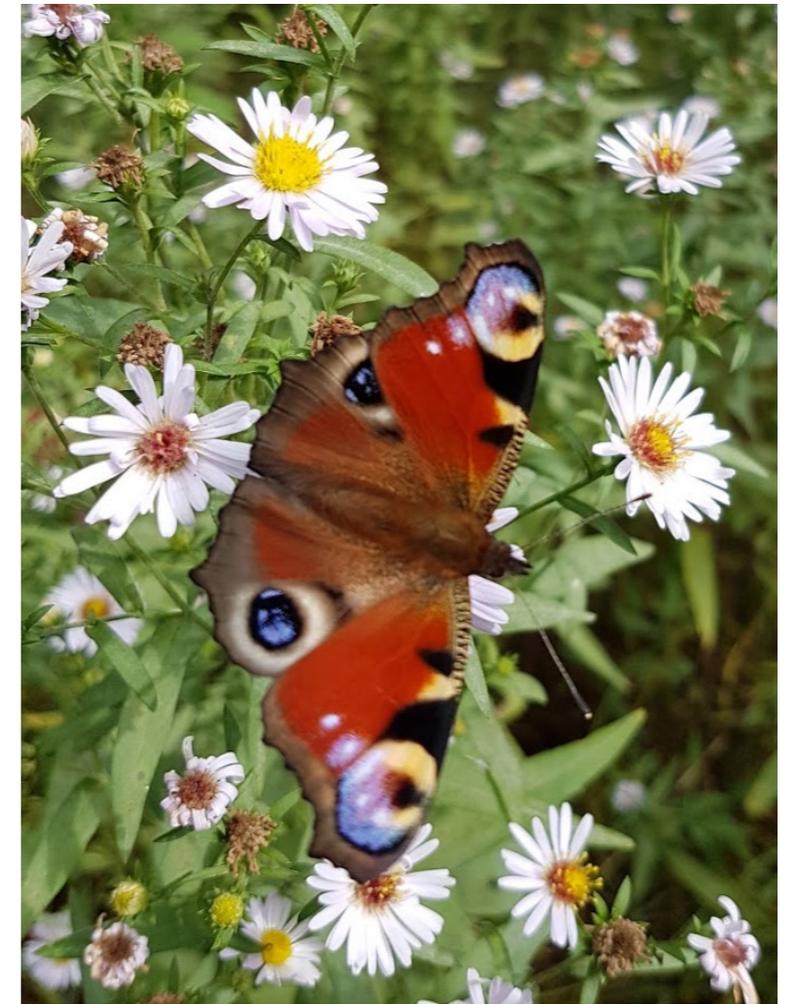
Number of flavours	m_{π^\pm}
	210
$N_f = 2 + 1 + 1$	260
	370
	470
$N_f = 2$	360
	430

Setup

$T = 0$ (ETMC) nomenclature	β	a [fm] [6]	N_σ^3	N_τ	T [MeV]	# confs.				
A60.24	1.90	0.0936(38)	24^3	5	422(17)	585				
				6	351(14)	1370				
				7	301(12)	341				
				8	263(11)	970				
				9	234(10)	577				
				10	211(9)	525				
				11	192(8)	227				
			32^3	12	176(7)	1052				
				13	162(7)	294				
				14	151(6)	1988				
				B55.32	1.95	0.0823(37)	32^3	5	479(22)	595
								6	400(18)	345
								7	342(15)	327
								8	300(13)	233
9	266(12)	453								
10	240(11)	295								
11	218(10)	667								
12	200(9)	1102								
13	184(8)	308								
14	171(8)	1304								
D45.32	2.10	0.0646(26)	32^3	15	160(7)	456				
				16	150(7)	823				
				40^3	6	509(20)	403			
					7	436(18)	412			
					8	382(15)	416			
					10	305(12)	420			
					12	255(10)	380			
					14	218(9)	793			
					16	191(8)	626			
					18	170(7)	599			
48^3	20	153(6)	582							

Results I

Gluonic (butterfly) operator
+
Gradient Flow Method



$$\mathcal{L}_\theta = \frac{1}{4} F_{\mu\nu}^a(x) F_{\mu\nu}^a(x) - i\theta \frac{g^2}{64\pi^2} \epsilon_{\mu\nu\rho\sigma} F_{\mu\nu}^a(x) F_{\rho\sigma}^a(x),$$

$$\exp[-VF(\theta)] = \int [dA] \exp\left(-\int d^4x \mathcal{L}_\theta\right)$$

Gradient flow

Lüscher, Lüscher Weisz

Evolve the link variables in a fictitious flow time:

$$\dot{V}_{x,\mu}(t) = -g_0^2 \left[\partial_{x,\mu} S_{\text{Wilson}}(V(t)) \right] V_{x,\mu}(t);$$

Monitor $\langle E \rangle = \frac{1}{2N_\tau N_\sigma^3} \sum_{x,\mu,\nu} \text{Tr}[F_{\mu\nu}(x) F^{\mu\nu}(x)]$ as a function of t

Stop flowing when $t^2 \langle E \rangle \big|_{t=t_0} = 0.3$

Observables $\langle O(t) \rangle$ renormalized at $\mu = 1/\sqrt{8t}$



Continuum limit of $\langle O(t) \rangle$ is independent on the chosen reference value

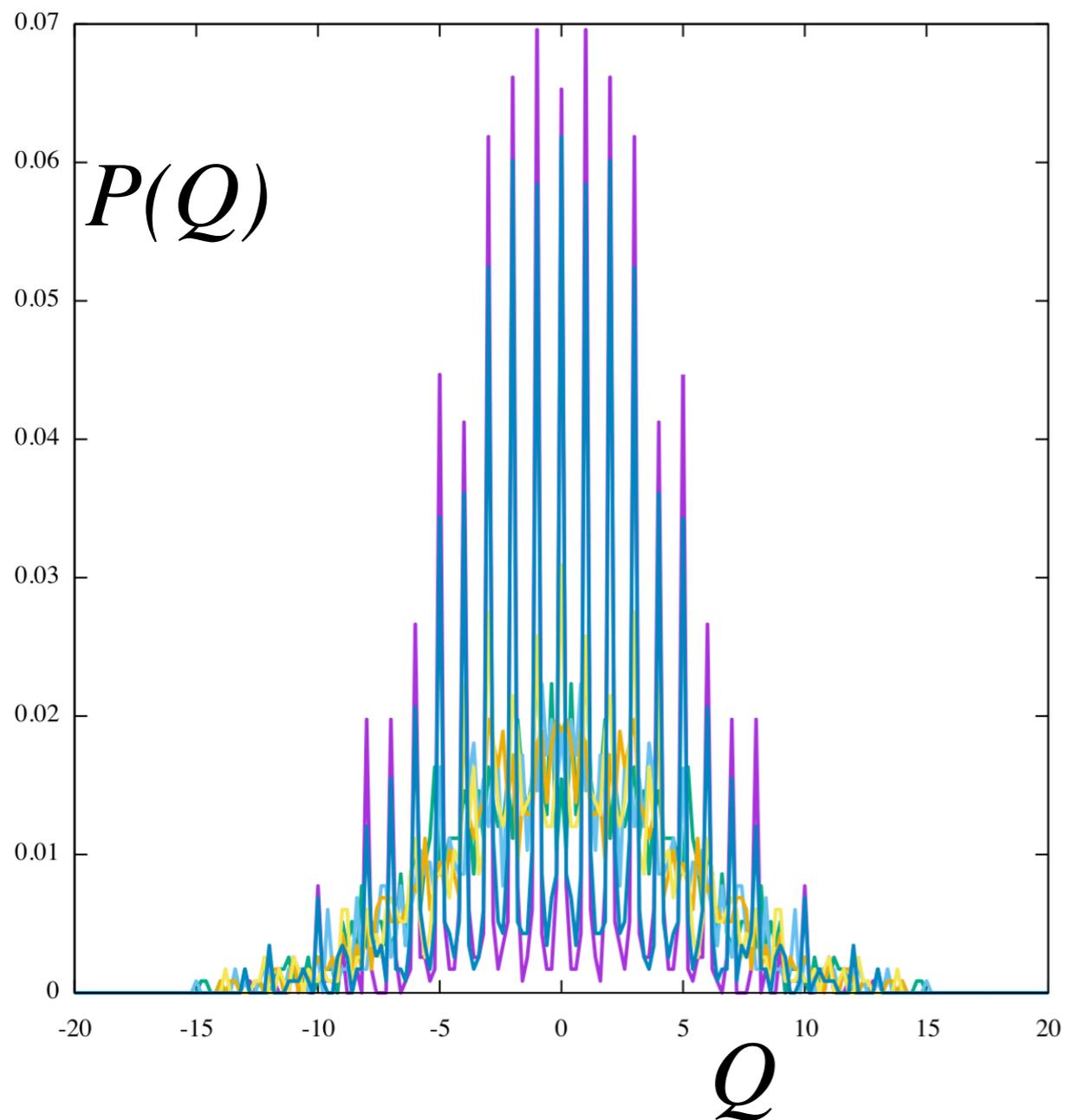
Caveat: note comments by Kanaya et al.

Distribution of the topological charge $P(Q)$

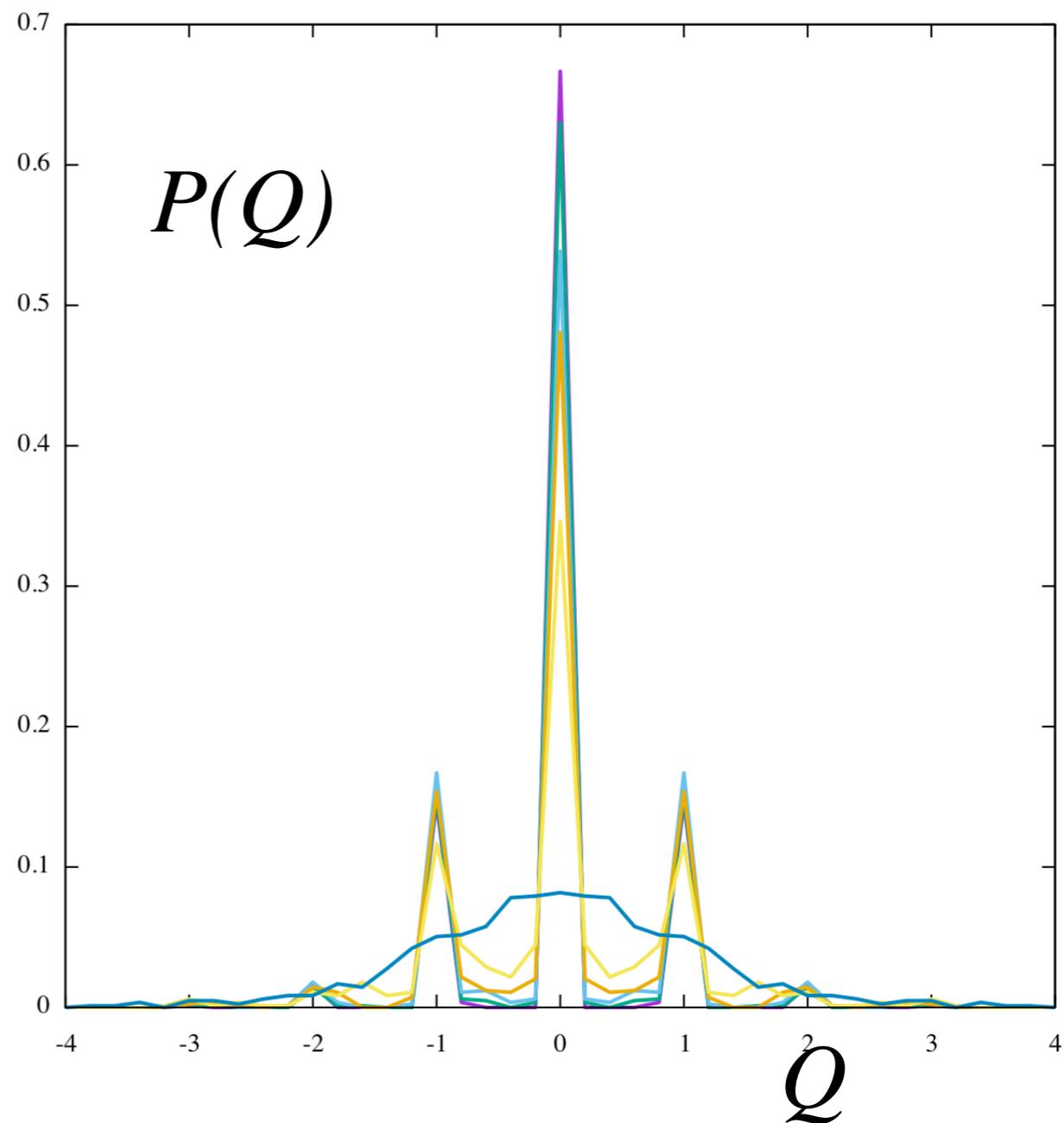
cluster around integers as cooling proceeds

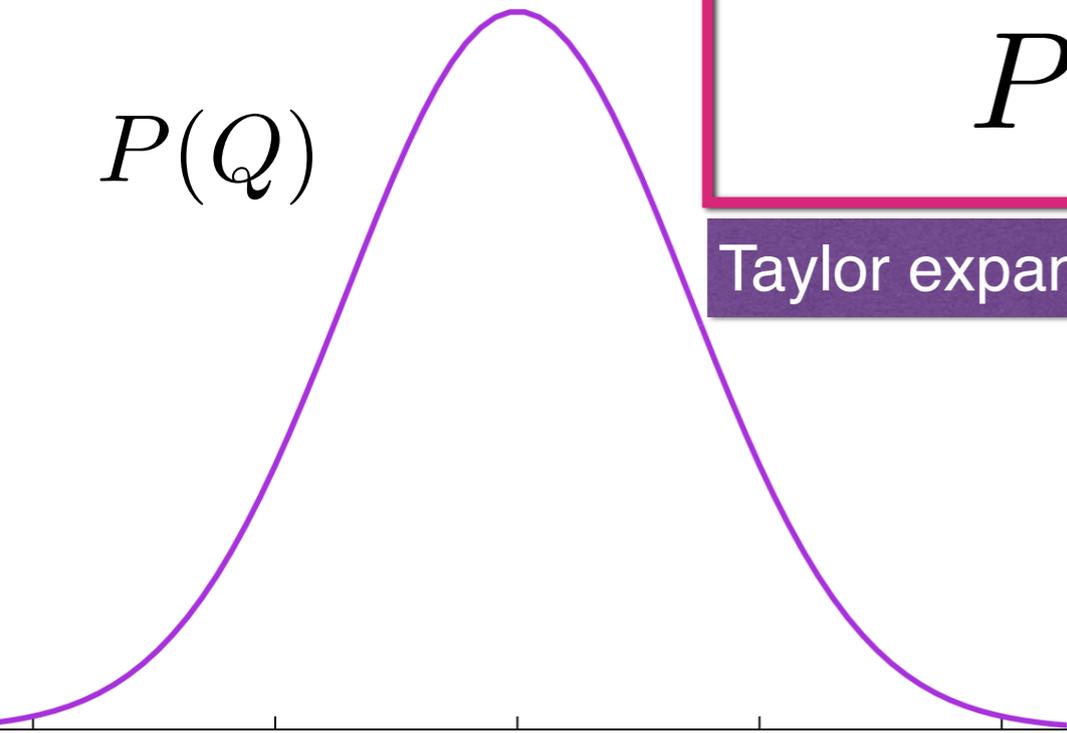
(results for $a = 0.06$ fm)

T=153 MeV



T=253 MeV



$P(Q)$  Q
 $P(Q)$ and $F(\theta)$

Taylor expansion, and cumulants of the topological charge distribution

$$e^{-F(\theta)} = \langle e^{i\theta Q} \rangle$$

$$P_\nu = \int_{-\pi}^{\pi} \frac{d\theta}{2\pi} e^{-i\theta\nu} e^{-F(\theta)} \quad Q = \nu$$

$$C_n = (-1)^{n+1} \frac{1}{V} \frac{d^{2n}}{d\theta^{2n}} F(\theta) \Big|_{\theta=0} \equiv \langle Q^{2n} \rangle_{conn}$$

$$F(\theta) = V \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\theta^{2n}}{(2n)!} C_n$$

$$P_\nu = \frac{e^{-\frac{\nu^2}{2\sigma^2}}}{\sqrt{2\pi\sigma^2}} \left[1 + \frac{1}{4!} \frac{\tau}{\sigma^2} \text{He}_4(\nu/\sigma) \right]$$

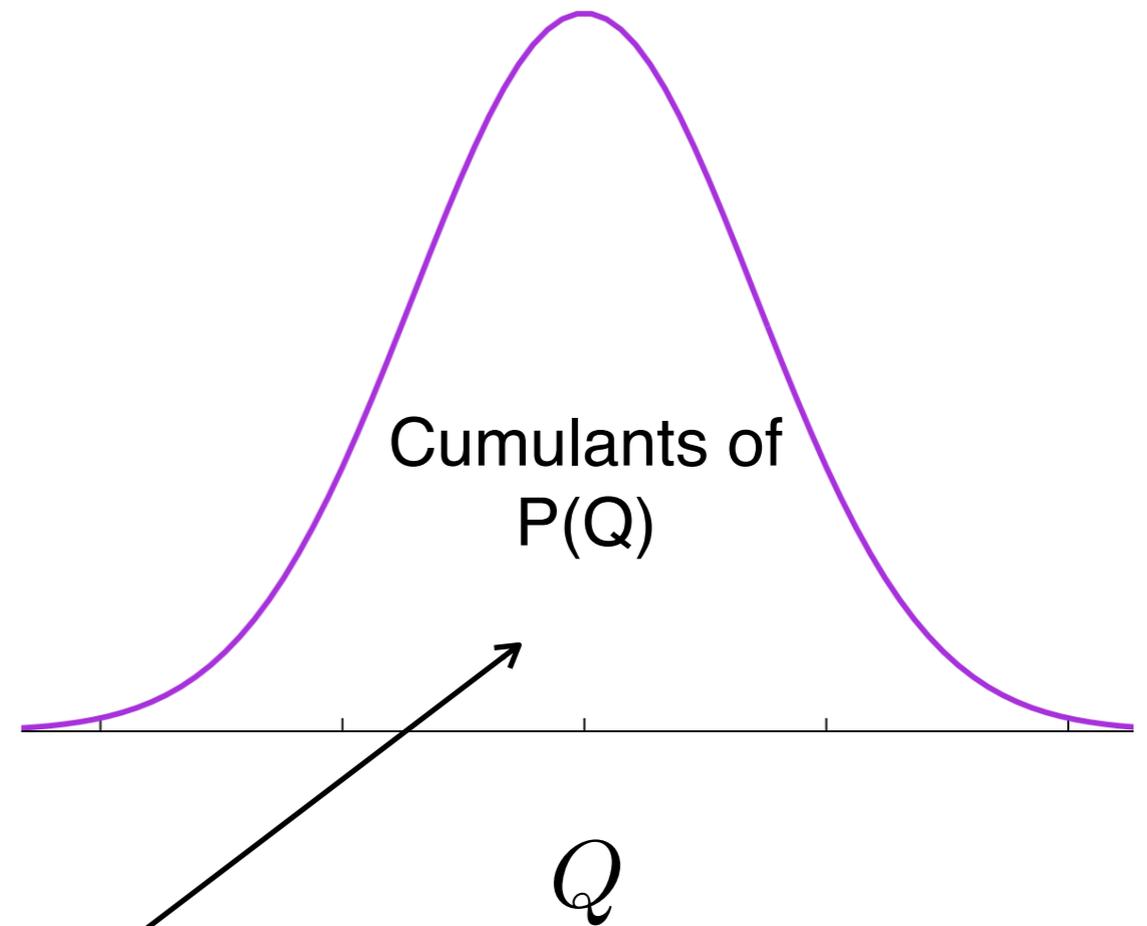
 $\sigma^2 = VC_1$ and $\tau = C_2/C_1$ $P(Q)$ is Gaussian for $V \rightarrow \infty$
 $F(\theta)$ is 'hidden' in $P(Q)$'s cumulants

In practice only the first two cumulants are accessible:

$$F(\theta, T) = 1/2 \chi(T) \theta^2 s(\theta, T)$$

$$s(\theta, T) = 1 + b_2(T) \theta^2 + \dots$$

$$b_2 = - \frac{\langle Q^4 \rangle - 3 \langle Q^2 \rangle^2}{12 \langle Q^2 \rangle}$$



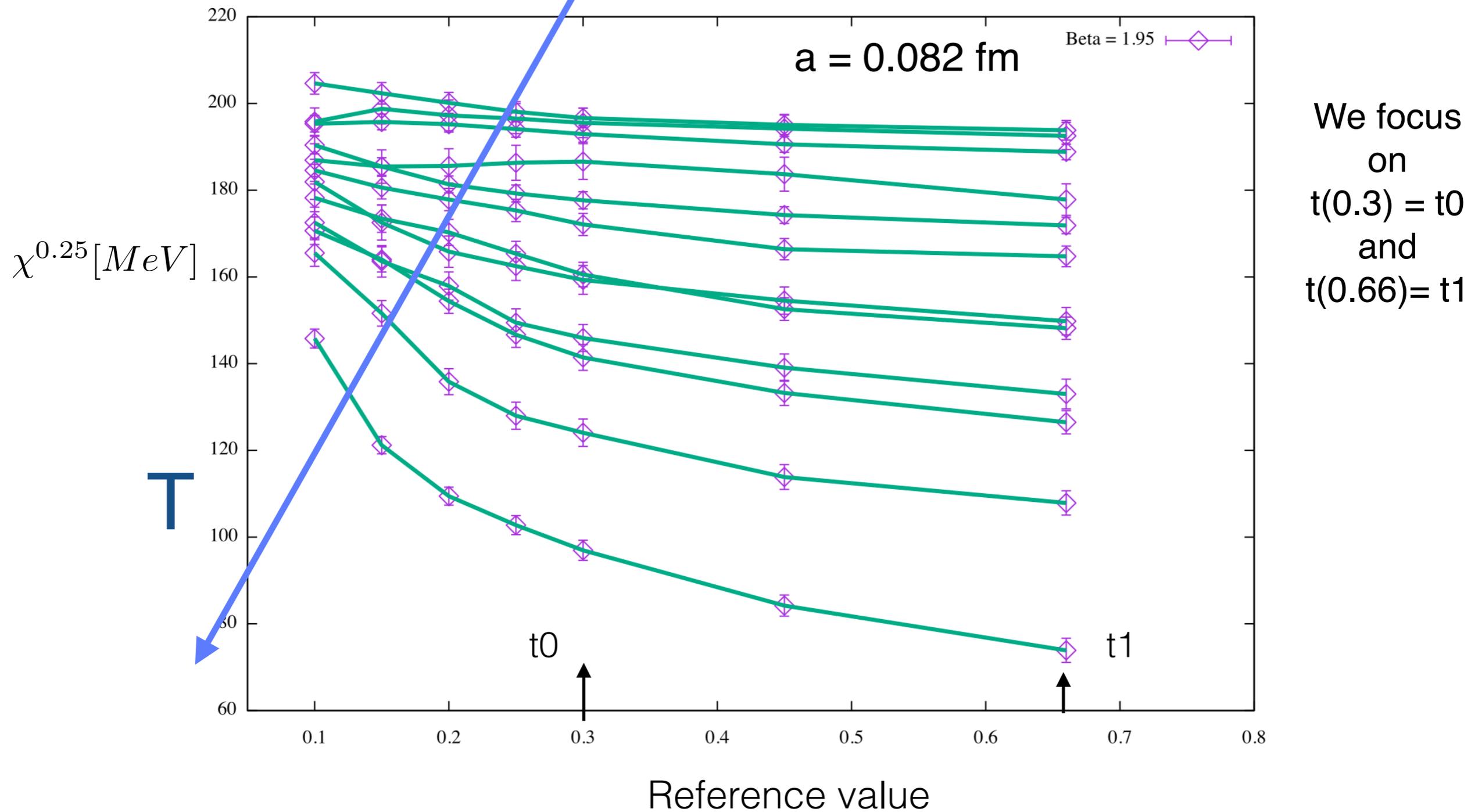
Taylor coefficients of $F(\theta, T)$

DIGA — at very high temperature — predicts

$$F(\theta, T) - F(0, T) = \chi(T)(1 - \cos(\theta)) \longrightarrow b_2 = -1/12$$

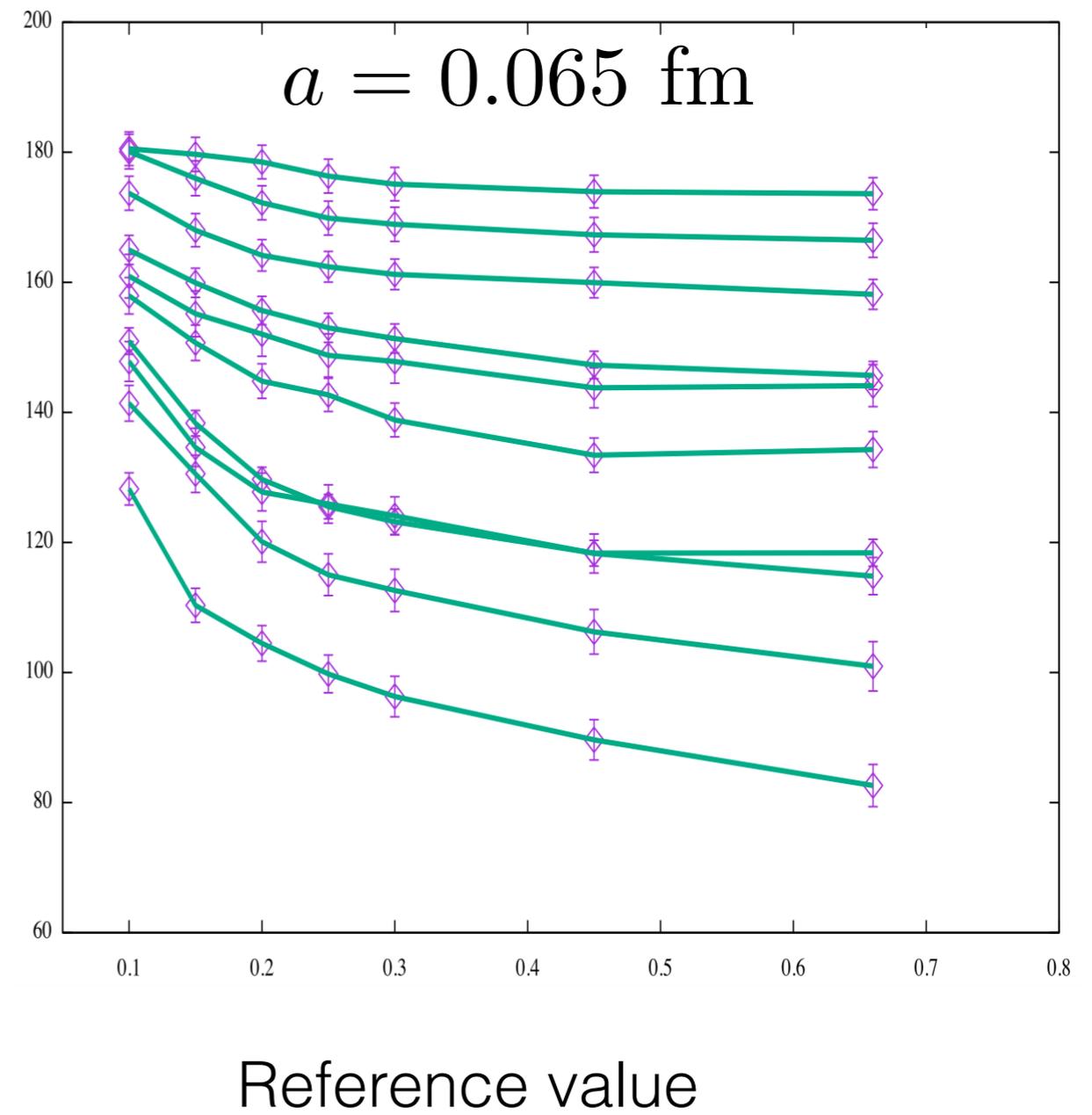
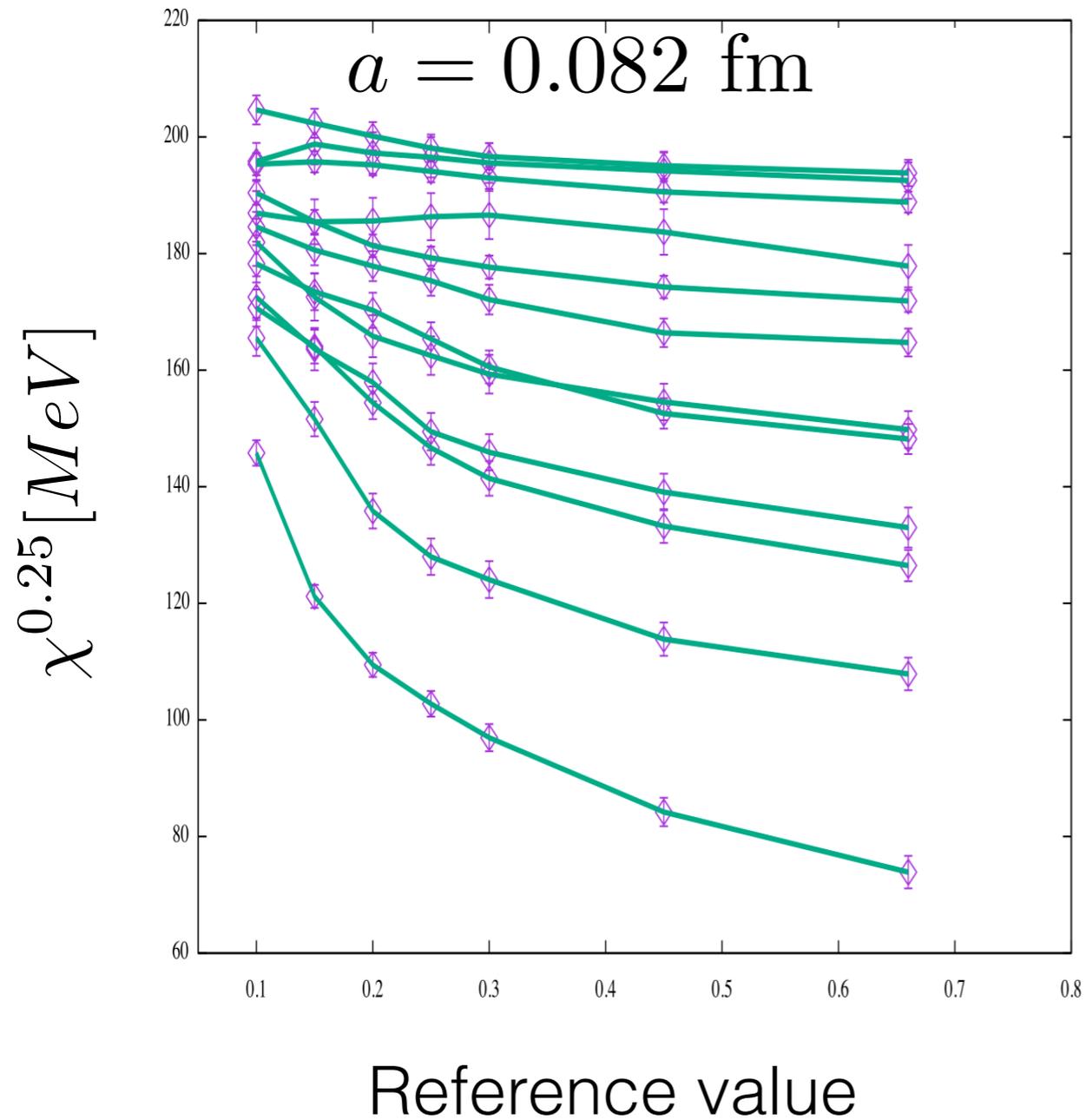
Flowing towards the plateau

$$t^2 \langle E \rangle |_{t=t_x, x=0-6} = (0.3, 0.66, 0.1, 0.15, 0.2, 0.25, 0.45)$$

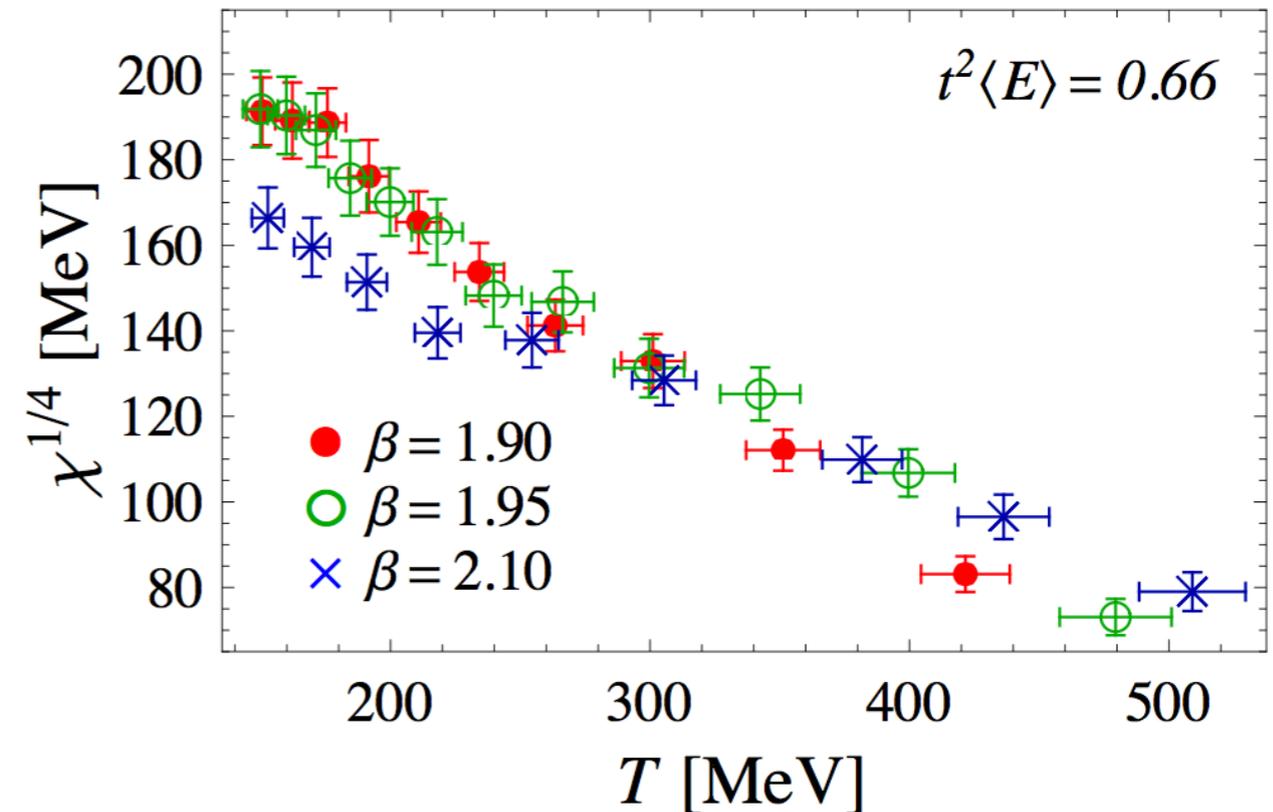
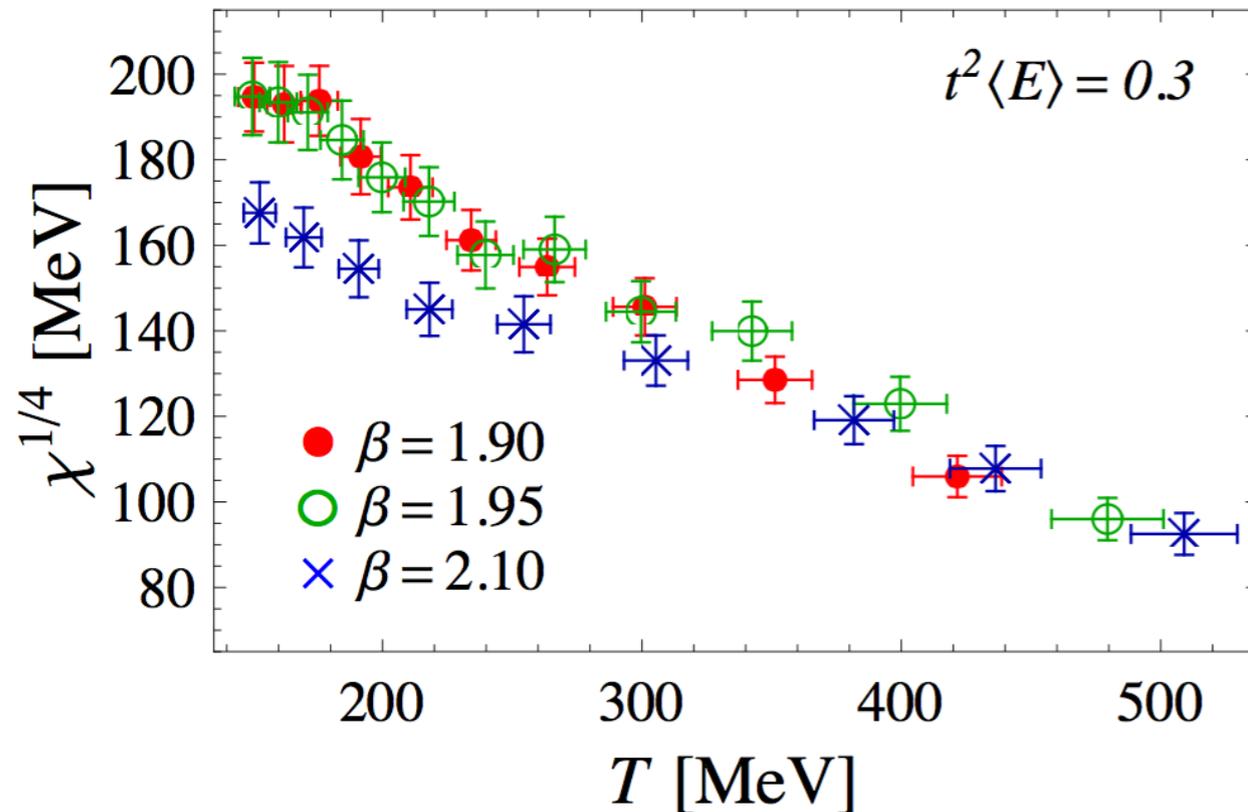


On finer lattices, plateau is almost reached:

Gradient method coincides with cooling



Results for the topological susceptibility for $M_\pi = 270$ MeV



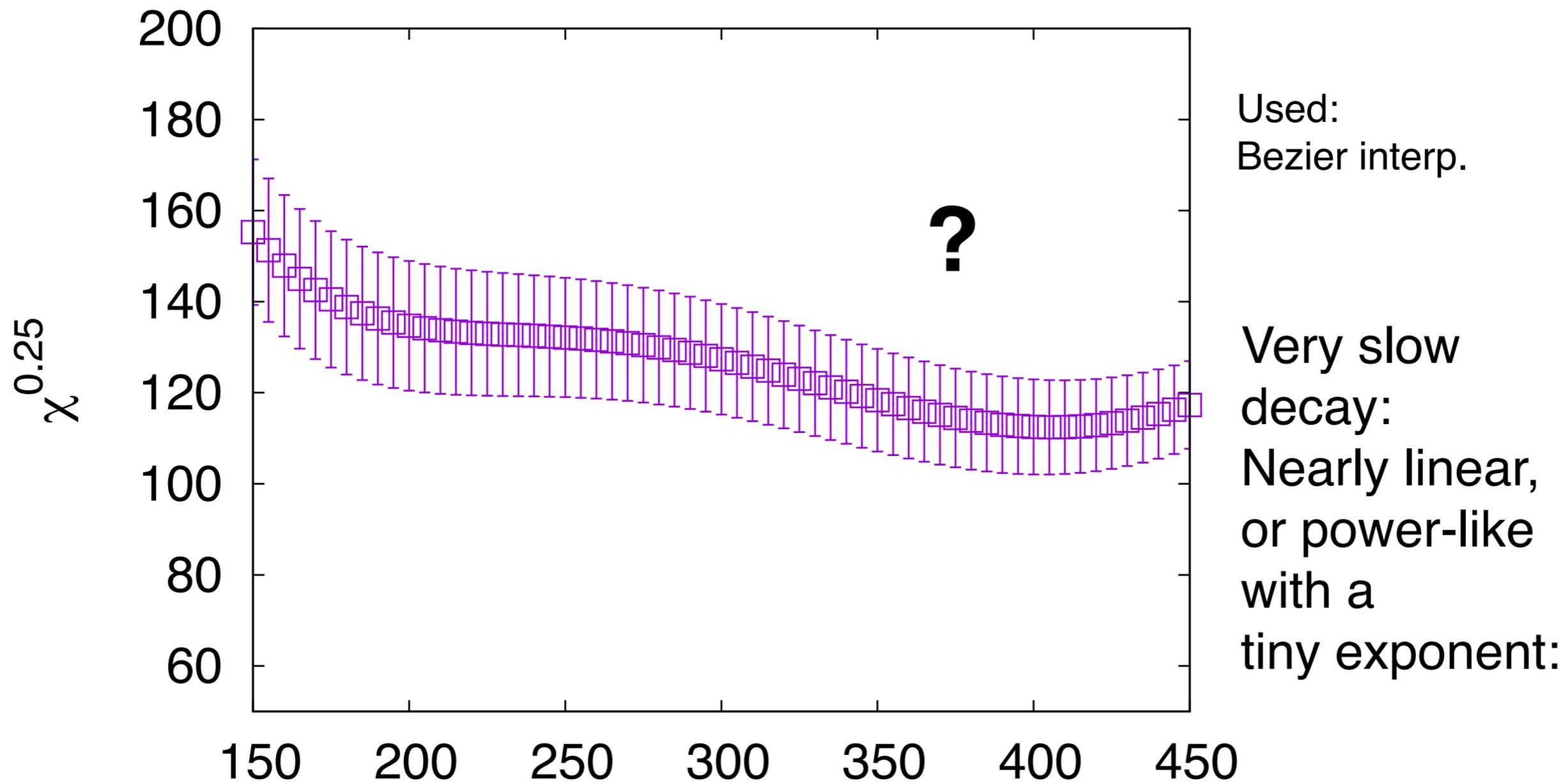
Continuum limit:

- in principle independent on flow limit
- we need to interpolate results at fixed scale to match T

$$\chi(T, m_\pi) = \lim_{a \rightarrow 0} \chi^{1/4}(T, a, m_\pi, t_x)$$

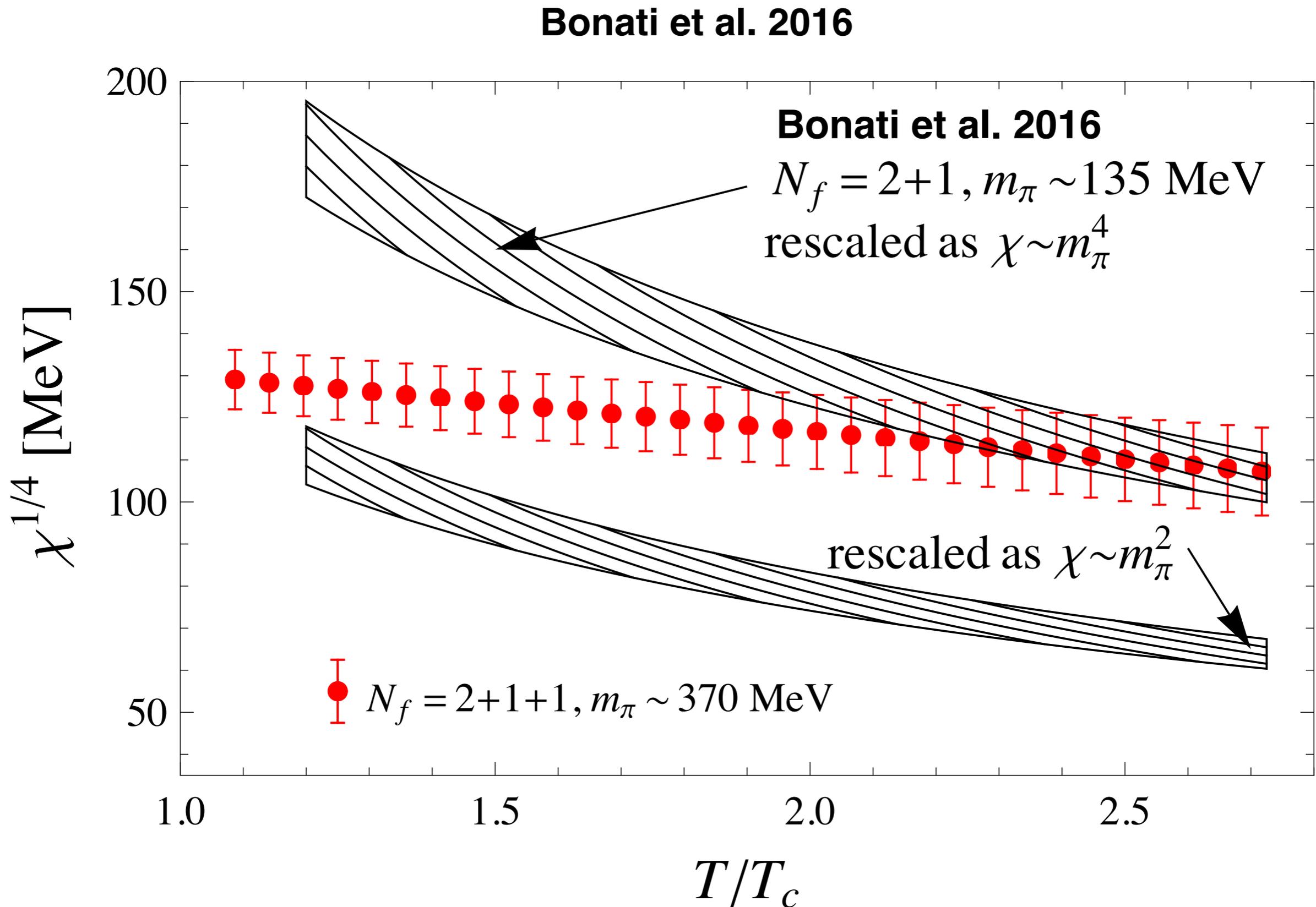
$$\chi^{1/4}(T, a, m_\pi, t_x) = \chi^{1/4}(T, m_\pi) + a^2 k(T, t_x)$$

Continuum results for $m_\pi = 370$ MeV

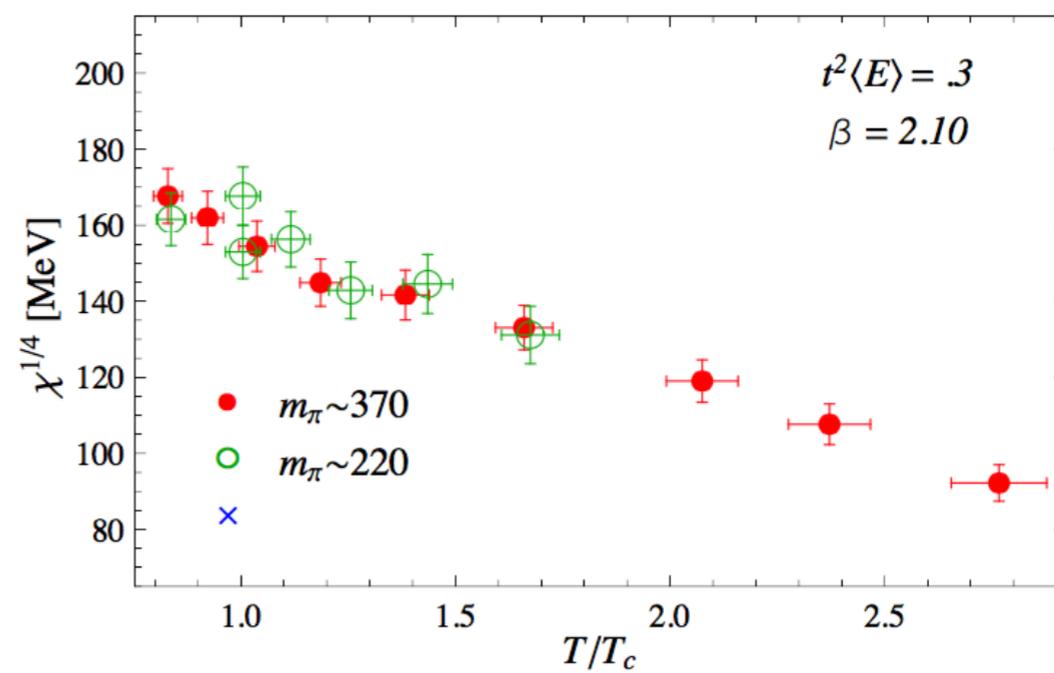
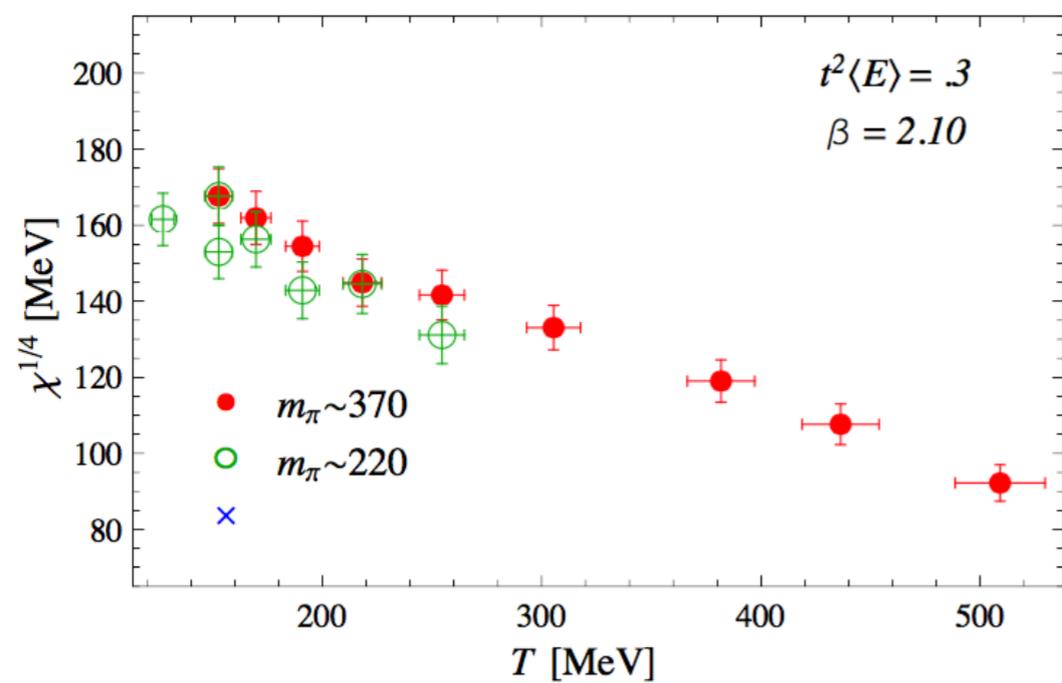
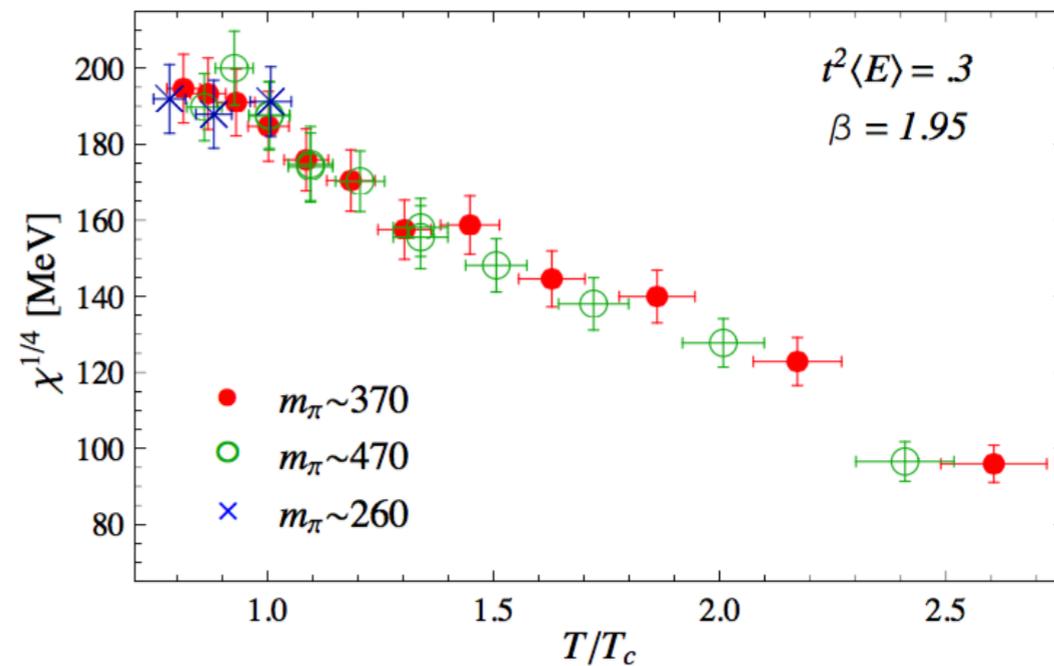
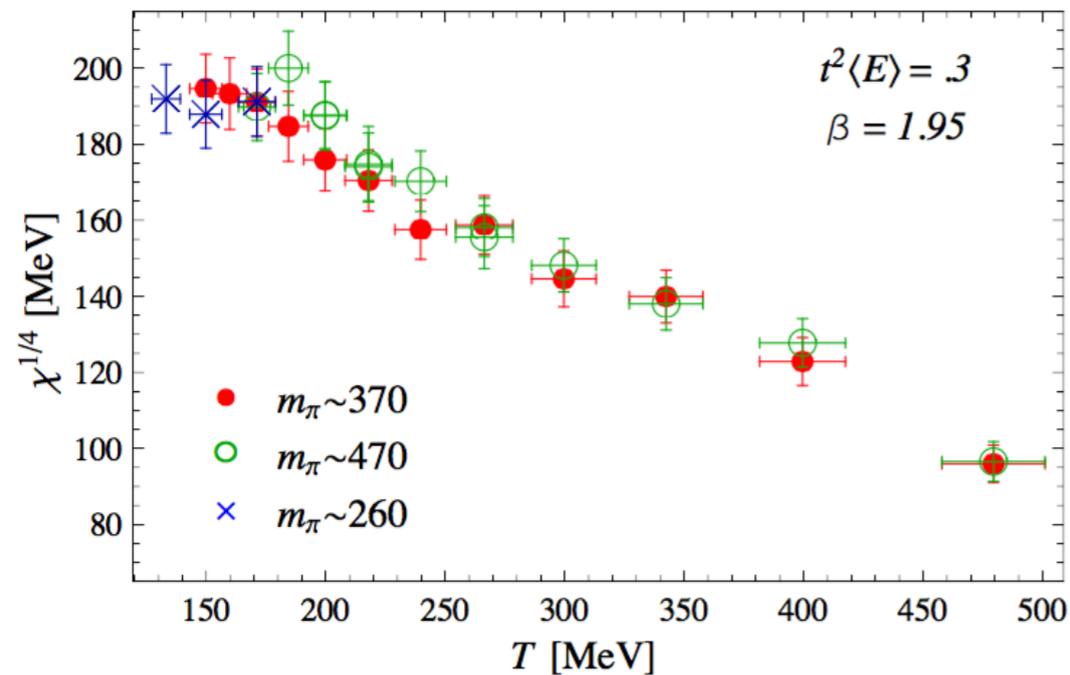


$$\chi(T)^{0.25} \simeq aT^{-0.26} \simeq T, T > 200 \text{ MeV}$$

A mass rescaling appears to work nicely



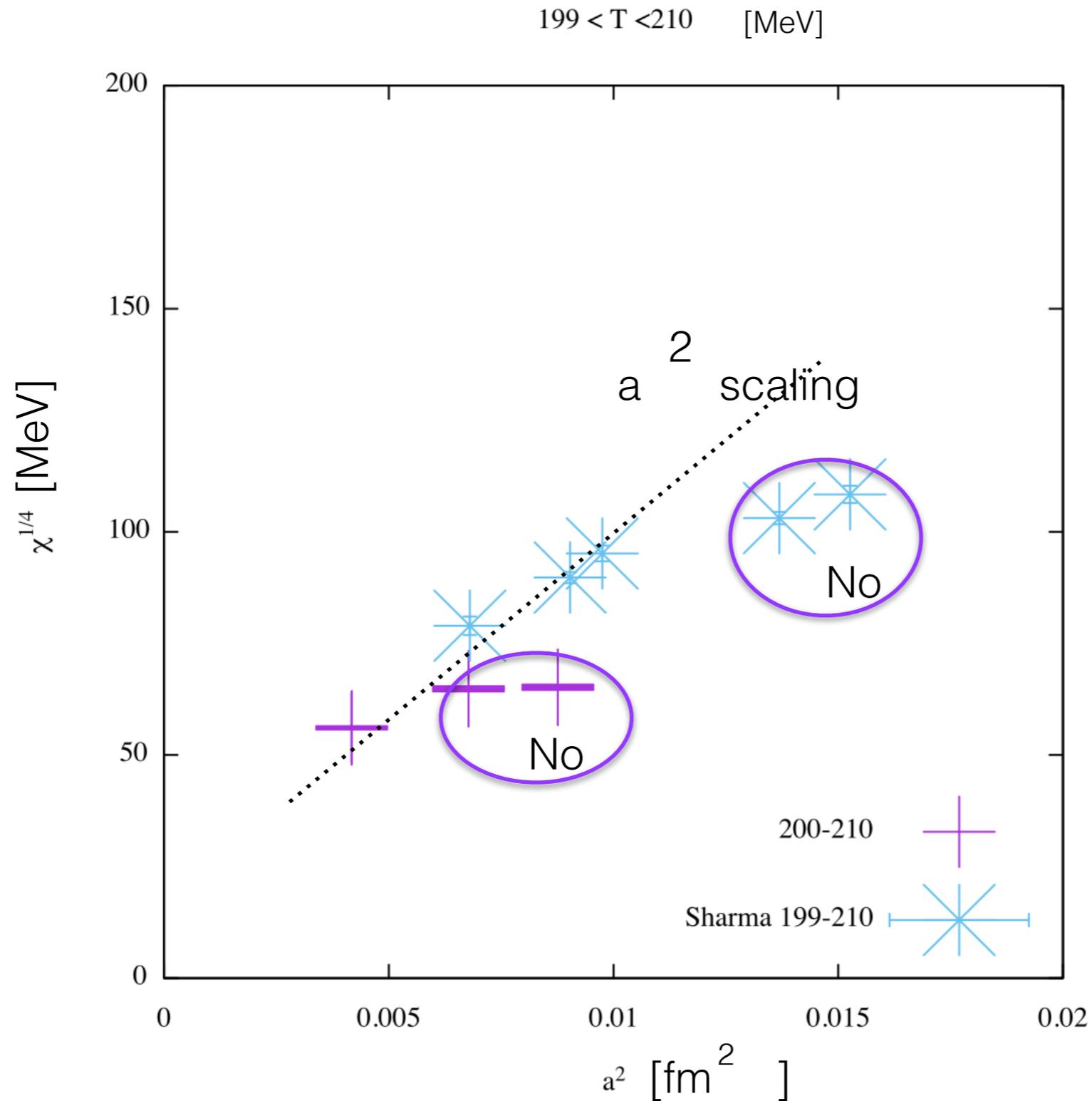
However: there is
no mass
dependence..



Possible explanation : strong scaling violations

Comparison with BNL results

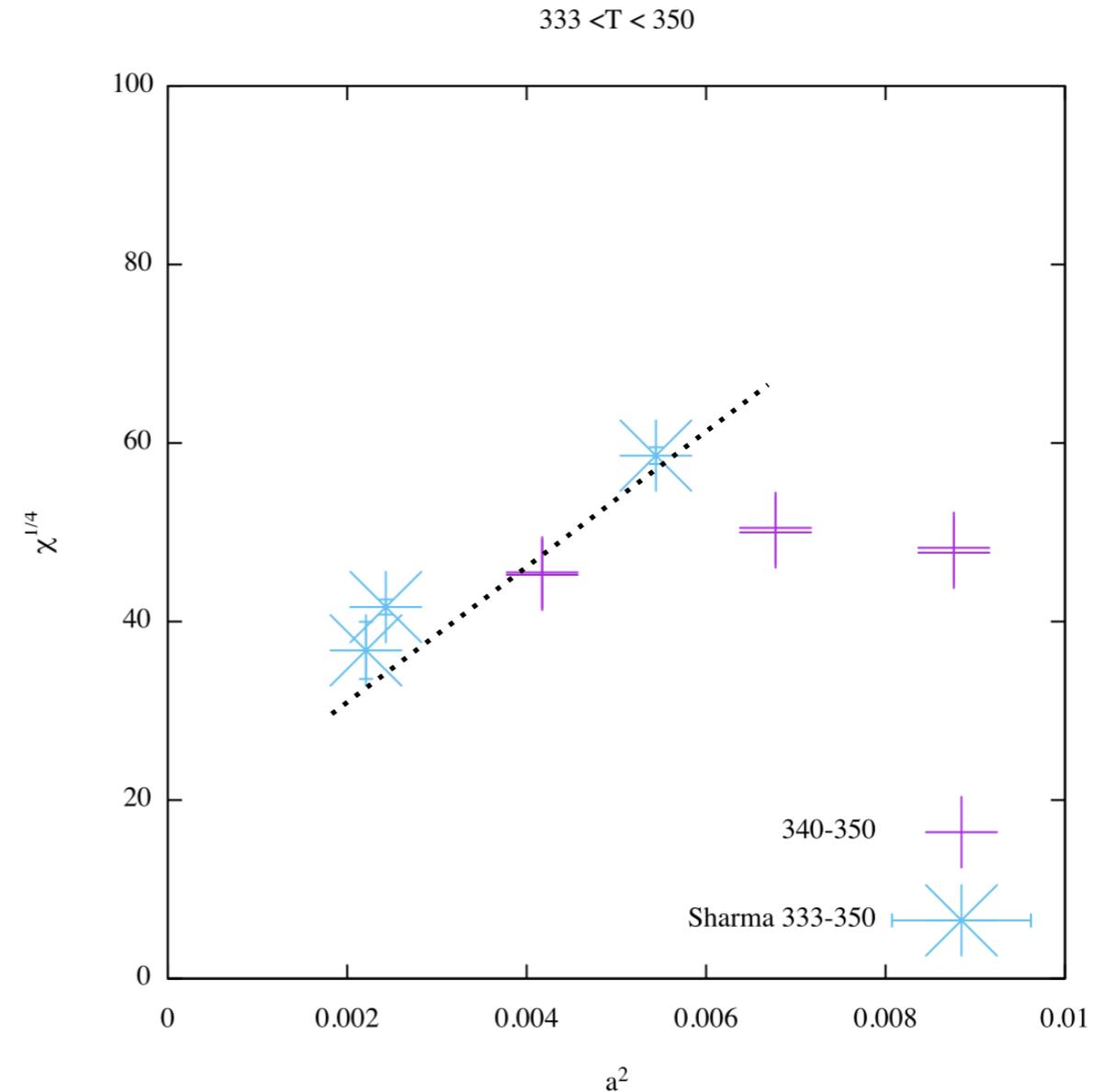
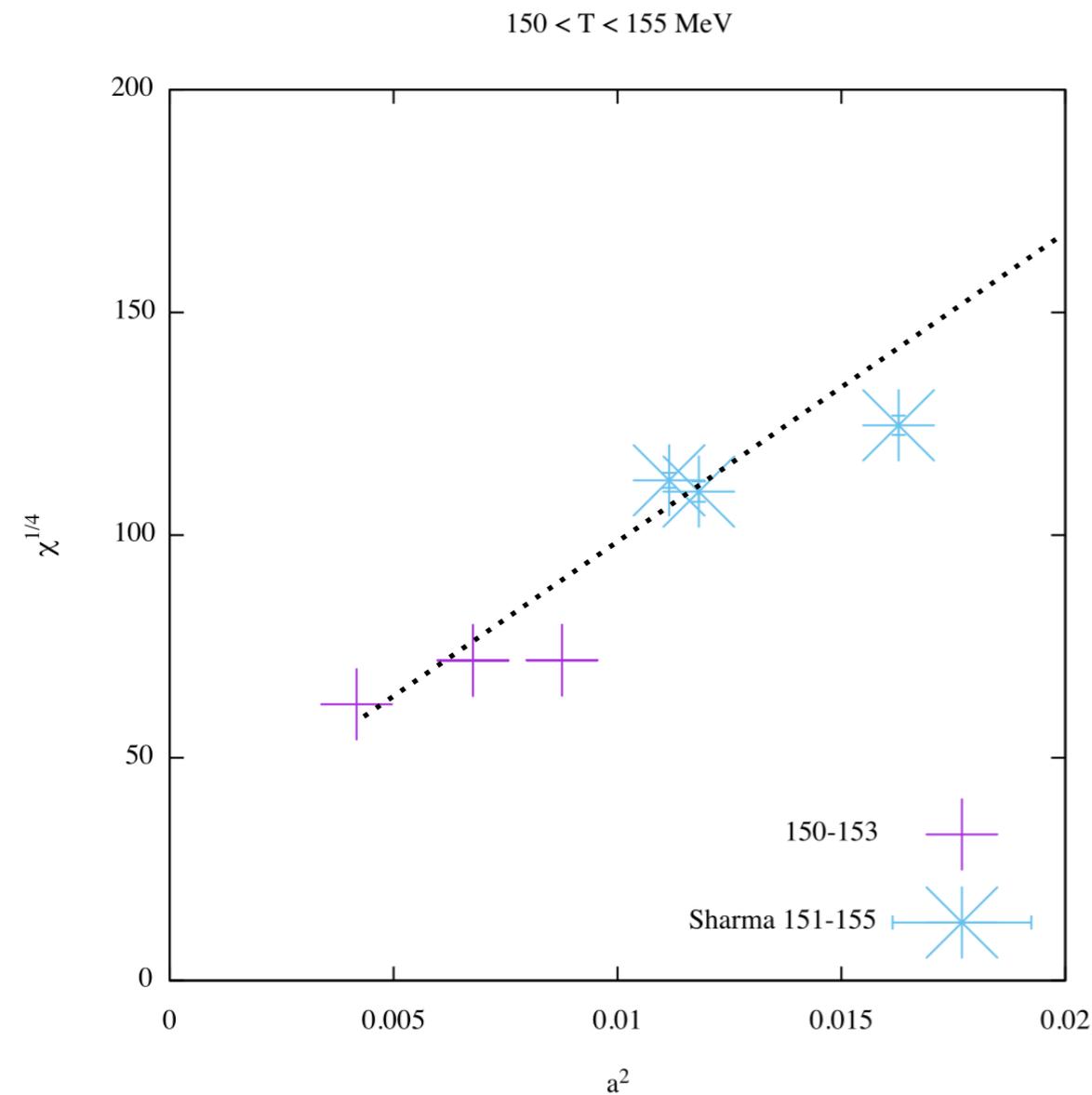
numerical data courtesy *S. Sharma*



Indication of corrections to a^2 scaling on our two coarser lattices

Comparison with BNL results (contn'd)

numerical data courtesy *S. Sharma*

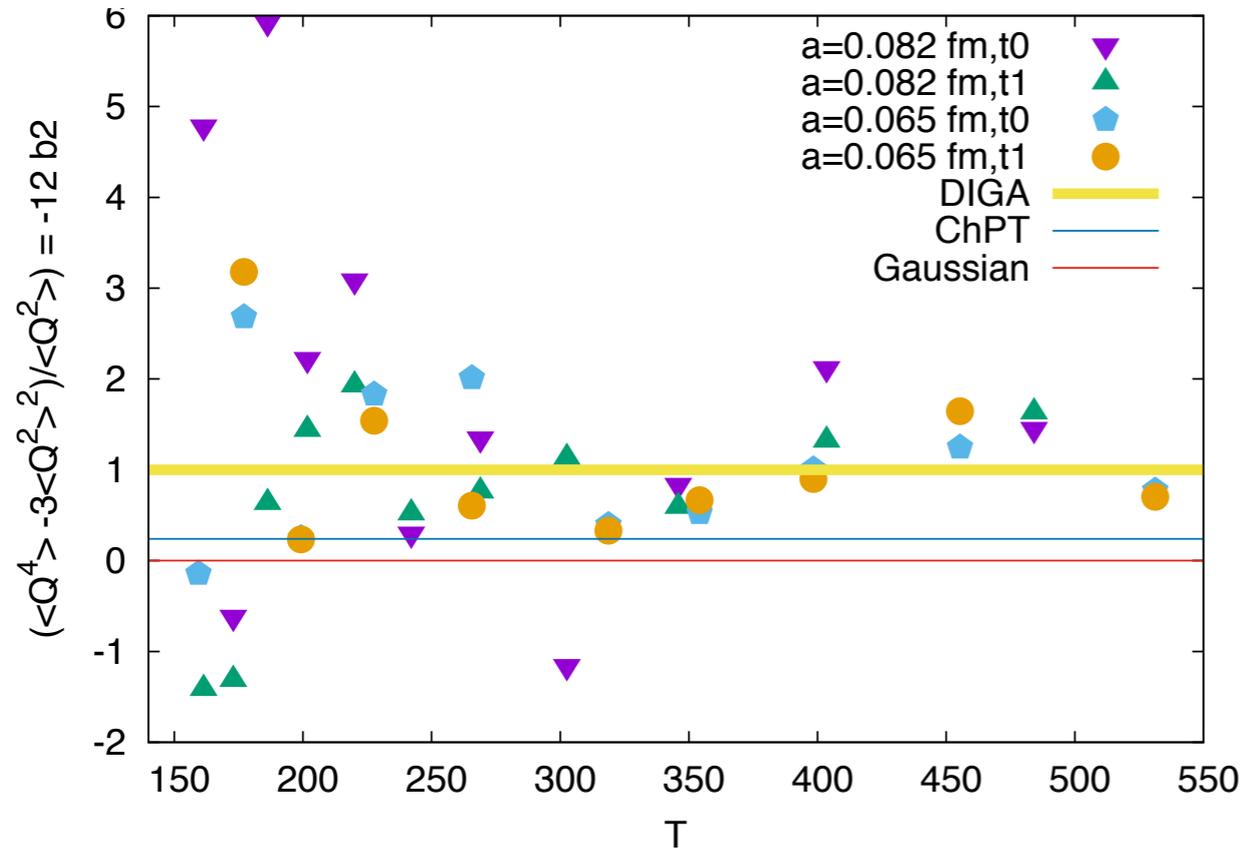


Consistent trend for other temperatures: on our finer lattice the corrections to a^2 scaling seem moderate

Instanton potential - cumulants' ratio b2

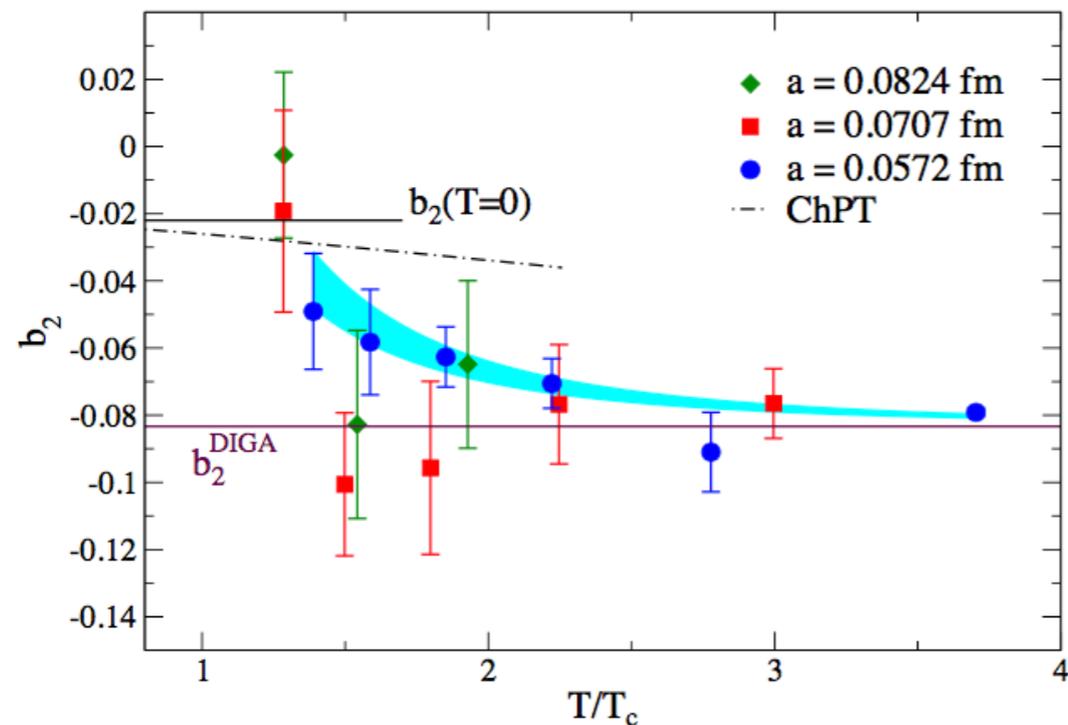
DIGA predicts

$$F(\theta, T) - F(0, T) = \chi(T)(1 - \cos(\theta)) \longrightarrow b_2 = -1/12$$



$$b_2 = -1/12$$

DIGA limit for $T > 350$ MeV



Consistent with Bonati et al.

Results II

Fermionic operator

$$n_L - n_R = Q_{top}$$

$$m \int d^4x \bar{\psi} \gamma_5 \psi = Q_{top}$$

Topological and chiral susceptibility

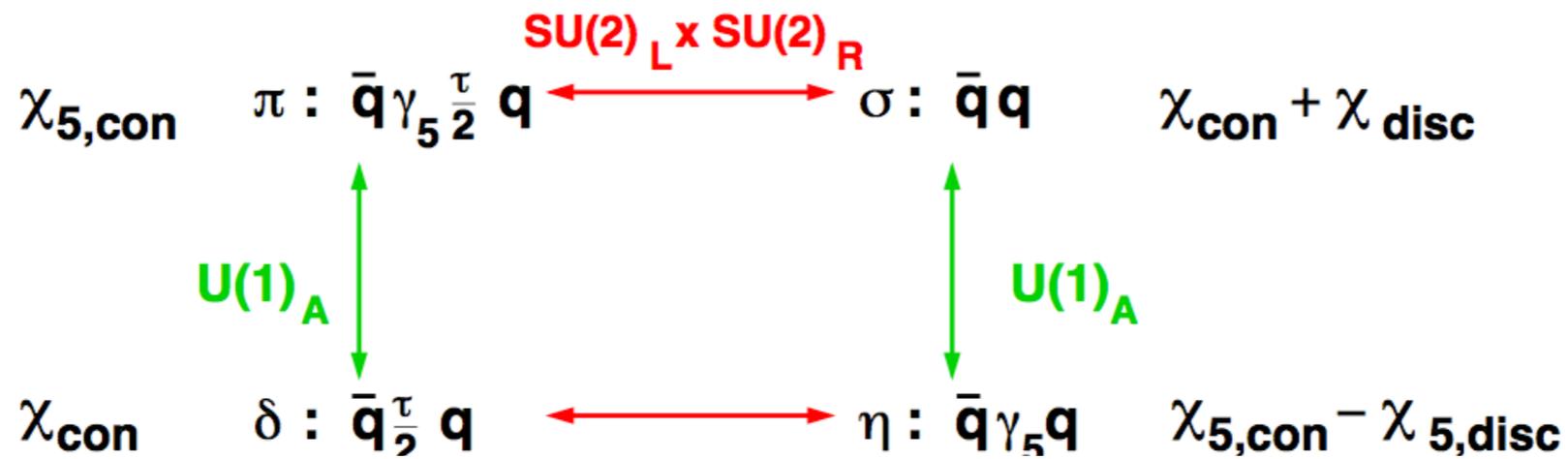
Kogut, Lagaë, Sinclair 1999

HotQCD, 2012

$$\chi_{top} = \langle Q_{top}^2 \rangle / V = m_l^2 \chi_{5,disc}$$

From:

$$m \int d^4x \bar{\psi} \gamma_5 \psi = Q_{top}$$

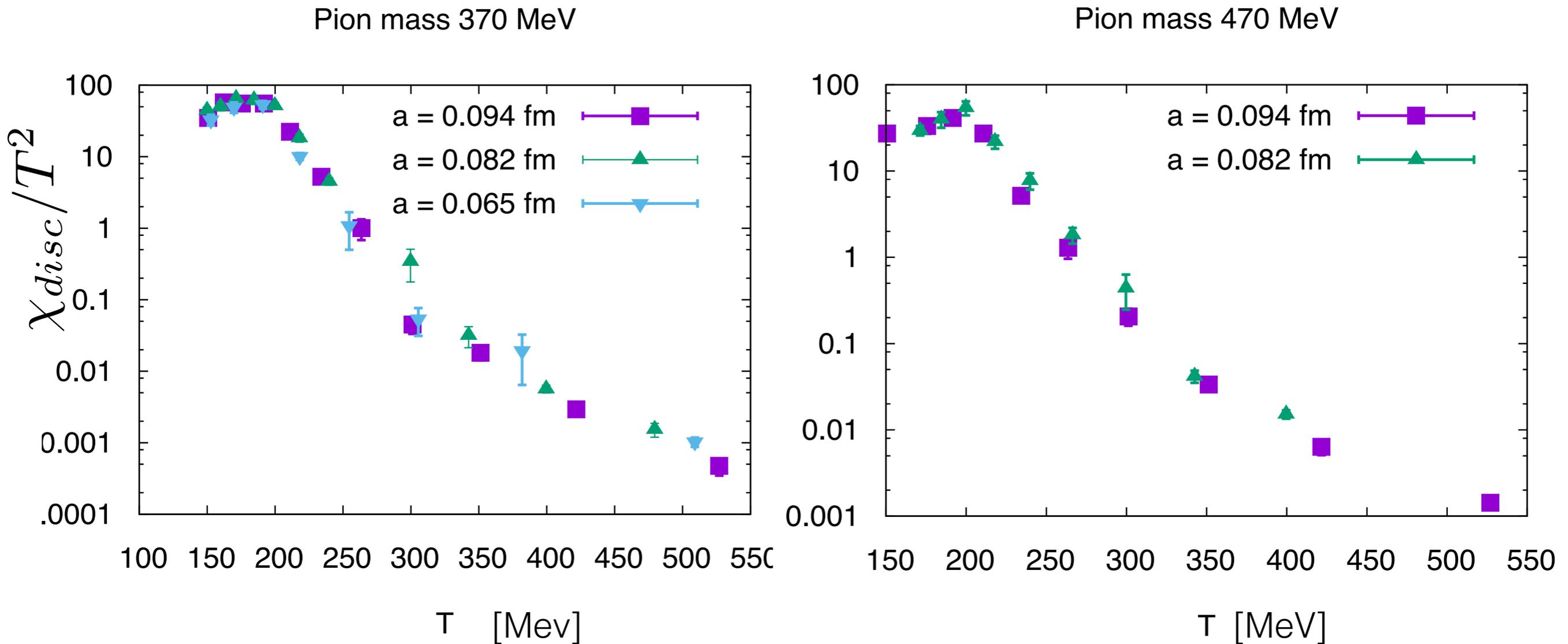


$$\chi_{\pi} - \chi_{\delta} = \chi_{disc} = \chi_{5,disc}, \quad \text{for } T \geq T_c, m_l \rightarrow 0$$

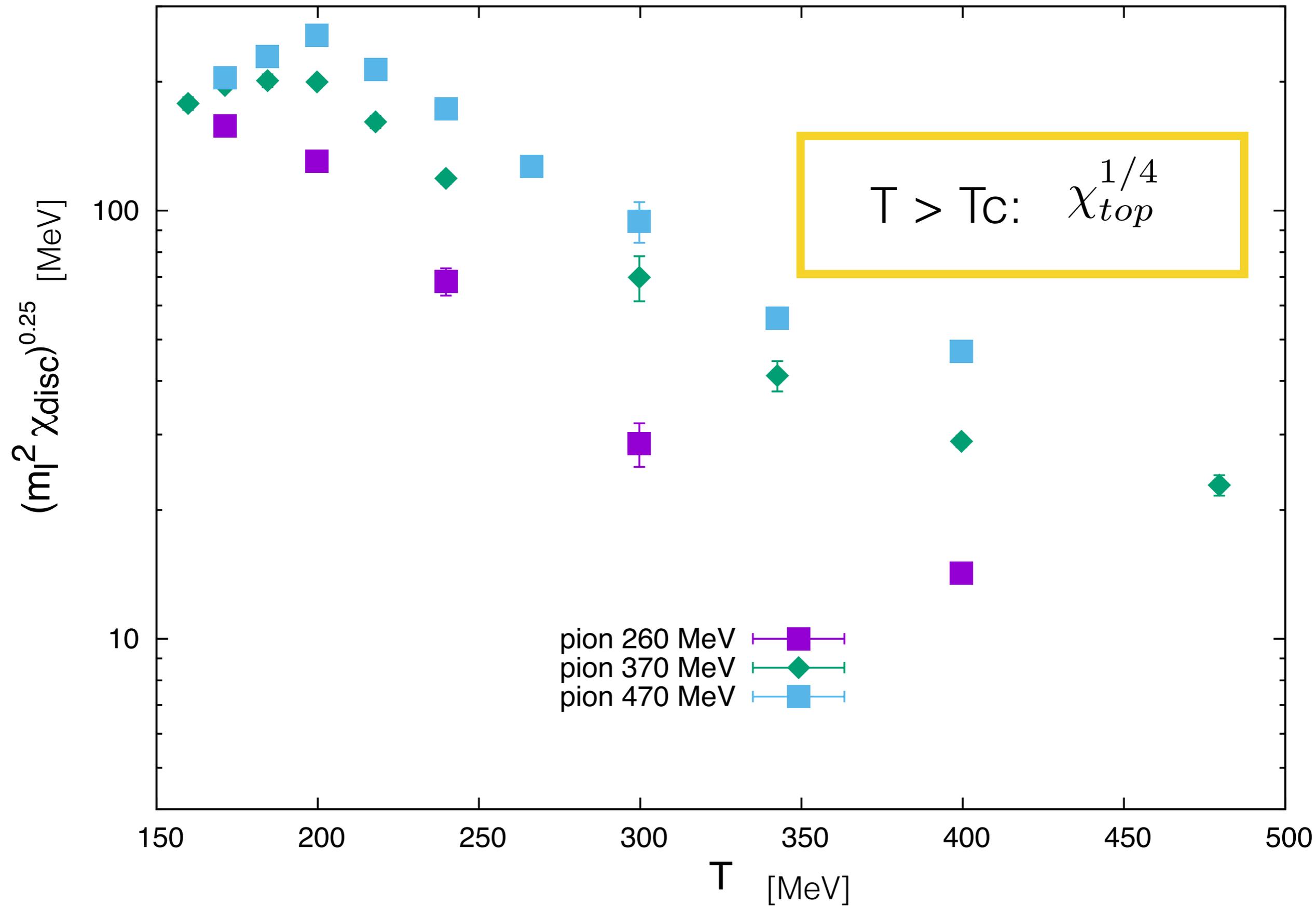
$$\chi_{top} = \langle Q_{top}^2 \rangle / V = m_l^2 \chi_{disc}$$

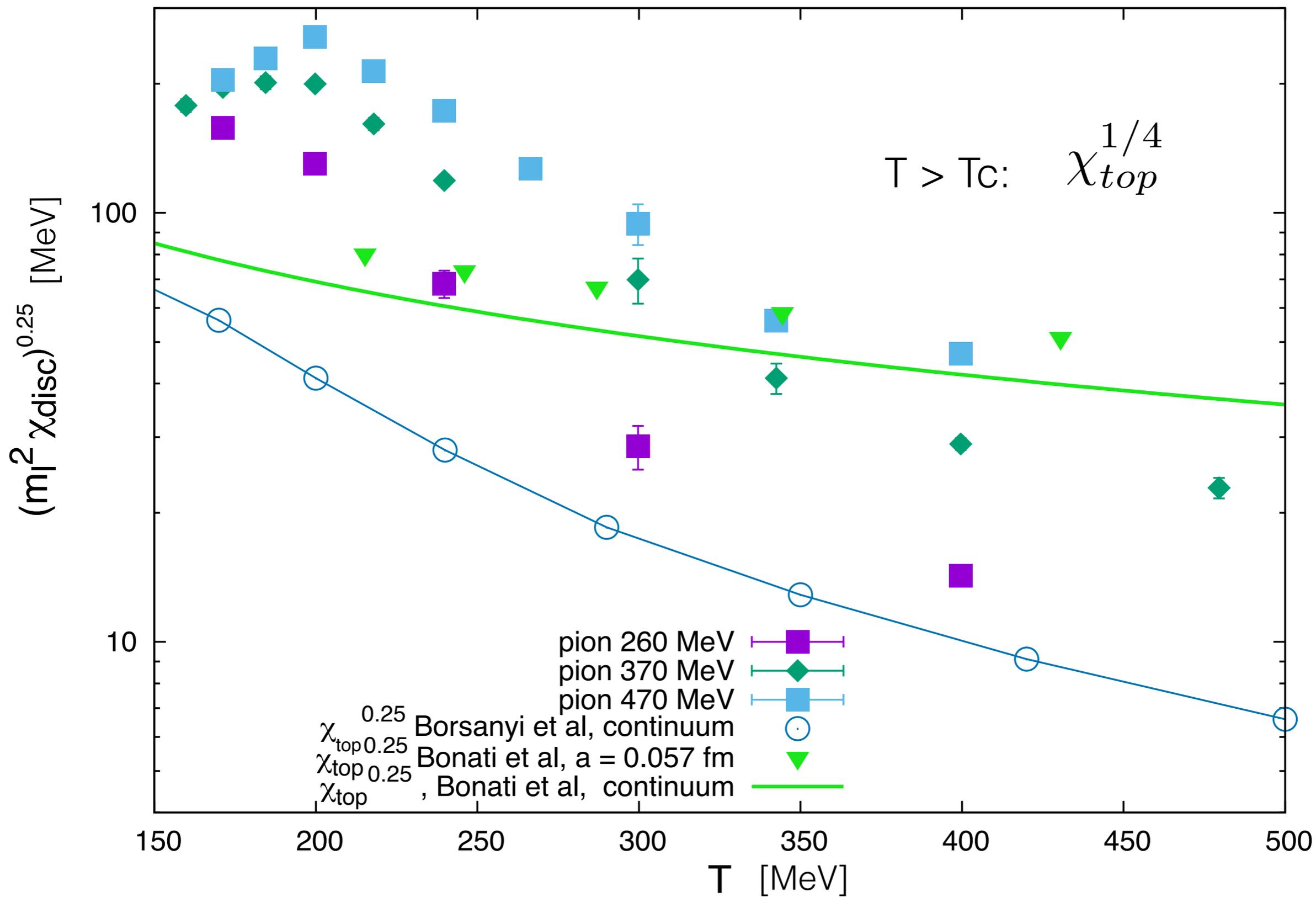
$$T > T_{U(1)_A} \simeq T_c$$

Chiral susceptibility

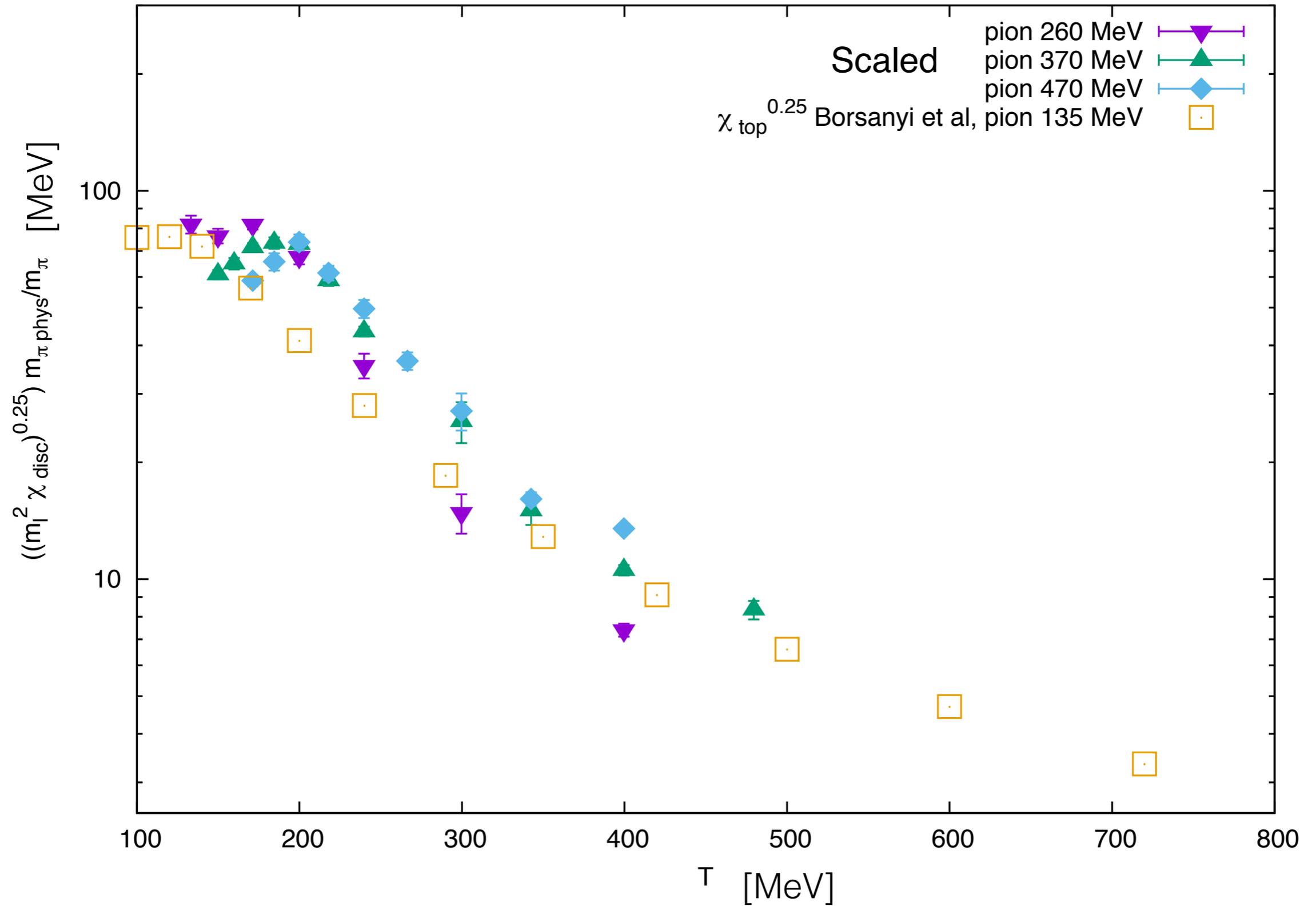


Within errors, no discernable spacing dependence





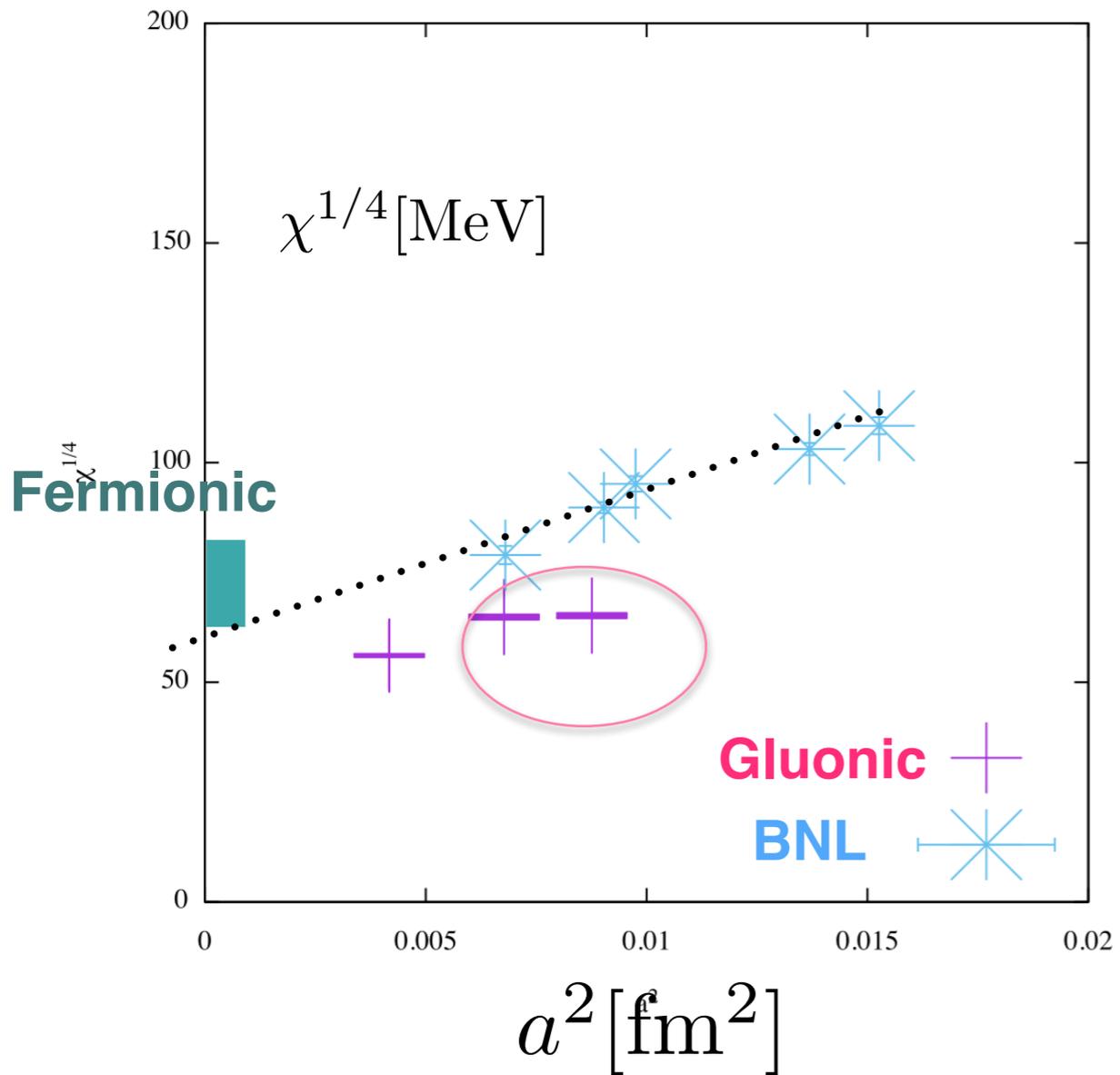
Results for physical pion mass



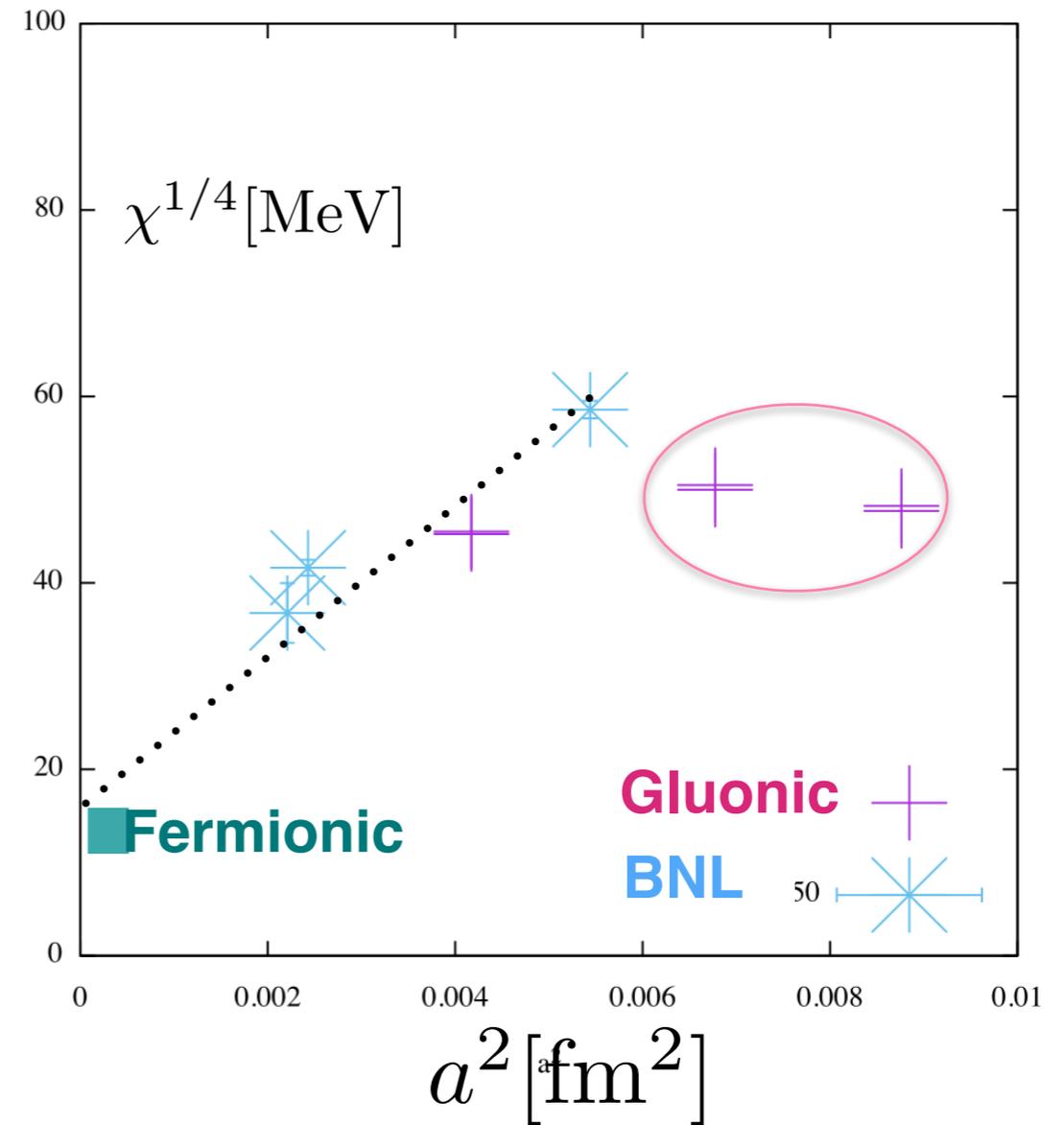
Comparison with BNL results including fermionic results

numerical data courtesy *S. Sharma*

199 < T < 210 MeV



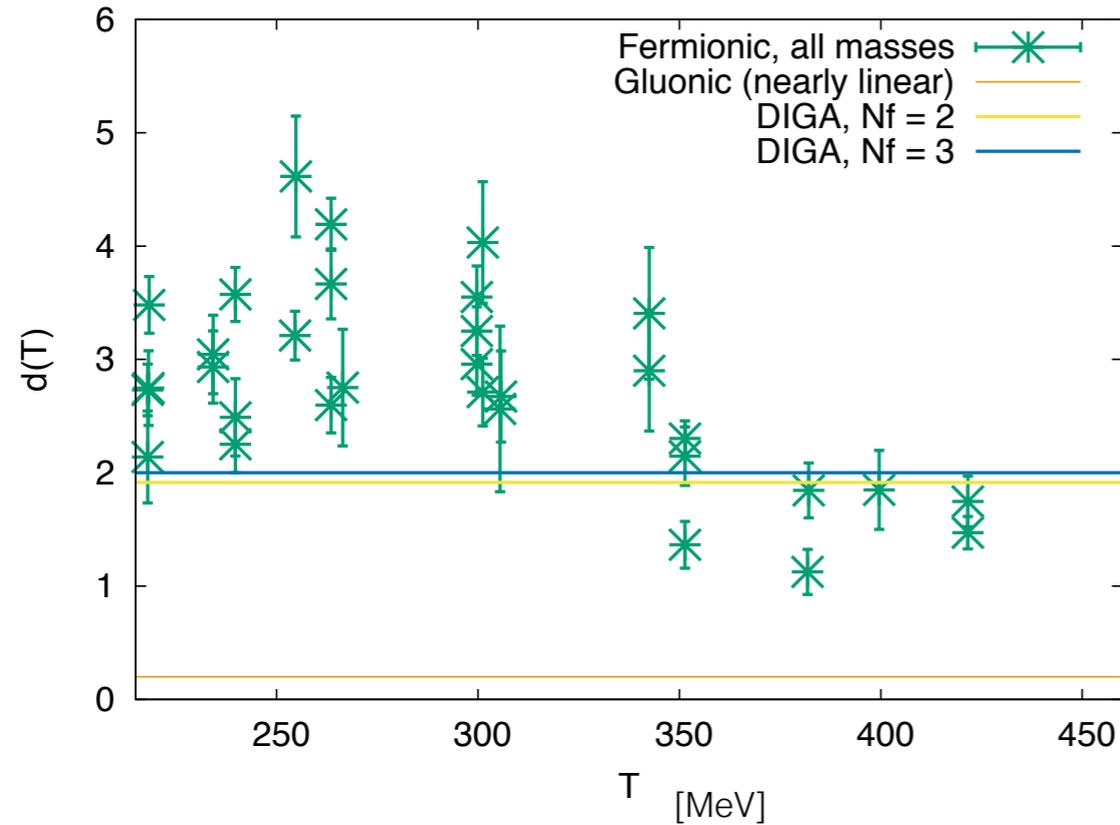
333 < T < 350 MeV



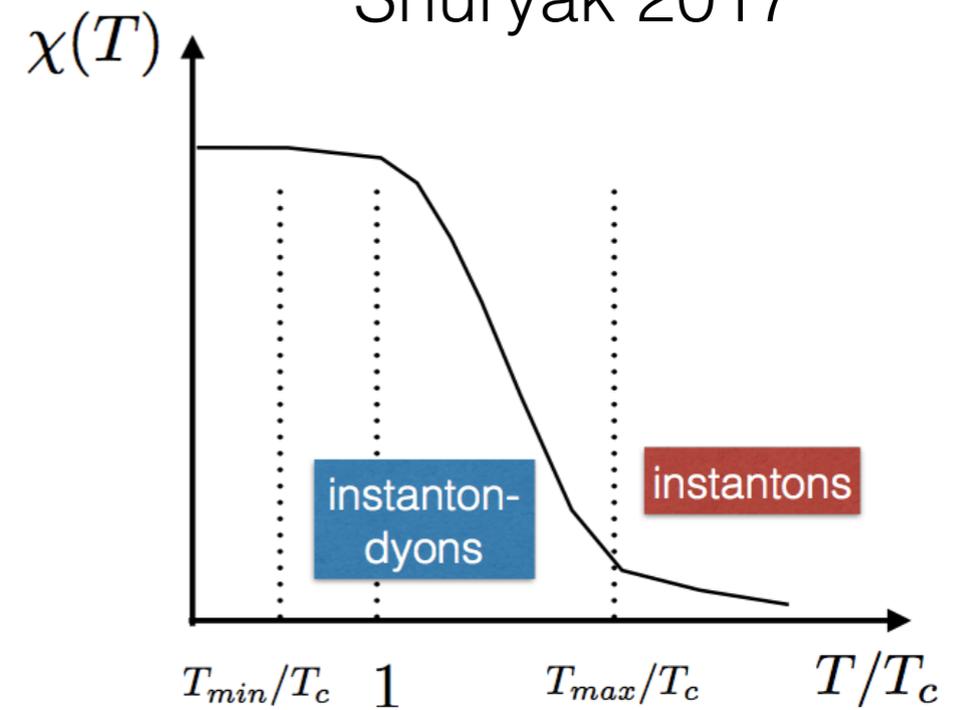
dotted lines to guide the eye

Effective exponent : $d(T) = -T \frac{d}{dT} \ln \chi^{0.25}(T)$

$$\chi^{0.25}(T) = aT^{-d(T)}$$



Possibly consistent with instant-dyon?
Shuryak 2017

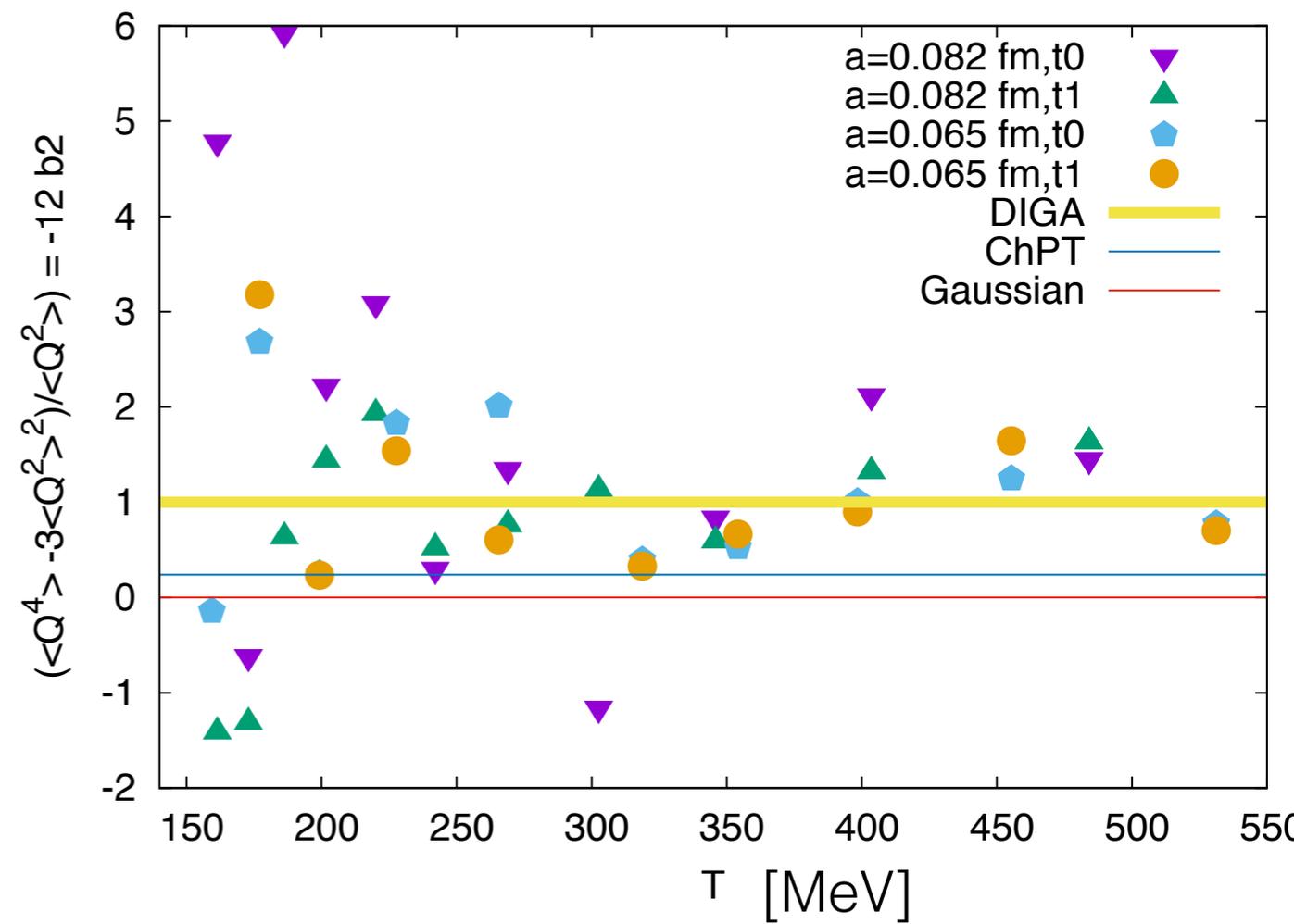
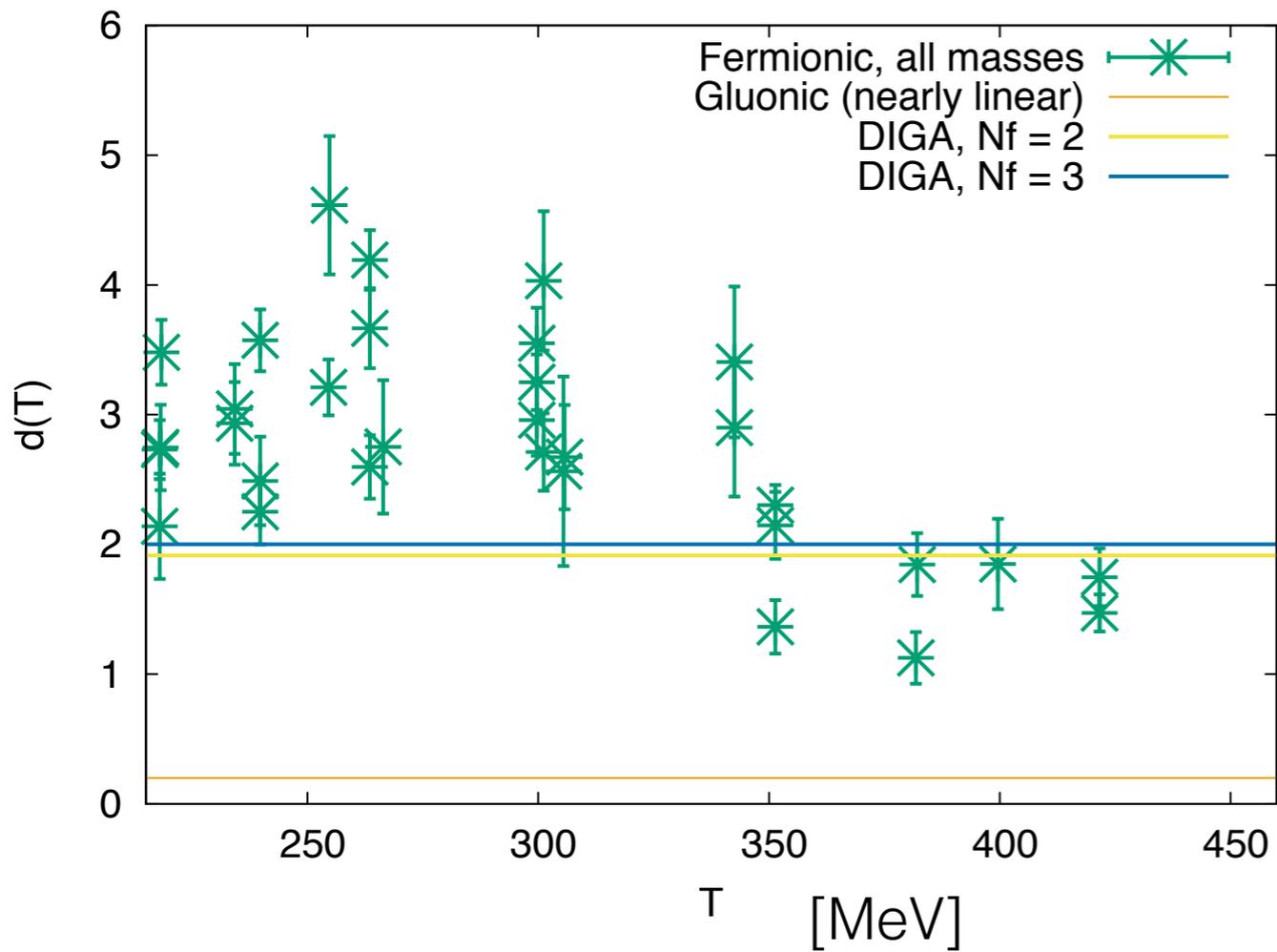


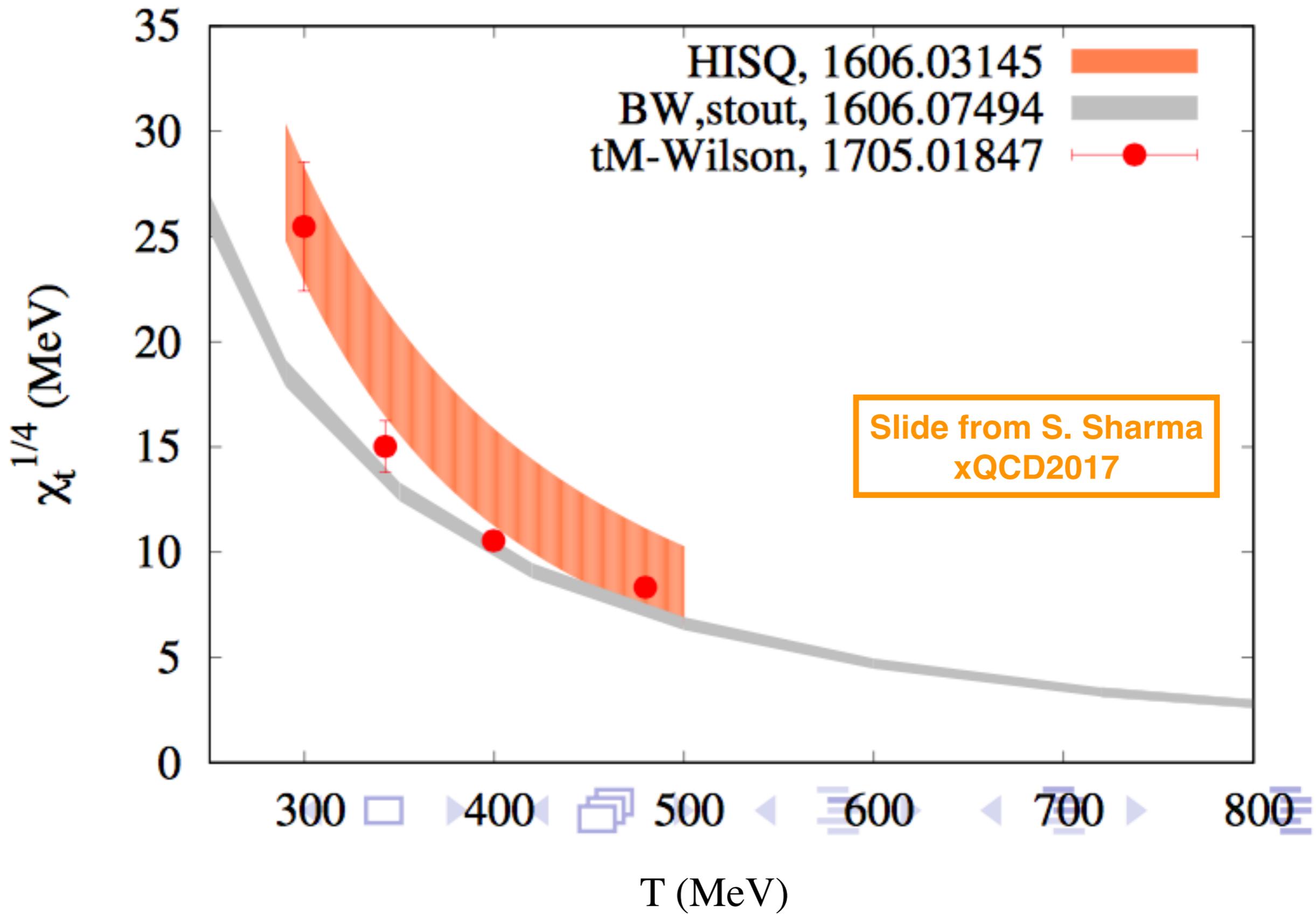
Faster decrease before DIGA sets in

Effective exponent :

$$\chi_{top}^{1/4} = aT^{-d(T)}$$

Same DIGA onset seen in $b_2 \approx 350$ MeV





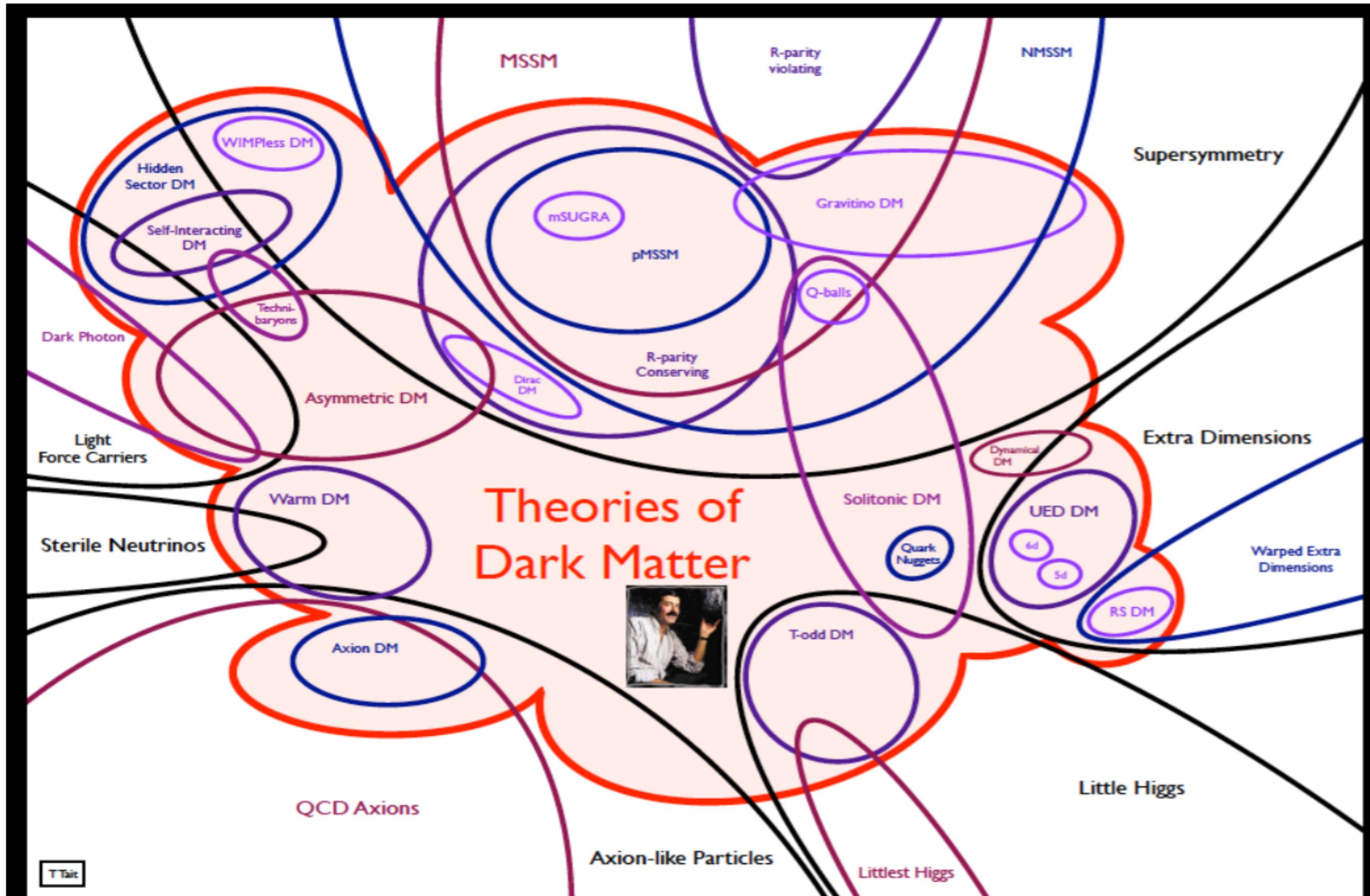


A window on the axions

Inspiring paper:

Berkowitz, Buchhoff, Rinaldi **Phys.Rev. D92 (2015) no.3, 034507**

Theory landscape (From Tim Tait, Snowmass)



The Equation of State of the Quark Gluon Plasma paves the way to Cosmology

Cold Dark Matter candidates might have been created after the inflation

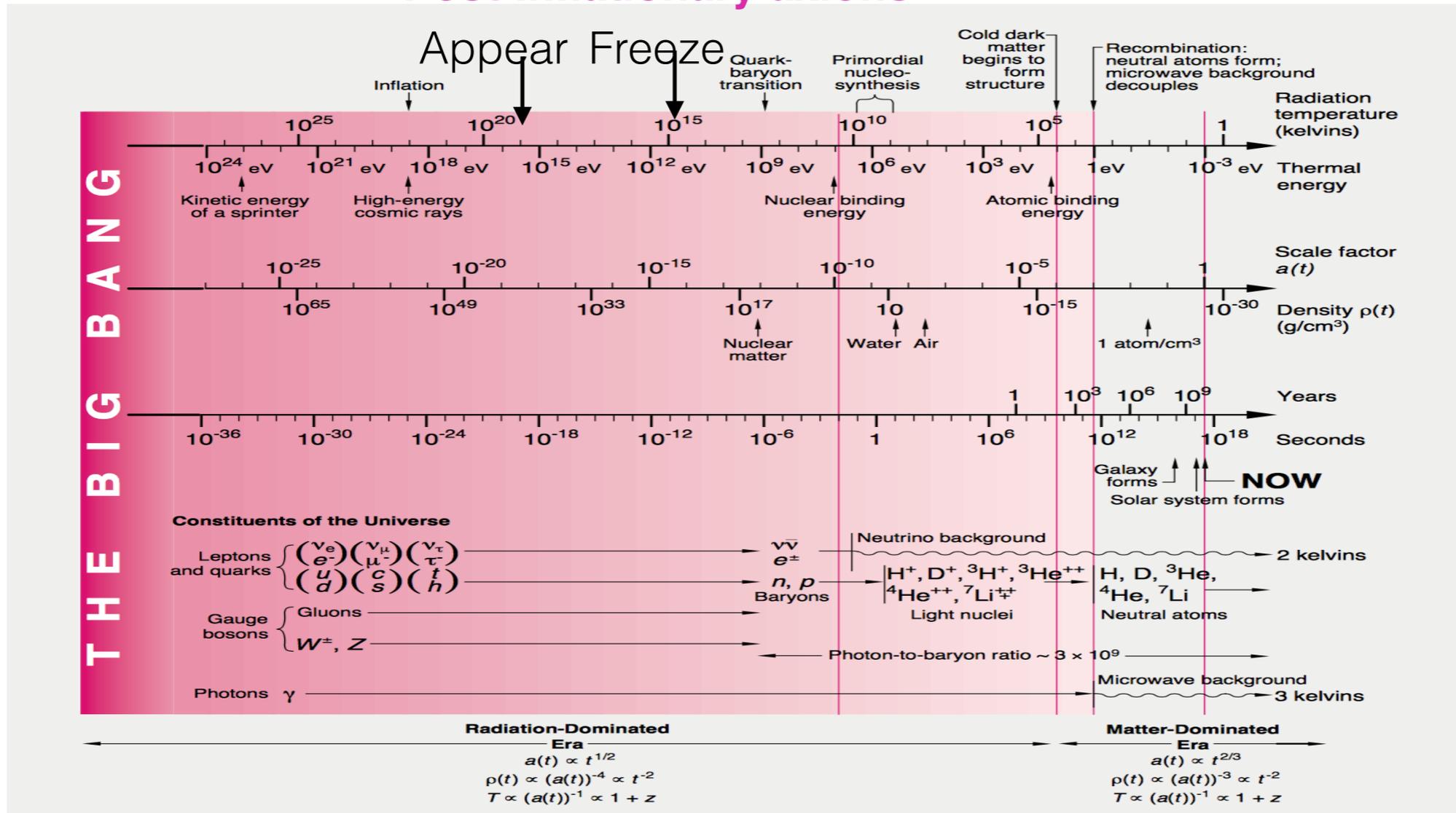
Several CDM candidates are highly speculative - but one, **the axion**, is

Theoretically well motivated in QCD

Amenable to quantitative estimates once QCD topological properties are known:

Post-inflationary axions

$$m_a(T) = \sqrt{\chi(T)} / f_a$$



Axions 'must' be there: solution to the strong CP problem

$$\mathcal{L}_{QCD}(\theta) = \mathcal{L}_{QCD} + \frac{g^2 \theta}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}^a F_{\rho\sigma}^a$$

Admitted but $\theta < 10^{-9}$

$$Q = \int d^4x \frac{g^2}{32\pi^2} \text{tr} F \tilde{F}$$

Postulate axions, coupled to Q:

$$\mathcal{L}_{\text{axions}} = \frac{1}{2} (\partial_\mu a)^2 + \left(\frac{a}{f_a} + \theta \right) \frac{1}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$$

$$Z_{QCD}(\theta, T) = \int [dA][d\psi][d\bar{\psi}] \exp \left(-T \sum_t d^3x \mathcal{L}_{QCD}(\theta) \right) = \exp[-V F(\theta, T)]$$

Axion potential

$$m_a^2(T) f_a^2 = \left. \frac{\partial^2 F(\theta, T)}{\partial \theta^2} \right|_{\theta=0} \equiv \chi(T),$$

Time from Big Bang

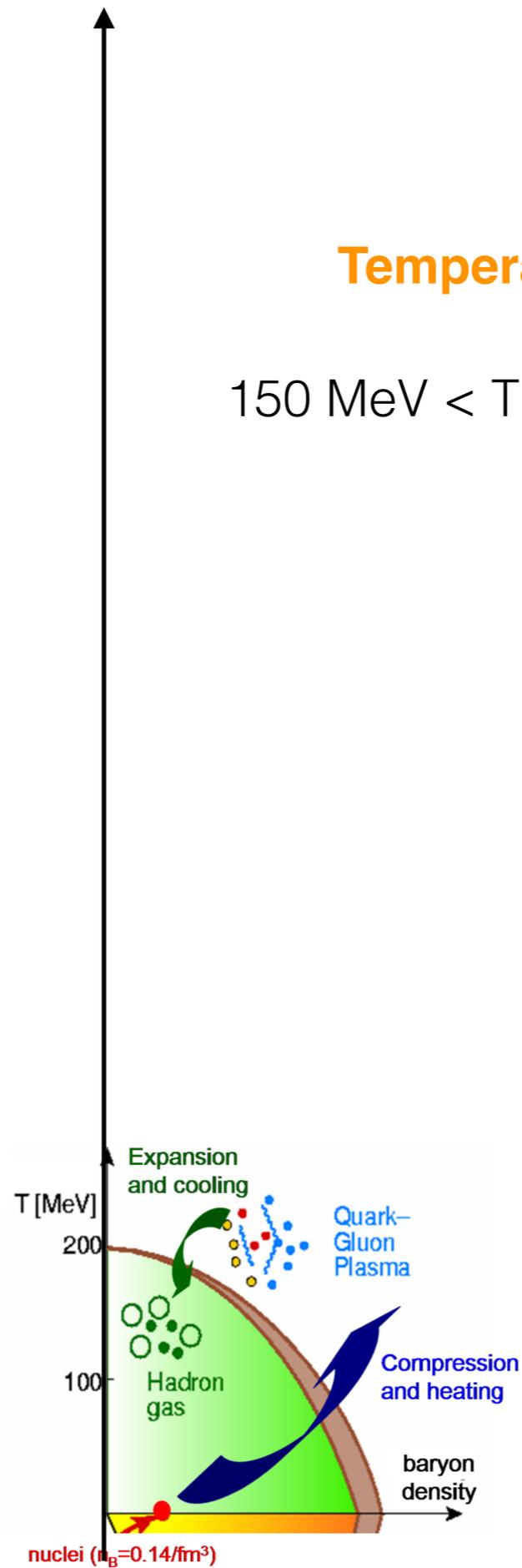
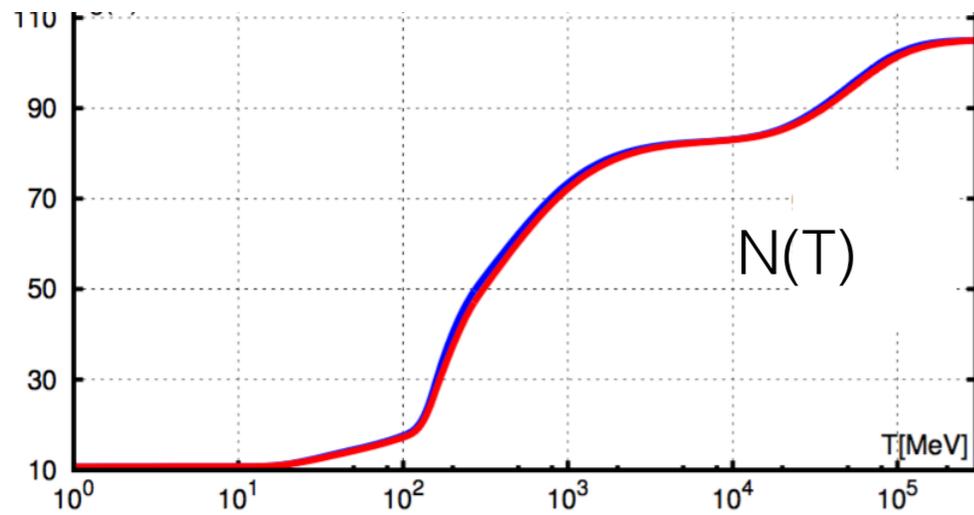


Temperatures

$$150 \text{ MeV} < T < 500 \text{ MeV}$$

..and beyond

Temperature and Time from BigBang are linked by the Equation of State

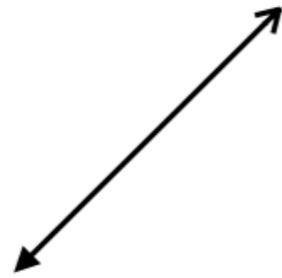


Time from Big Bang



Axions's freezout

$$3H(T) = m_a(T)$$



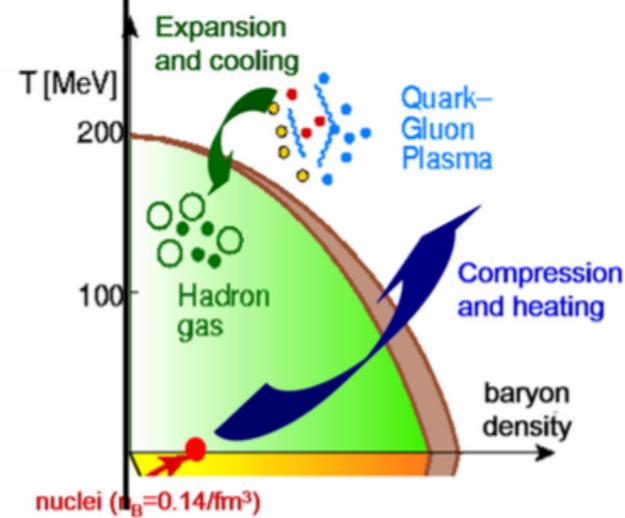
**Axions' mass
and density
today**

Temperature

Hubble parameter
 $H(T) \simeq T^2/M_P$

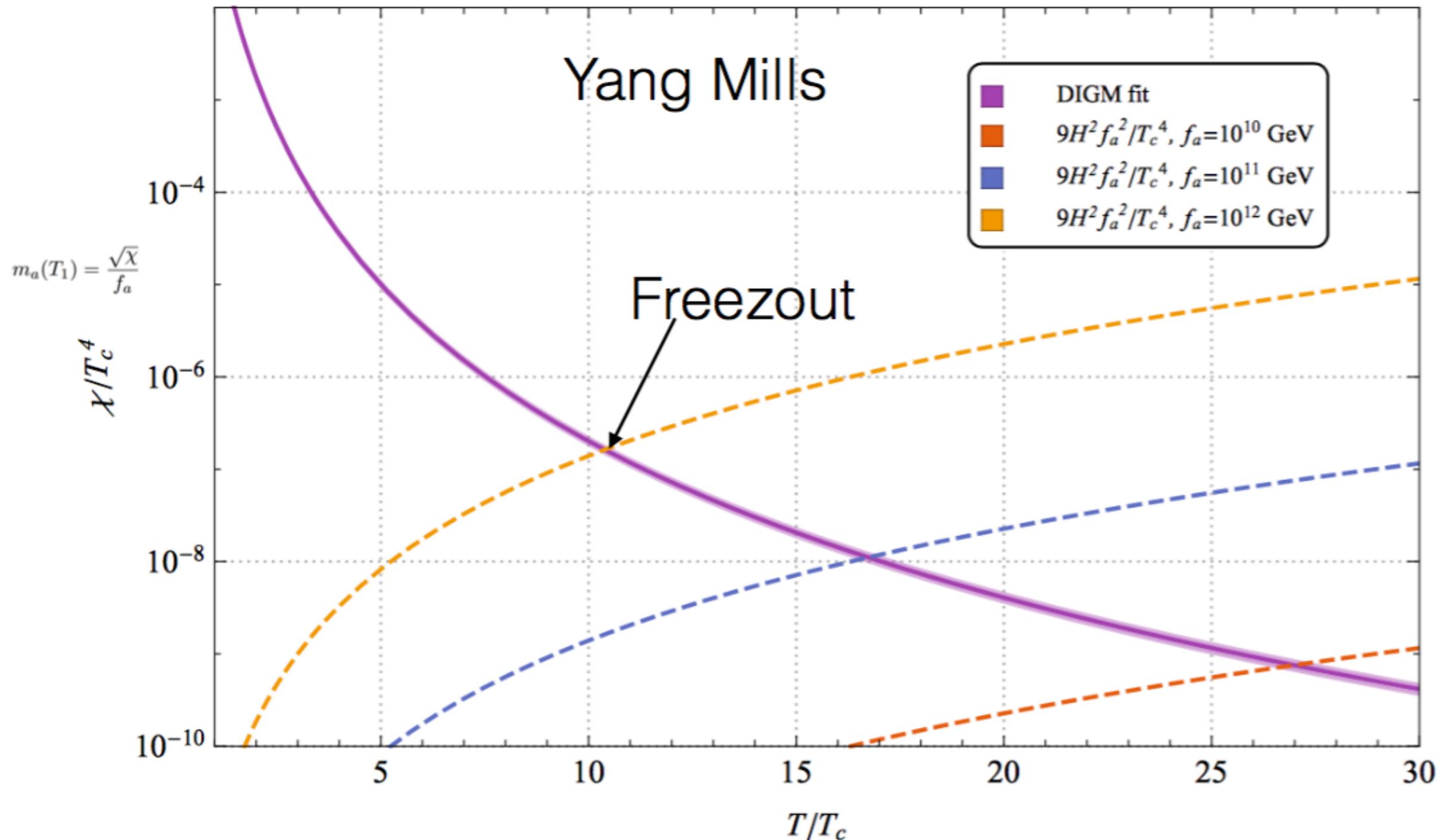
$$m_a(T) = \sqrt{\chi(T)}/f_a$$

**Quark Gluon Plasma:
Topology**



Axion freezout : $3H(T) = m_a(T) = \sqrt{\chi(T)}/f_a$

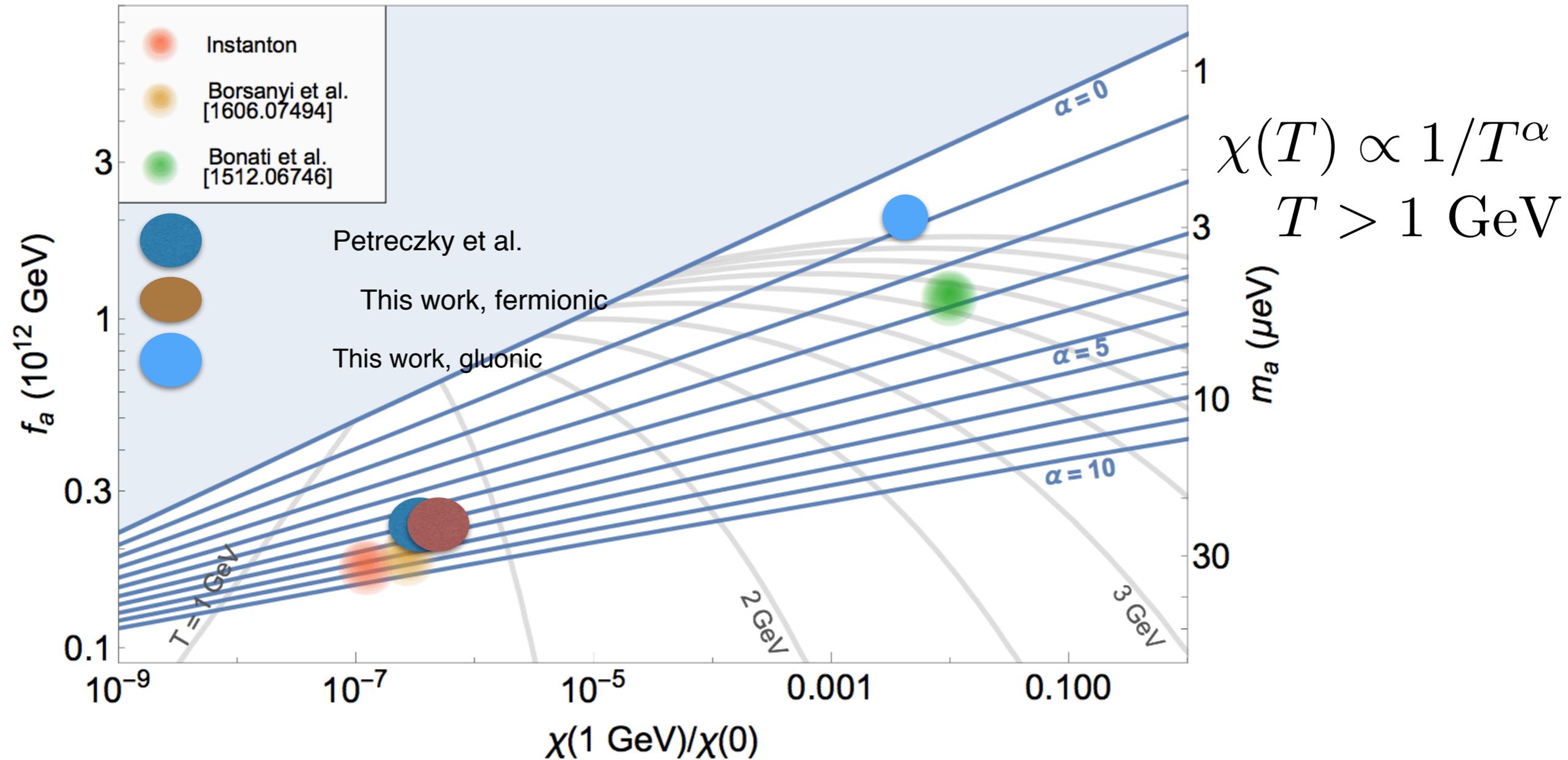
Berkowitz Buchoff Rinaldi 2015



Axion density at freezout controls axion density today

Needed assumption on
fraction of DM made of axions

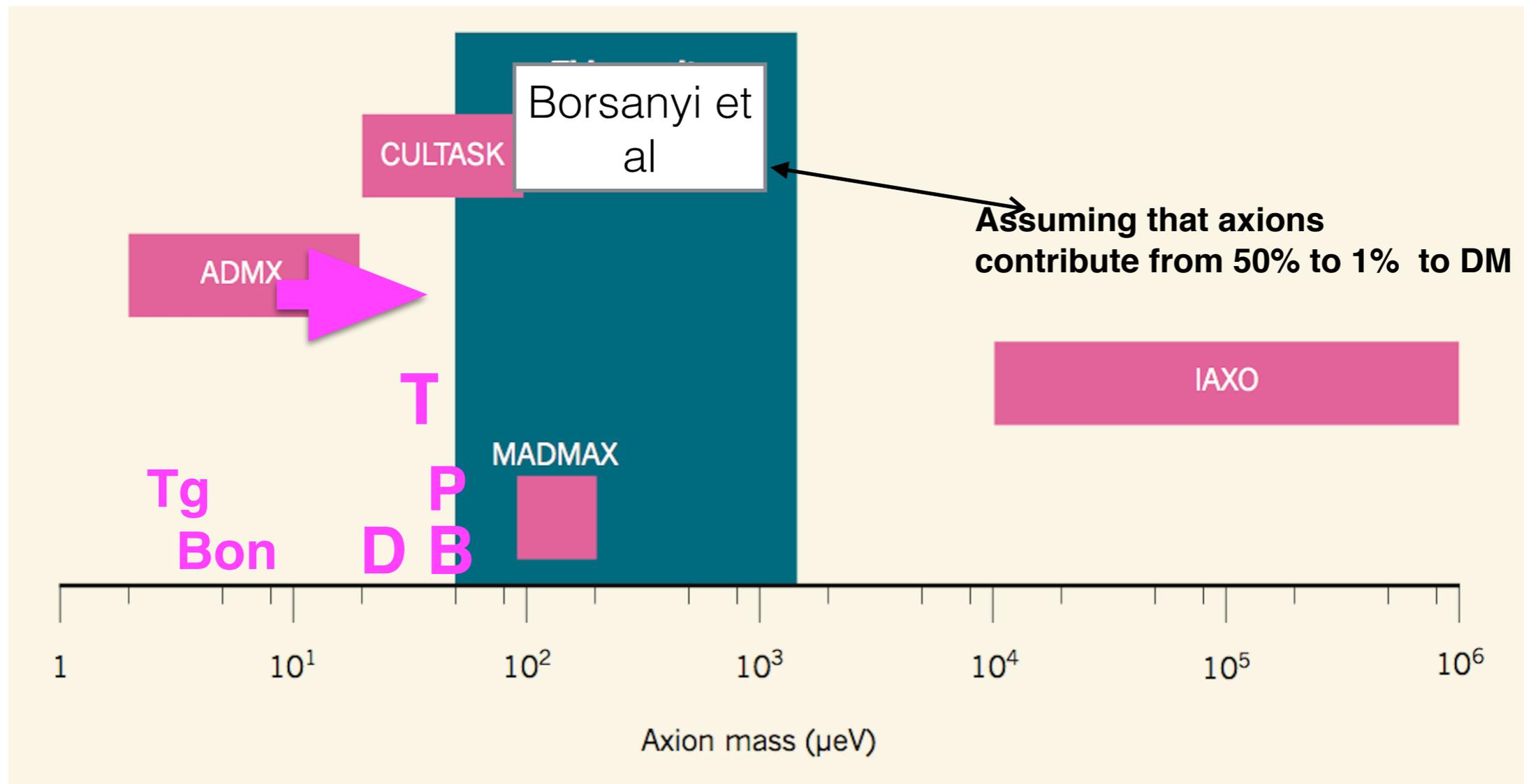
Assume: Axions make all of Dark Matter



PhD Thesis, G. Grilli di Cortona, Sissa 2016
(advisor G. Villadoro)

Lower limits on the axion mass assuming that axions make 100% of DM:

Tg: This work, gluonic; **Bon**: Bonati et al.; **D**: DIGA, **B**: Borsanyi et al.,
P: Petreczky et al., **T**: this work, fermionic



Updated from Nature N&V

Summary

- There is an emerging evidence that the QGP behaves as a DIGA for $T > 300$ MeV, but such evidence only comes from the exponent and b_2 . Could this agreement be accidental?
- Differences among different groups/actions should be understood and possibly resolved.
- The behavior around T_c is still under scrutiny, and should be clarified to better understand the approach to DIGA, the nature of the medium produced at the LHC, and the fate of the UA(1) symmetry.

..to get back to hadrons..

PHYSICAL REVIEW D

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1 MAY 1996

Return of the prodigal Goldstone boson

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(Received 14 July 1995)

We propose that the mass of the η' meson is a particularly sensitive probe of the properties of finite energy density hadronic matter and quark-gluon plasma. We argue that the mass of the η' excitation in hot and dense matter should be small, and, therefore, that the η' production cross section should be much increased relative to that for pp collisions. This may have observable consequences in dilepton and diphoton experiments.

Significant in-medium η' mass reduction in $\sqrt{s_{NN}} = 200$ GeV

Au+Au collisions at the BNL Relativistic Heavy Ion Collider

R. Vértési¹, T. Csörgő^{2,1} and J. Sziklai¹