

Fate of in-medium heavy quarks via a Lindblad equation

Davide De Boni

Swansea, 13 September 2017



Swansea University
Prifysgol Abertawe



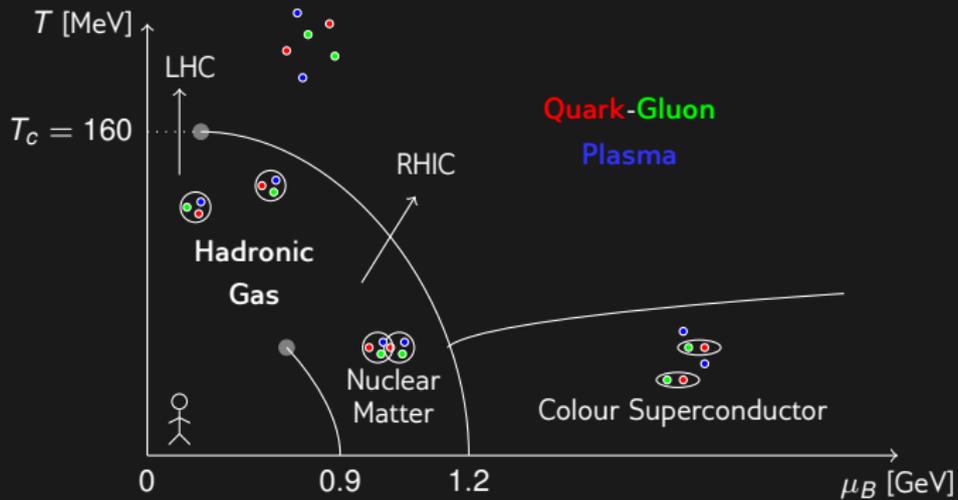
Contents

- ① Quark-gluon plasma and its signatures
- ② Abelian model and open quantum systems
- ③ Lindblad equation for the heavy quarks
- ④ Numerical results

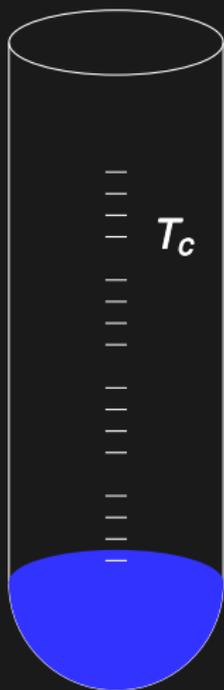
DDB, JHEP08 (2017) 064, 1705.03567

Blaizot, DDB, Faccioli, Garberoglio, Nuc. Phys. A 946(2016) 49,
1503.03857

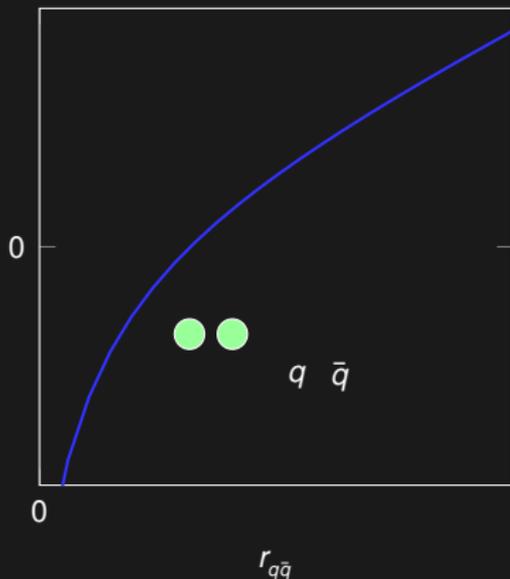
QCD phase diagram



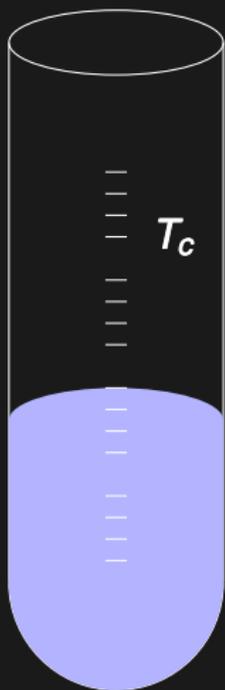
Signatures of QGP: Heavy-quark bound states



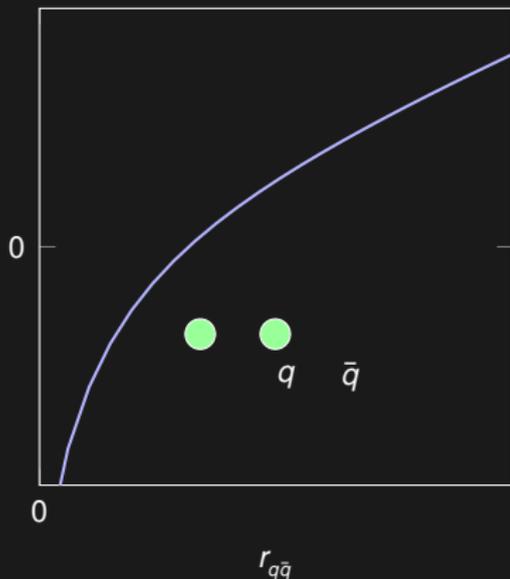
Interquark potential



Signatures of QGP: Heavy-quark bound states



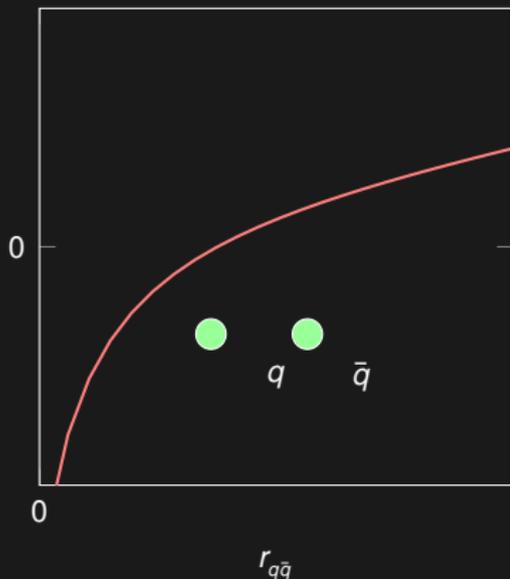
Interquark potential



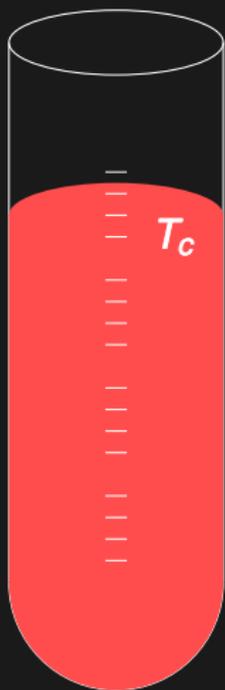
Signatures of QGP: Heavy-quark bound states



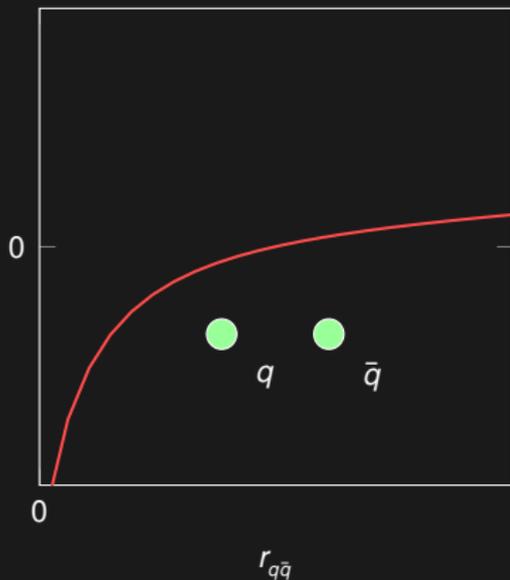
Interquark potential



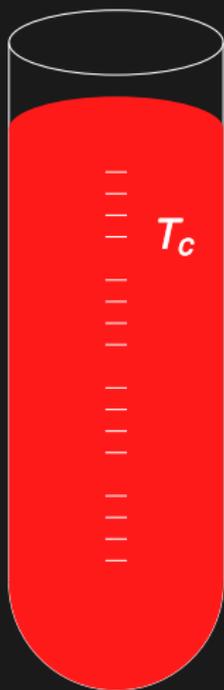
Signatures of QGP: Heavy-quark bound states



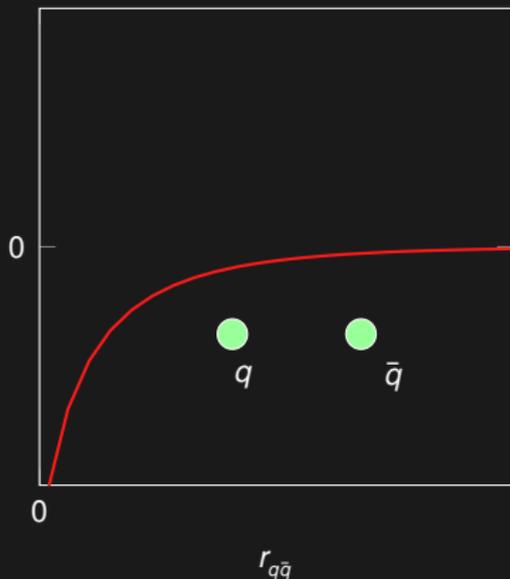
Interquark potential



Signatures of QGP: Heavy-quark bound states



Interquark potential



Interquark potential can be calculated using Lattice QCD,
perturbative techniques, holographic approach

Interquark potential can be calculated using Lattice QCD, perturbative techniques, holographic approach

- Potential gets **screened** at high $T \Rightarrow$ **Dissociation**
 - studied via Schrödinger equation
- Potential develops an **imaginary part** \Rightarrow **Dissociation**
 - Schrödinger eqn not appropriate when deviation from unitary evolution is large

Interquark potential can be calculated using Lattice QCD, perturbative techniques, holographic approach

- Potential gets **screened** at high $T \Rightarrow$ **Dissociation**
 - studied via Schrödinger equation
- Potential develops an **imaginary part** \Rightarrow **Dissociation**
 - Schrödinger eqn not appropriate when deviation from unitary evolution is large
- High mobility of heavy quarks in the QGP \Rightarrow **Regeneration**
 - \rightarrow Need for dynamical treatment
 - static Schrödinger approach not enough

Interquark potential can be calculated using Lattice QCD, perturbative techniques, holographic approach

- Potential gets **screened** at high $T \Rightarrow$ **Dissociation**
 - studied via Schrödinger equation
- Potential develops an **imaginary part** \Rightarrow **Dissociation**
 - Schrödinger eqn not appropriate when deviation from unitary evolution is large
- High mobility of heavy quarks in the QGP \Rightarrow **Regeneration**
 - \rightarrow Need for dynamical treatment
 - static Schrödinger approach not enough

Goal: To provide a unified framework for studying both processes:

- starting off from the underlying gauge theory
- controlling the real-time dynamics of the heavy quarks in the plasma

The model

We consider

- an Abelian plasma made of **relativistic light fermions** (quarks and antiquarks) in thermal equilibrium

The model

We consider

- an Abelian plasma made of **relativistic light fermions** (quarks and antiquarks) in thermal equilibrium
- N heavy quarks and antiquarks propagating out of equilibrium in the plasma
 - **non relativistic heavy particles** ($v \ll c = 1$)
 - \Rightarrow we can neglect magnetic forces acting on them
 - \Rightarrow their number is fixed

The model

We consider

- an Abelian plasma made of **relativistic light fermions** (quarks and antiquarks) in thermal equilibrium
- N heavy quarks and antiquarks propagating out of equilibrium in the plasma
 - **non relativistic heavy particles** ($v \ll c = 1$)
 \Rightarrow we can neglect magnetic forces acting on them
 \Rightarrow their number is fixed
- Scales setting: (light quarks) $m \ll T \ll M$ (heavy quarks)

- Heavy particles are treated non-relativistically using *first quantization*
- Light particles of the plasma are treated within *thermal field theory*

- Heavy particles are treated non-relativistically using *first quantization*
- Light particles of the plasma are treated within *thermal field theory*

Using the Coulomb gauge, the Hamiltonian of the system reads

$$\begin{aligned}
 H_{\text{tot}} = & \frac{1}{2M} \sum_{i=1}^N \left(\mathbf{p}_i^2 + \bar{\mathbf{p}}_i^2 \right) + \int d\mathbf{x} \bar{\psi}(\mathbf{x}) (-i\gamma^i \partial_i + m) \psi(\mathbf{x}) + \\
 & + \underbrace{\frac{1}{2} \iint d\mathbf{x} d\mathbf{y} j_{\text{tot}}^0(\mathbf{x}) \frac{1}{4\pi|\mathbf{x}-\mathbf{y}|} j_{\text{tot}}^0(\mathbf{y})}_{\text{Coulomb interactions}}
 \end{aligned}$$

- Heavy particles are treated non-relativistically using *first quantization*
- Light particles of the plasma are treated within *thermal field theory*

Using the Coulomb gauge, the Hamiltonian of the system reads

$$H_{\text{tot}} = \frac{1}{2M} \sum_{i=1}^N (\mathbf{p}_i^2 + \bar{\mathbf{p}}_i^2) + \int d\mathbf{x} \bar{\psi}(\mathbf{x}) (-i\gamma^i \partial_i + m) \psi(\mathbf{x}) + \underbrace{\frac{1}{2} \iint d\mathbf{x} d\mathbf{y} j_{\text{tot}}^0(\mathbf{x}) \frac{1}{4\pi|\mathbf{x}-\mathbf{y}|} j_{\text{tot}}^0(\mathbf{y})}_{\text{Coulomb interactions}}$$

$$j_{\text{tot}}^0(\mathbf{x}) = \underbrace{g \sum_{i=1}^N (\delta(\mathbf{x} - \mathbf{q}_i) - \delta(\mathbf{x} - \bar{\mathbf{q}}_i))}_{\text{heavy quarks+antiquarks}} + \underbrace{g \bar{\psi}(\mathbf{x}) \gamma^0 \psi(\mathbf{x})}_{\text{light particles}}$$

Open quantum systems

“Schrödinger” equation for closed quantum system (heavy particles + plasma)

$$i\hbar \frac{d\rho_{\text{tot}}}{dt}(t) = [H_{\text{tot}}, \rho_{\text{tot}}(t)] \quad H_{\text{tot}} = H \otimes \mathbb{I}_{\text{env}} + \mathbb{I} \otimes H_{\text{env}} + H_{\text{int}}$$

- ρ_{tot} is the density operator of the total (closed) system
- $\rho(t) = |\psi(t)\rangle\langle\psi(t)|$ for a pure state

Master equation for open quantum system (heavy particles)

$$\begin{aligned} i\hbar \frac{d\rho}{dt}(t) &= \text{Tr}_{\text{env}} \{ [H_{\text{tot}}, \rho_{\text{tot}}(t)] \} \\ &= [H, \rho(t)] + \text{Tr}_{\text{env}} \{ [\mathbb{I} \otimes H_{\text{env}} + H_{\text{int}}, \rho_{\text{tot}}(t)] \} \\ &\equiv [H, \rho(t)] + i\mathcal{D}\rho(t) \end{aligned}$$

$$\rho \equiv \text{Tr}_{\text{env}} \rho_{\text{tot}}$$

Lindblad equation

Most general master equation in the **Markovian limit** (negligible memory effects)

$$\dot{\rho} = -\frac{i}{\hbar}[H, \rho] + \frac{1}{2\hbar} \sum_{\mu} \left([L_{\mu}\rho, L_{\mu}^{\dagger}] + [L_{\mu}, \rho L_{\mu}^{\dagger}] \right)$$

$L_{\mu}, L_{\mu}^{\dagger}$ are the Lindblad operators

Path integral from Trotter decomposition

$$\langle q|\hat{\rho}(t+\Delta t)|q'\rangle = \langle q|\hat{\rho}(t)|q'\rangle - \frac{i}{\hbar} \langle q|[\hat{H}, \hat{\rho}(t)]|q'\rangle + \text{Lindbladian terms}$$

$$\Rightarrow \rho(t, q, q') = \int dq_0 \int dq'_0 P(q, q', t | q_0, q'_0, t_0) \rho(t_0, q_0, q'_0)$$

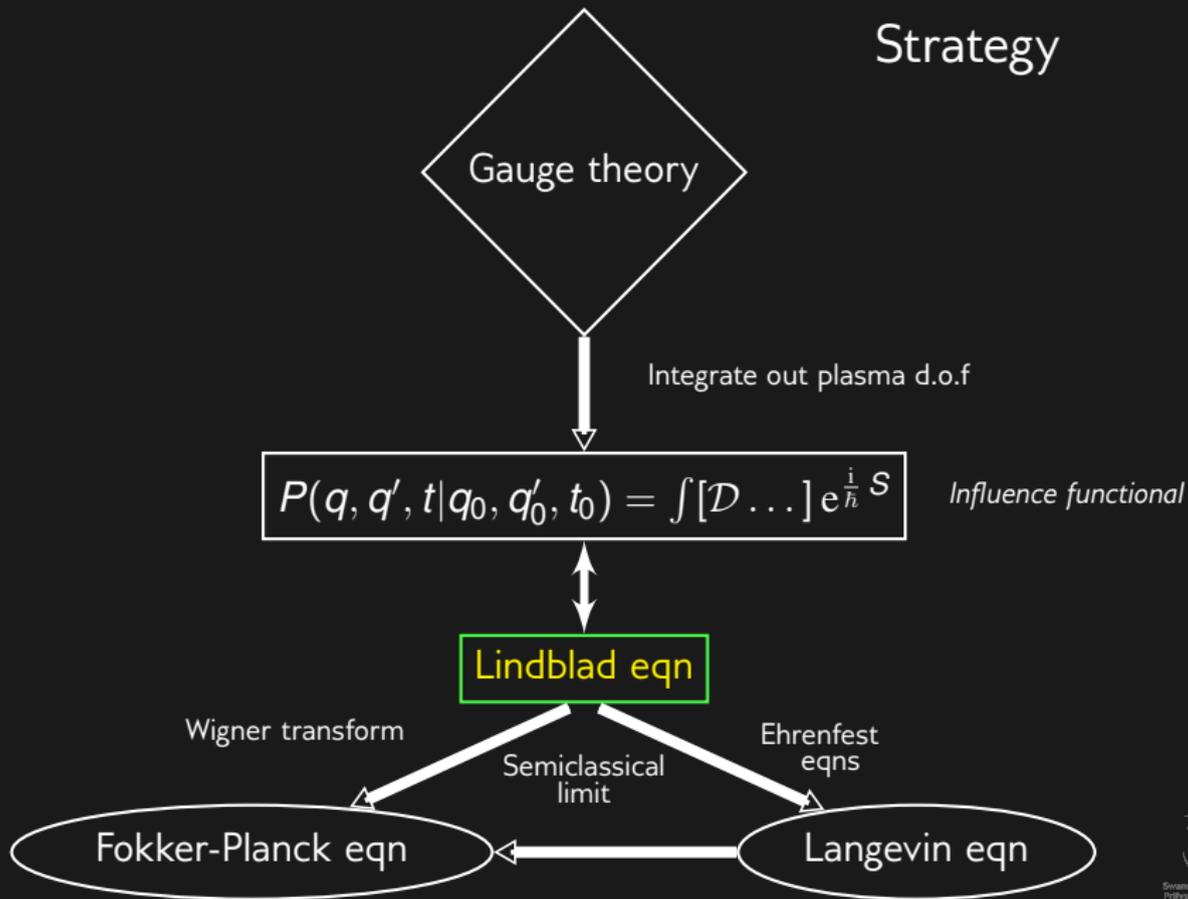
Path integral

$$P(q, q', t | q_0, q'_0, t_0) = \int_{(q_0, t_0)}^{(q, t)} \mathcal{D}q \mathcal{D}p \int_{(q'_0, t_0)}^{(q', t)} \mathcal{D}q' \mathcal{D}p' \exp \left[\frac{i}{\hbar} S[q, p; q', p'] \right]$$

$$S[q, p; q', p'] = \int_{t_0}^t d\tau \left[\dot{q}p - H_{\text{eff}}(q, p) - \dot{q}'p' + H_{\text{eff}}^*(q', p') - i \sum_{\mu} L_{\mu}(q, p) L_{\mu}^*(q', p') \right]$$

$$H_{\text{eff}} = H - \frac{i}{2} \sum_{\mu} L_{\mu}^{\dagger} L_{\mu}$$

Strategy

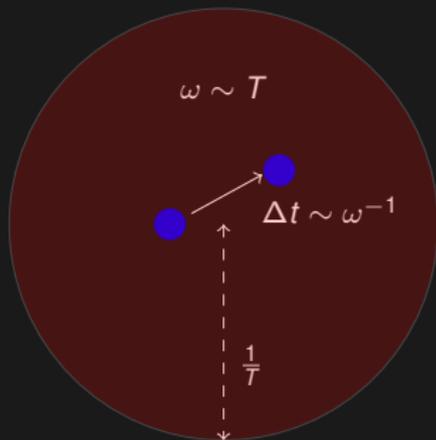


Main approximations

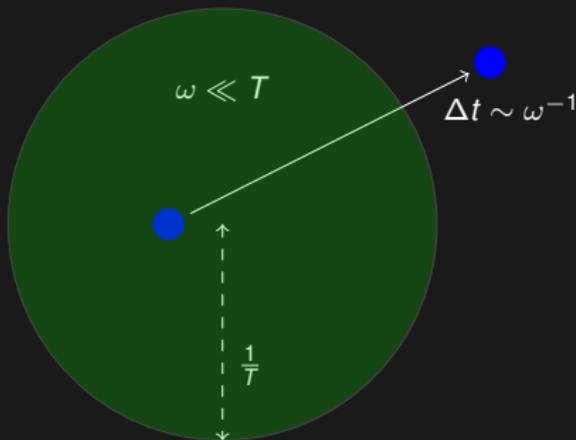
Markovian approximation: Low frequency expansion ($\omega/T \ll 1$)

ω = plasma frequency

Frozen dynamics



Interesting dynamics

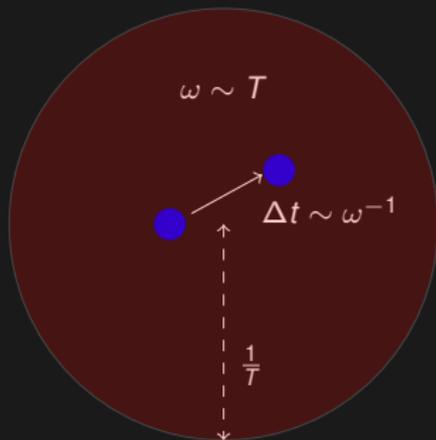


Main approximations

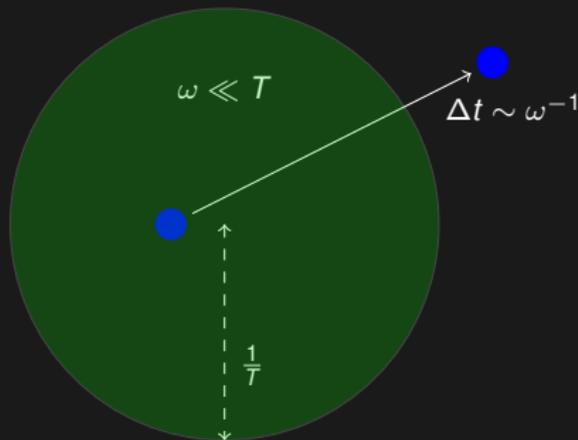
Markovian approximation: Low frequency expansion ($\omega/T \ll 1$)

ω = plasma frequency

Frozen dynamics



Interesting dynamics



Perturbative approximation: Truncation up to order g^2

Lindblad eqn for a single heavy quark

$$\mathbf{r} = \frac{1}{2}(\mathbf{q} + \mathbf{q}'), \quad \mathbf{y} = \mathbf{q} - \mathbf{q}'$$

- $W(\mathbf{r})$ is the imaginary part of the interquark potential
- The Lindblad matrix elements depend only on W

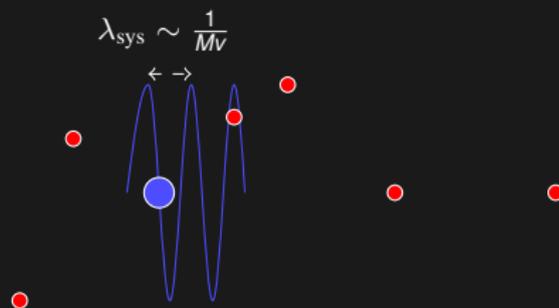
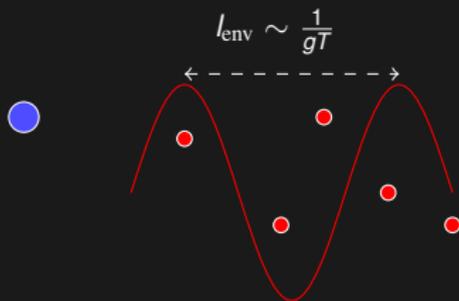
$$\frac{\partial \rho(t, \mathbf{r}, \mathbf{y})}{\partial t} = \left(\begin{array}{c} \text{Schrödinger} \\ \frac{i\hbar}{M} \frac{\partial}{\partial \mathbf{r}} \cdot \frac{\partial}{\partial \mathbf{y}} - \frac{i}{\hbar} (V_{\text{ext}}(\mathbf{r} + \mathbf{y}/2) - V_{\text{ext}}(\mathbf{r} - \mathbf{y}/2)) \\ \\ \underbrace{-\frac{g^2}{\hbar} (W(\mathbf{y}) - W(0))}_{\text{diffusion, decoherence}} - \underbrace{\frac{g^2 \hbar}{2MT} \frac{\partial W(\mathbf{y})}{\partial \mathbf{y}} \cdot \frac{\partial}{\partial \mathbf{y}}}_{\text{dissipation}} \end{array} \right) \rho(t, \mathbf{r}, \mathbf{y})$$

Resolution of the system by the plasma

The (irreversible) interaction between the system and the plasma is determined by $W(\mathbf{r})$, which has a characteristic correlation length l_{env}

Light quarks (plasma) ●

Heavy quarks ●



Resolution of the system by the plasma

- $l_{\text{env}} \gg \lambda_{\text{sys}} \Rightarrow$ Reversible (Schrödinger) dynamics
No quantum decoherence

Resolution of the system by the plasma

- $I_{\text{env}} \gg \lambda_{\text{sys}} \Rightarrow$ Reversible (Schrödinger) dynamics
No quantum decoherence
- $I_{\text{env}} \gtrsim \lambda_{\text{sys}} \Rightarrow$ Open-system dynamics
Langevin (classical) dynamics after a decoherence time

Resolution of the system by the plasma

- $I_{\text{env}} \gg \lambda_{\text{sys}} \Rightarrow$ Reversible (Schrödinger) dynamics
No quantum decoherence
- $I_{\text{env}} \gtrsim \lambda_{\text{sys}} \Rightarrow$ Open-system dynamics
Langevin (classical) dynamics after a decoherence time
- $I_{\text{env}} \lesssim \lambda_{\text{sys}} \Rightarrow$ Medium as a bound-state sieve

Resolution of the system by the plasma

- $I_{\text{env}} \gg \lambda_{\text{sys}} \Rightarrow$ Reversible (Schrödinger) dynamics
No quantum decoherence
- $I_{\text{env}} \gtrsim \lambda_{\text{sys}} \Rightarrow$ Open-system dynamics
Langevin (classical) dynamics after a decoherence time
- $I_{\text{env}} \lesssim \lambda_{\text{sys}} \Rightarrow$ Medium as a bound-state sieve
- $I_{\text{env}} \ll \lambda_{\text{sys}} \Rightarrow$ Medium heavily perturbs the system
All bound states decay at the same rate

Fokker-Planck and Langevin equations

Wigner function

$$\rho(t, \mathbf{r}, \mathbf{p}) = \int d\mathbf{y} \rho(t, \mathbf{r}, \mathbf{y}) e^{-\frac{i}{\hbar} \mathbf{p} \cdot \mathbf{y}}$$

Fokker-Planck equation for one heavy quark (semiclassical limit):

$$\left[\partial_t + \frac{\mathbf{p}}{M} \cdot \partial_{\mathbf{r}} - \partial_{\mathbf{r}} V_{\text{ext}}(\mathbf{r}) \cdot \partial_{\mathbf{p}} \right] \rho(t, \mathbf{r}, \mathbf{p}) = \gamma \left[MT \nabla_{\mathbf{p}}^2 + \partial_{\mathbf{p}} \cdot \mathbf{p} \right] \rho(t, \mathbf{r}, \mathbf{p})$$

Fokker-Planck and Langevin equations

Wigner function

$$\rho(t, \mathbf{r}, \mathbf{p}) = \int d\mathbf{y} \rho(t, \mathbf{r}, \mathbf{y}) e^{-\frac{i}{\hbar} \mathbf{p} \cdot \mathbf{y}}$$

Fokker-Planck equation for one heavy quark (semiclassical limit):

$$\left[\partial_t + \frac{\mathbf{p}}{M} \cdot \partial_{\mathbf{r}} - \partial_{\mathbf{r}} V_{\text{ext}}(\mathbf{r}) \cdot \partial_{\mathbf{p}} \right] \rho(t, \mathbf{r}, \mathbf{p}) = \gamma \left[MT \nabla_{\mathbf{p}}^2 + \partial_{\mathbf{p}} \cdot \mathbf{p} \right] \rho(t, \mathbf{r}, \mathbf{p})$$

Corresponding Langevin equation for one heavy quark:

$$M \ddot{\mathbf{r}} + M \gamma \dot{\mathbf{r}} + \nabla_{\mathbf{r}} V_{\text{ext}}(\mathbf{r}) = \boldsymbol{\eta}(\mathbf{r}, t) \quad \gamma \sim W''(\mathbf{r} = 0)$$

(γ is space-depent in the many-quark case)

Noise vector corresponds to a stochastic force

$$\langle \boldsymbol{\eta}(\mathbf{r}, t) \rangle = 0, \quad \langle \eta_i(\mathbf{r}, t) \eta_j(\mathbf{r}, t') \rangle = 2M\gamma T \delta_{ij} \delta(t - t')$$

Langevin dynamics

- High temperature \leftrightarrow melted $q\bar{q}$ pairs
- Low temperature \leftrightarrow $q\bar{q}$ pairs strongly bound
- Medium temperature \leftrightarrow $q\bar{q}$ pairs faintly bound

Langevin dynamics

Pros



- Cheap simulations of many $q\bar{q}$ pairs
- Easy to study pair dissociation and formation

Cons



- Initial conditions are *classical*, hence the bound states are not quantum
- Dynamics is correct only in the semiclassical limit

Lindblad eqn for a $q\bar{q}$ pair (CoM frame)

$$\mathbf{r} = \frac{1}{2}(\mathbf{q} + \mathbf{q}'), \quad \mathbf{y} = \frac{1}{2}(\mathbf{q} - \mathbf{q}')$$

$$\frac{\partial \rho(t, \mathbf{r}, \mathbf{y})}{\partial t} = \left(\underbrace{\left[\frac{i\hbar}{M} \frac{\partial}{\partial \mathbf{r}} \cdot \frac{\partial}{\partial \mathbf{y}} - \frac{i}{\hbar} (V(\mathbf{r} + \mathbf{y}/2) - V(\mathbf{r} - \mathbf{y}/2)) \right]}_{\text{Schrödinger}} \right. \\ \left. - \underbrace{\frac{g^2}{\hbar} (2W(\mathbf{y}) - 2W(\mathbf{r}) + W(\mathbf{r} + \mathbf{y}) + W(\mathbf{r} - \mathbf{y}) - 2W(0))}_{\text{diffusion, decoherence}} \right. \\ \left. - \underbrace{\frac{g^2 \hbar}{2MT} \left(\frac{\partial W(\mathbf{y})}{\partial \mathbf{y}} \cdot \frac{\partial}{\partial \mathbf{y}} - \frac{\partial W(\mathbf{r})}{\partial \mathbf{r}} \cdot \frac{\partial}{\partial \mathbf{r}} - \frac{\partial^2 W(\mathbf{r})}{\partial \mathbf{r}^2} \right)}_{\text{dissipation}} \right) \rho(t, \mathbf{r}, \mathbf{y})$$

Quantities of interest

- Probability of having the state $|\psi\rangle$ at time t

$$P(\psi, t | \psi_0, t_0) = \int dq \int dq' \psi(q') \psi^*(q) \rho(t, q, q')$$

- Linear entropy (proxy of thermal entropy $S = -\text{Tr} [\rho \ln \rho]$)

$$S_L = \text{Tr} \rho - \text{Tr} \rho^2 = 1 - \text{Tr} \rho^2$$

Quantities of interest

- Probability of having the state $|\psi\rangle$ at time t

$$P(\psi, t | \psi_0, t_0) = \int dq \int dq' \psi(q') \psi^*(q) \rho(t, q, q')$$

- Linear entropy (proxy of thermal entropy $S = -\text{Tr} [\rho \ln \rho]$)

$$S_L = \text{Tr} \rho - \text{Tr} \rho^2 = 1 - \text{Tr} \rho^2$$

Pure states $\rho = \rho^2 \Rightarrow S_L = 0$

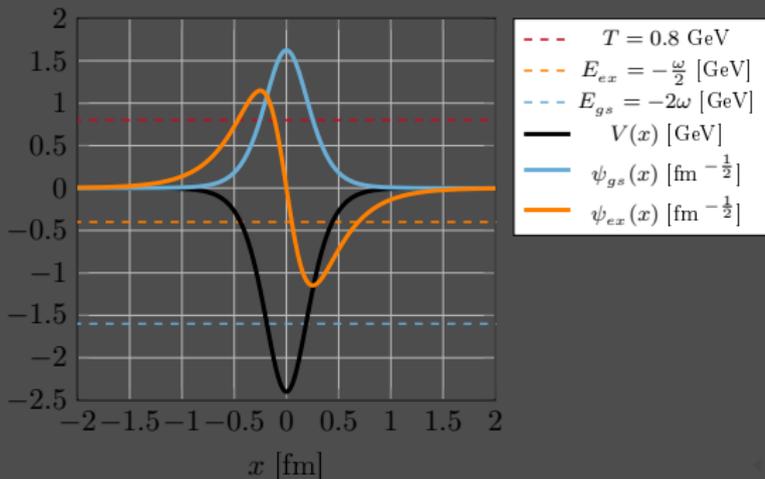
Non-pure states $\rho \neq \rho^2 \Rightarrow 0 < S_L \leq 1$

Numerical results for a $q\bar{q}$ pair in 1D

Pöschl-Teller potential:

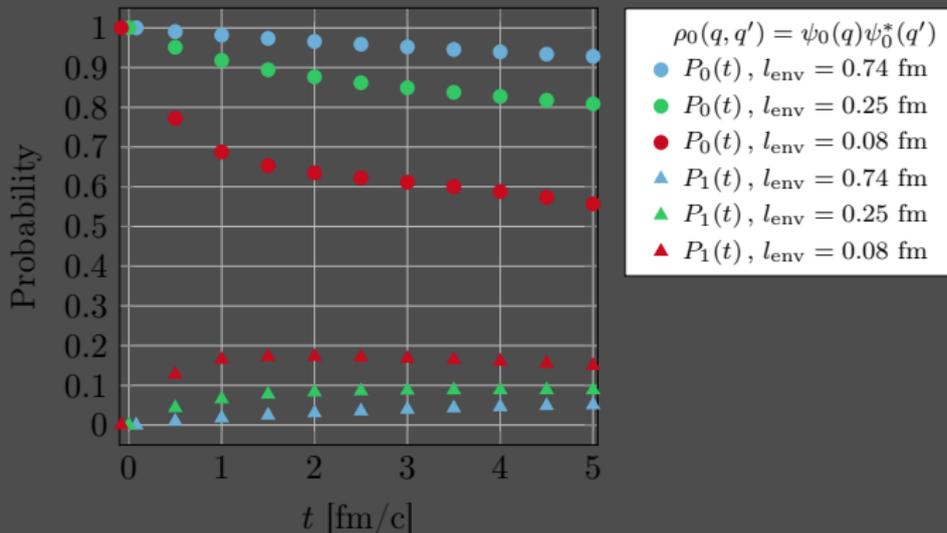
$$V(x) = -\frac{\omega}{2}j(j+1)\text{sech}^2\left[\sqrt{\frac{M\omega}{2\hbar^2}}x\right] \quad j = 2 \text{ (bound states)}$$

$$W(x) = -\frac{T}{2} \exp\left[-\frac{1}{2}\left(\frac{x}{l_{\text{env}}}\right)^2\right] \quad l_{\text{env}} \sim \frac{1}{gT}$$



Starting off with the ground state

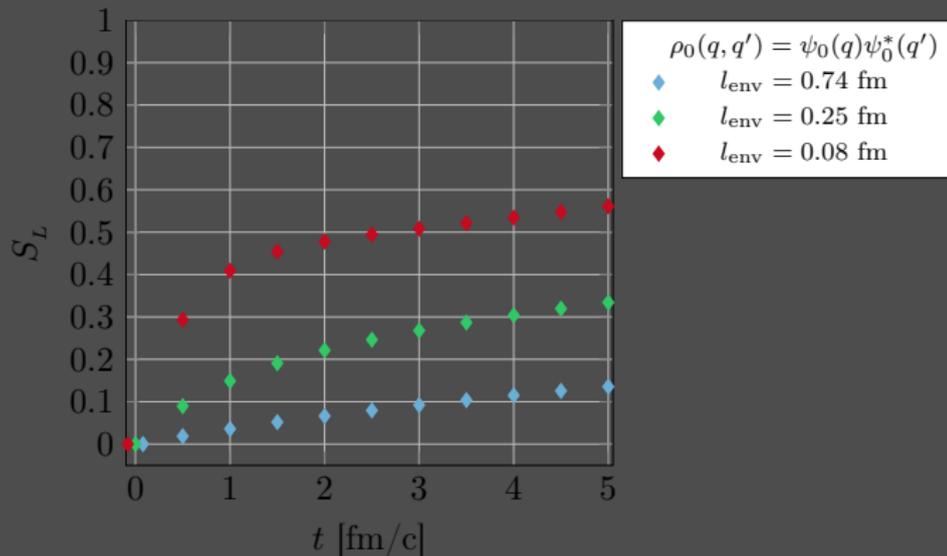
$$\lambda_{\text{sys}} = 0.16 \text{ fm}$$



Ground state melts with $P = 1 - P_0 - P_1 \sim 10\%$ after $\Delta t = 5 \text{ fm/c}$ when $l_{\text{env}} = 0.25 \text{ fm}$

Starting off with the ground state

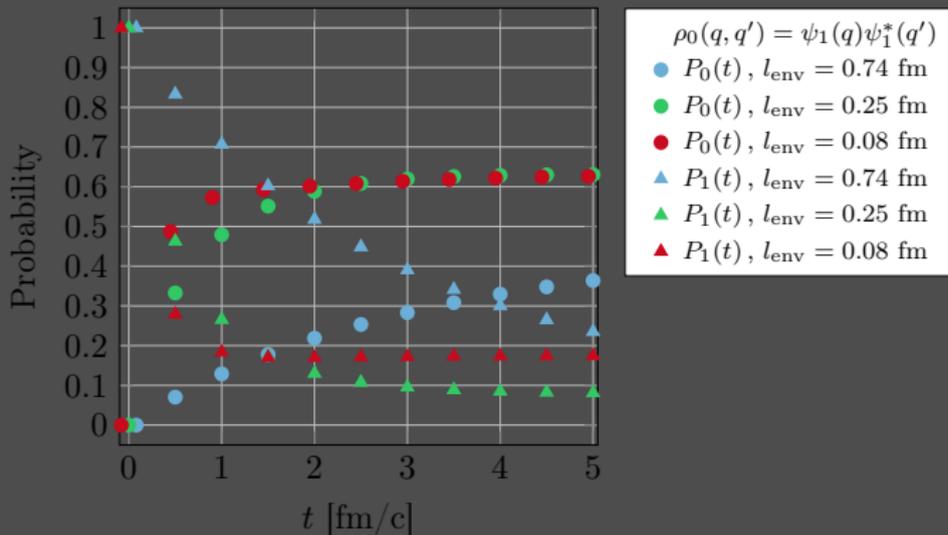
$$\lambda_{\text{sys}} = 0.16 \text{ fm}$$



The smaller l_{env} , the more rapidly the linear entropy initially increases

Starting off with the excited state

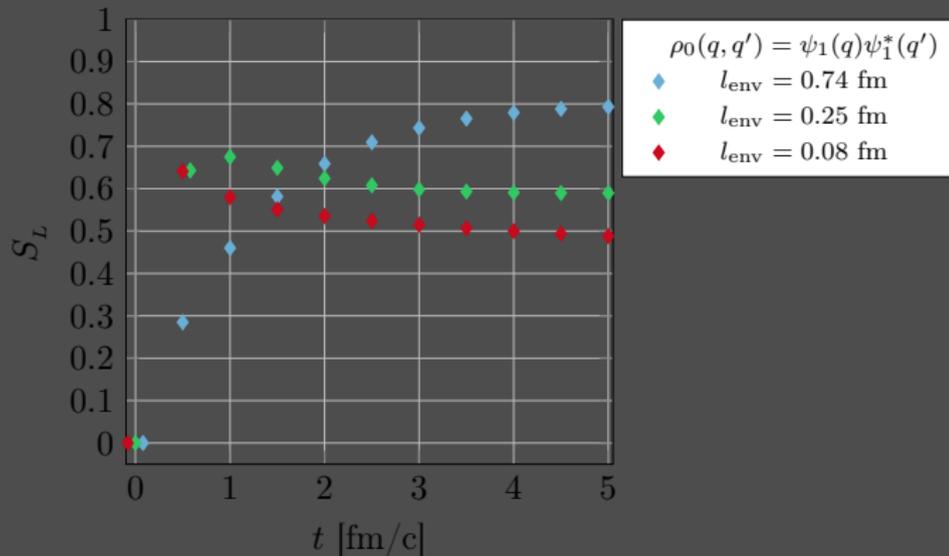
$$\lambda_{\text{sys}} = 0.38 \text{ fm}$$



Excited state melts with $P = 1 - P_0 - P_1 \sim 30\%$ after $\Delta t = 5 \text{ fm/c}$ when $l_{\text{env}} = 0.25 \text{ fm}$

Starting off with the excited state

$$\lambda_{\text{sys}} = 0.38 \text{ fm}$$



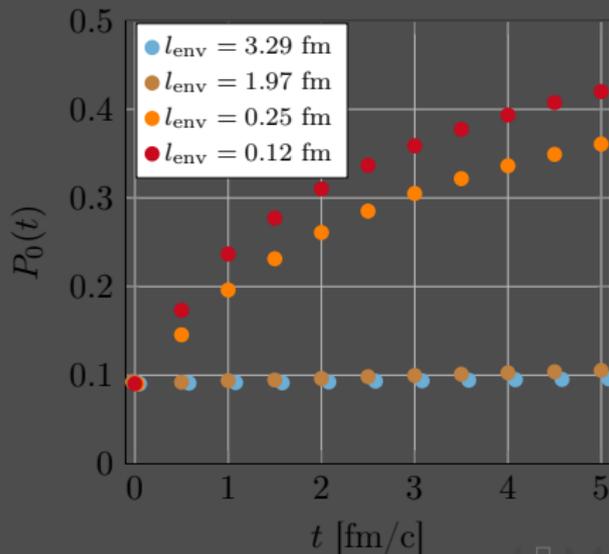
Notice that the linear entropy does not increase monotonically like the thermal entropy

Starting off with a thermal scattering state

$$\psi_{\text{scatt}}(x) \sim e^{-\frac{1}{2}\left(\frac{x}{\delta}\right)^2 + \frac{i}{\hbar}xp} \quad \delta = \sqrt{2\langle\hat{x}^2\rangle} = \sqrt{2}\lambda_{\text{sys}}$$

$$\lambda_{\text{sys}} = \frac{h}{p_{\text{th}}} = \frac{\sqrt{2}h}{\sqrt{mT}} = 1.77 \text{ fm} \quad m = 1.2 \text{ GeV}$$

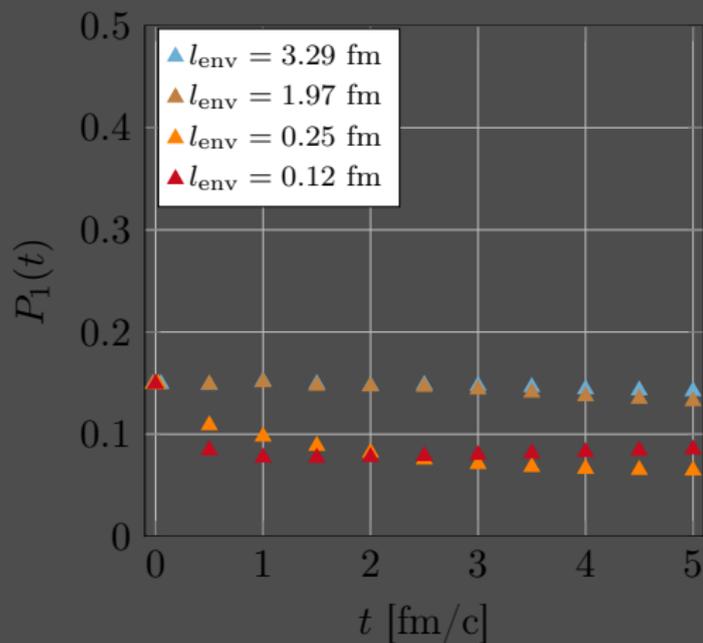
Recombination effects!



Starting off with a thermal scattering state

$$\psi_{\text{scatt}}(x) \sim e^{-\frac{1}{2}\left(\frac{x}{\delta}\right)^2 + \frac{i}{\hbar} x p} \quad \delta = \sqrt{2\langle \hat{x}^2 \rangle} = \sqrt{2} \lambda_{\text{sys}}$$

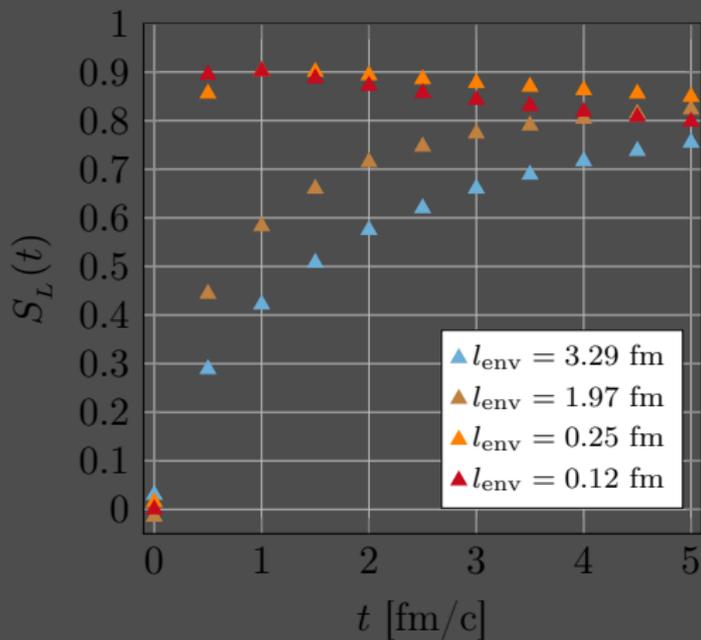
$$\lambda_{\text{sys}} = 1.77 \text{ fm}$$



Starting off with a thermal scattering state

$$\psi_{\text{scatt}}(x) \sim e^{-\frac{1}{2}\left(\frac{x}{\delta}\right)^2 + \frac{i}{\hbar} x p} \quad \delta = \sqrt{2\langle \hat{x}^2 \rangle} = \sqrt{2} \lambda_{\text{sys}}$$

$$\lambda_{\text{sys}} = 1.77 \text{ fm}$$



Time evolution of the density matrix

- $l_{\text{env}} \sim \lambda_{\text{sys}}$

- $l_{\text{env}} < \lambda_{\text{sys}}$

- $l_{\text{env}} \ll \lambda_{\text{sys}}$

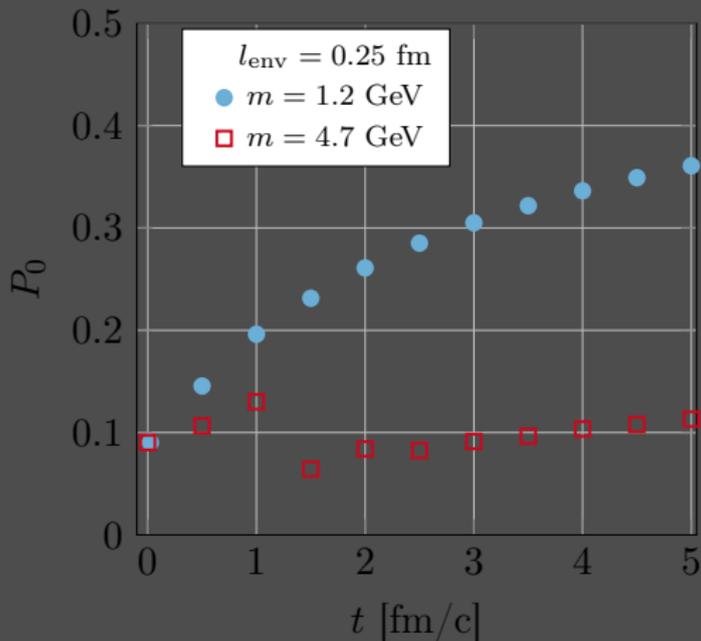
Mass dependence of recombination

Experimental fact: $\Upsilon (b\bar{b})$ recombination \ll $J/\Psi (c\bar{c})$ recombination

$$\lambda = \frac{h}{p_{\text{th}}} = \frac{\sqrt{2}h}{\sqrt{mT}}$$

$$\lambda_b = 0.89 \text{ fm}$$

$$\lambda_c = 1.77 \text{ fm}$$



Conclusions and outlook

- A Lindblad equation for a heavy $q\bar{q}$ pair can be derived from the gauge theory
- This equation allows us to study dissociation, recombination and quantum decoherence of bound states
- Solution goes beyond Langevin/Fokker-Planck dynamics

Conclusions and outlook

- A Lindblad equation for a heavy $q\bar{q}$ pair can be derived from the gauge theory
- This equation allows us to study dissociation, recombination and quantum decoherence of bound states
- Solution goes beyond Langevin/Fokker-Planck dynamics

Next steps:

- Solve the Lindblad equation in 3D (and for more than 2 particles)
- Is it possible to implement the initial quantum conditions (maybe by weighting somehow the classical paths) in a Langevin equation ?
- Derive the Lindblad equation in QCD

```
\huge{Thank you}
```

```
\end{document}
```

Backup material

Keldysh formalism

- Trotter decomposition brings to

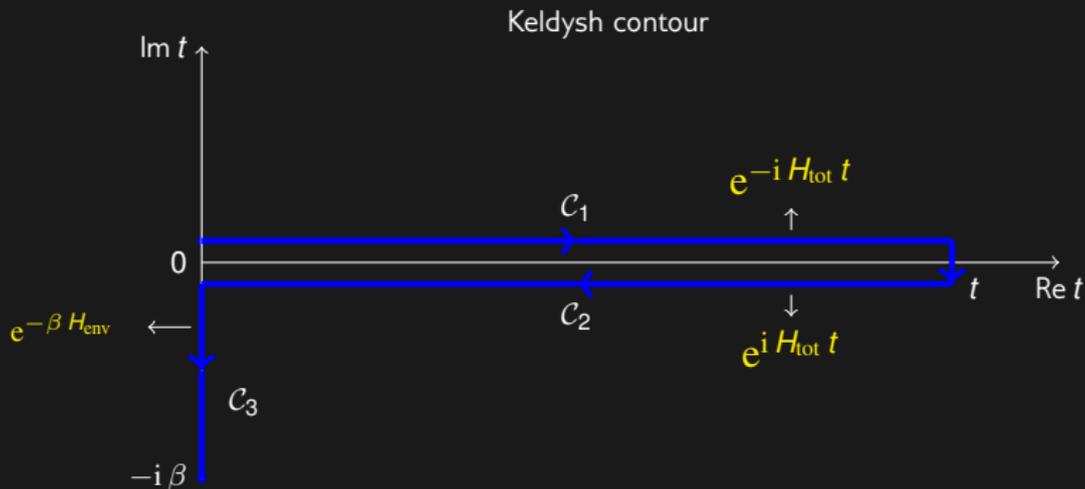
$$P(\mathbf{Q}_f, t | \mathbf{Q}_i, 0) = \int_{\mathbf{Q}_i}^{\mathbf{Q}_f} \mathcal{D}\mathbf{Q} \int \mathcal{D}(\bar{\psi}\psi) e^{iS[\mathbf{Q}, \psi, \bar{\psi}]}$$

The action of the system is defined on the Keldysh contour

$$S[\mathbf{Q}, \psi, \bar{\psi}] = \int_c dt \left[\frac{M}{2} \sum_{j=1}^N (\dot{\mathbf{q}}_j^2 + \dot{\bar{\mathbf{q}}}_j^2) + \int d\mathbf{x} \bar{\psi}(\mathbf{x}, t) (i\gamma^\mu \partial_\mu - m) \psi(\mathbf{x}, t) + \right. \\ \left. - \frac{1}{2} \iint d\mathbf{x} d\mathbf{y} j_{\text{tot}}^0(\mathbf{x}, t) \frac{1}{4\pi|\mathbf{x} - \mathbf{y}|} j_{\text{tot}}^0(\mathbf{y}, t) \right]$$

Keldysh formalism

$$\rho_{\text{tot}}(t) = e^{-i H_{\text{tot}} t} \rho_{\text{tot}}(0) e^{i H_{\text{tot}} t}; \quad \rho_{\text{tot}}(0) = |\mathbf{Q}_0\rangle\langle \mathbf{Q}_0| \otimes \frac{e^{-\beta H_{\text{env}}}}{Z_{\text{env}}}$$



Eliminating plasma degrees of freedom

1st step

Hubbard-Stratonovich transformation

→ Introducing a Coulomb field A_0

$$\exp \left[-\frac{i}{2} j_{\text{tot}}^0 \cdot K \cdot j_{\text{tot}}^0 \right] = \mathcal{N} \int DA_0 \exp \left[\frac{i}{2} A_0 \cdot K^{-1} \cdot A_0 - i A_0 \cdot j_{\text{tot}}^0 \right]$$

where

$$K(\mathbf{x} - \mathbf{y}) = \frac{1}{4\pi|\mathbf{x} - \mathbf{y}|}; \quad K^{-1}(\mathbf{x} - \mathbf{y}) = -\delta(\mathbf{x} - \mathbf{y}) \nabla_{\mathbf{y}}^2$$

2nd step

To eliminate the field of the light quarks by performing the Gaussian integral below

$$\begin{aligned} & \int \mathcal{D}(\bar{\psi}\psi) \exp \left[i \int_c d^4x \bar{\psi}(x) \left(i\gamma^\mu \partial_\mu - m - g\gamma^0 A_0(x) \right) \psi(x) \right] = \\ & = \det(i\gamma^\mu \partial_\mu - m - g\gamma^0 A_0) = \\ & = \exp \left[\text{Tr} \ln \left[i\gamma^\mu \partial_\mu - m - g\gamma^0 A_0 \right] \right] \end{aligned}$$

3rd step (first approximation)

Expand

$$\det \left[i\gamma^\mu \partial_\mu - m - g\gamma^0 A_0 \right]$$

to second order in g ($g \ll 1$) and perform the Gaussian integral over A_0

$$\int \mathcal{D}A_0 \exp \left[-\frac{1}{2} A_0 \cdot \Delta_c^{-1} \cdot A_0 - i A_0 \cdot j^0 \right] = \exp \left[\frac{1}{2} j^0 \cdot \Delta_c \cdot j^0 \right]$$

where

$$-\Delta_c^{-1}(x-y) = \delta_c(t_x^c - t_y^c) K^{-1}(\mathbf{x} - \mathbf{y}) + \Pi_{00}^c(x-y)$$

$$\Delta_c(x-y) = i \langle T_c [A_0(x)A_0(y)] \rangle$$

We obtain the *Feynman-Vernon influence functional*

$$P(\mathbf{Q}_f, t | \mathbf{Q}_i, 0) = \int_{\mathbf{Q}_i}^{\mathbf{Q}_f} \mathcal{D}\mathbf{Q} \exp \left[i \left(\Phi[\mathbf{Q}] + \frac{M}{2} \int_{c_1 \cup c_2} dt \sum_{j=1}^N (\dot{\mathbf{q}}_j^2 + \ddot{\mathbf{q}}_j^2) \right) \right]$$

$$\begin{aligned} \Phi[\mathbf{Q}] &= \frac{1}{2} \iint_c d^4x d^4y \rho(\mathbf{x}) \Delta_c(\mathbf{x} - \mathbf{y}) \rho(\mathbf{y}) \\ &= \frac{1}{2} \int_0^t dt_x dt_y \int d\mathbf{x} d\mathbf{y} (-1)^{a+b} j_a^0(t_x, \mathbf{x}) \Delta_{ab}(t_x - t_y, \mathbf{x} - \mathbf{y}) j_b^0(t_y, \mathbf{y}) \end{aligned}$$

with

$$j_a^0(t_x, \mathbf{x}) = g \sum_{i=1}^N (\delta(\mathbf{x} - \mathbf{q}_{i,a}) - \delta(\mathbf{x} - \bar{\mathbf{q}}_{i,a}))$$

Second approximation: low frequency expansion of the gluon correlation function

$$\Delta(\omega) \approx \Delta(\omega = 0) + \omega \Delta'(\omega = 0)$$

$$\Delta(t_x - t_y) \approx \delta(t_x - t_y) \Delta(\omega = 0) + i \frac{d}{dt_x} \delta(t_x - t_y) \Delta'(\omega = 0)$$

Performing this expansion up to $o(\omega)$ in

$$\Phi[\mathbf{Q}] = \Phi_{\alpha\alpha}[\mathbf{Q}] + \Phi_{\bar{\alpha}\bar{\alpha}}[\mathbf{Q}] + \Phi_{\alpha\bar{\alpha}}[\mathbf{Q}]$$

and introducing the two real quantities

$$V(\mathbf{r}) \equiv -\Delta^R(\omega = 0, \mathbf{r}); \quad W(\mathbf{r}) = -\Delta^<(\omega = 0, \mathbf{r})$$

where $\Delta^R = \Delta_{11} - i\Delta^<$ and $\Delta^< = -i\Delta_{12}$, we get

$$\begin{aligned} \Phi_{\alpha\alpha}[\mathbf{Q}] = & \frac{g^2}{2} \sum_{i,j=1}^N \int_{t_i}^{t_f} dt \left[V(\mathbf{q}_{j,2} - \mathbf{q}_{i,2}) - V(\mathbf{q}_{j,1} - \mathbf{q}_{i,1}) \right. \\ & - iW(\mathbf{q}_{j,2} - \mathbf{q}_{i,2}) - iW(\mathbf{q}_{j,1} - \mathbf{q}_{i,1}) + 2iW(\mathbf{q}_{j,1} - \mathbf{q}_{i,2}) \\ & \left. + \frac{\beta}{2} (\dot{\mathbf{q}}_{i,2} + \dot{\mathbf{q}}_{j,1}) \cdot \frac{\partial}{\partial \mathbf{q}_{i,2}} W(\mathbf{q}_{j,1} - \mathbf{q}_{i,2}) \right] \end{aligned}$$

and similarly for $\Phi_{\bar{\alpha}\bar{\alpha}}[\mathbf{Q}]$ and $\Phi_{\alpha\bar{\alpha}}[\mathbf{Q}]$

→ the conditional probability depends only on V and W

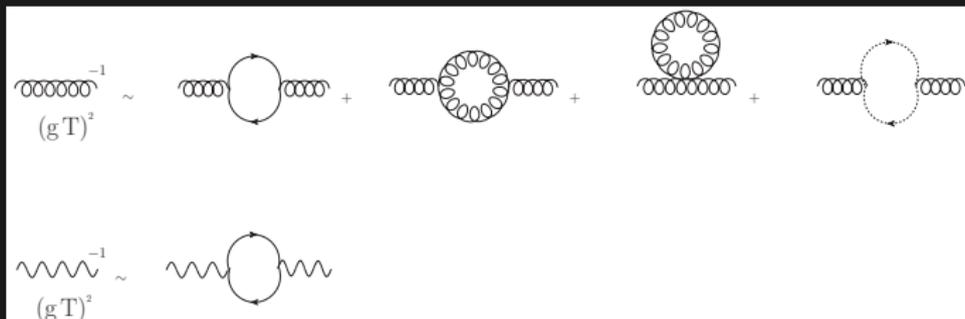
3rd approximation: Hard thermal loop

Slow heavy particles exchange **soft** gluons, which have momentum $|\mathbf{k}| \lesssim gT$ and frequency $\omega \lesssim gT$

(consistency with low frequency approximation and with the expansion of the fermionic determinant up to $o(g^2)$)

Inverse propagators of soft particles are of the **same order** of 1-loop self energies

$$(gT)^2 \sim \Delta_{\mu\nu}^{-1}(k \sim gT) \sim \Pi_{\mu\nu}(k \sim gT)$$



Gluon self-energy contains the information about collision effects and the screening of the interactions in the plasma

Similarity between HTL 1-loop **gluon** and **photon** self-energy

$$\Pi_{\mu\nu}^{\text{ab}}(\omega, \mathbf{q}) = \frac{g^2 T^2}{3} \overbrace{\left(N_c + \frac{N_f}{2} \right)}{=4} \delta^{\text{ab}} \int \frac{d\Omega}{4\pi} \left(\frac{i\omega \hat{K}_\mu \hat{K}_\nu}{i\omega + \mathbf{q} \cdot \hat{\mathbf{k}}} + \delta_{\mu 4} \delta_{\nu 4} \right)$$

$$\Pi_{\mu\nu}(\omega, \mathbf{q}) = \frac{e^2 T^2}{3} \int \frac{d\Omega}{4\pi} \left(\frac{i\omega \hat{K}_\mu \hat{K}_\nu}{i\omega + \mathbf{q} \cdot \hat{\mathbf{k}}} + \delta_{\mu 4} \delta_{\nu 4} \right) ; \quad \hat{K} \equiv (\hat{\mathbf{k}}, -i)$$

We work with a QED plasma, eventually going to QCD by changing the constant in front of the integral

Using the HTL approximation we obtain

$$V(r) = \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{\mathbf{k}^2 + m_D^2} = \frac{e^{-m_D r}}{4\pi r}$$
$$W(r) = - \int \frac{d\mathbf{k}}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{r}} \frac{\pi T m_D^2}{|\mathbf{k}|(\mathbf{k}^2 + m_D^2)^2}$$

where $\Pi_{00}^{\text{HTL}}(0, \mathbf{k}) = m_D^2 = \frac{4}{3}g^2 T^2$

- $V(r)$ is the screening potential between the heavy quarks
- $W(r)$ originates from the collisions between the light fermions of the plasma and the heavy particles

Indeed $-\frac{g^2 T}{2} W(r=0)$ is the rate of collisions between a heavy quark and the particles of the plasma