Attractive and repulsive hadronic interactions: Do we need more states?

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Extreme Hadrons 2017
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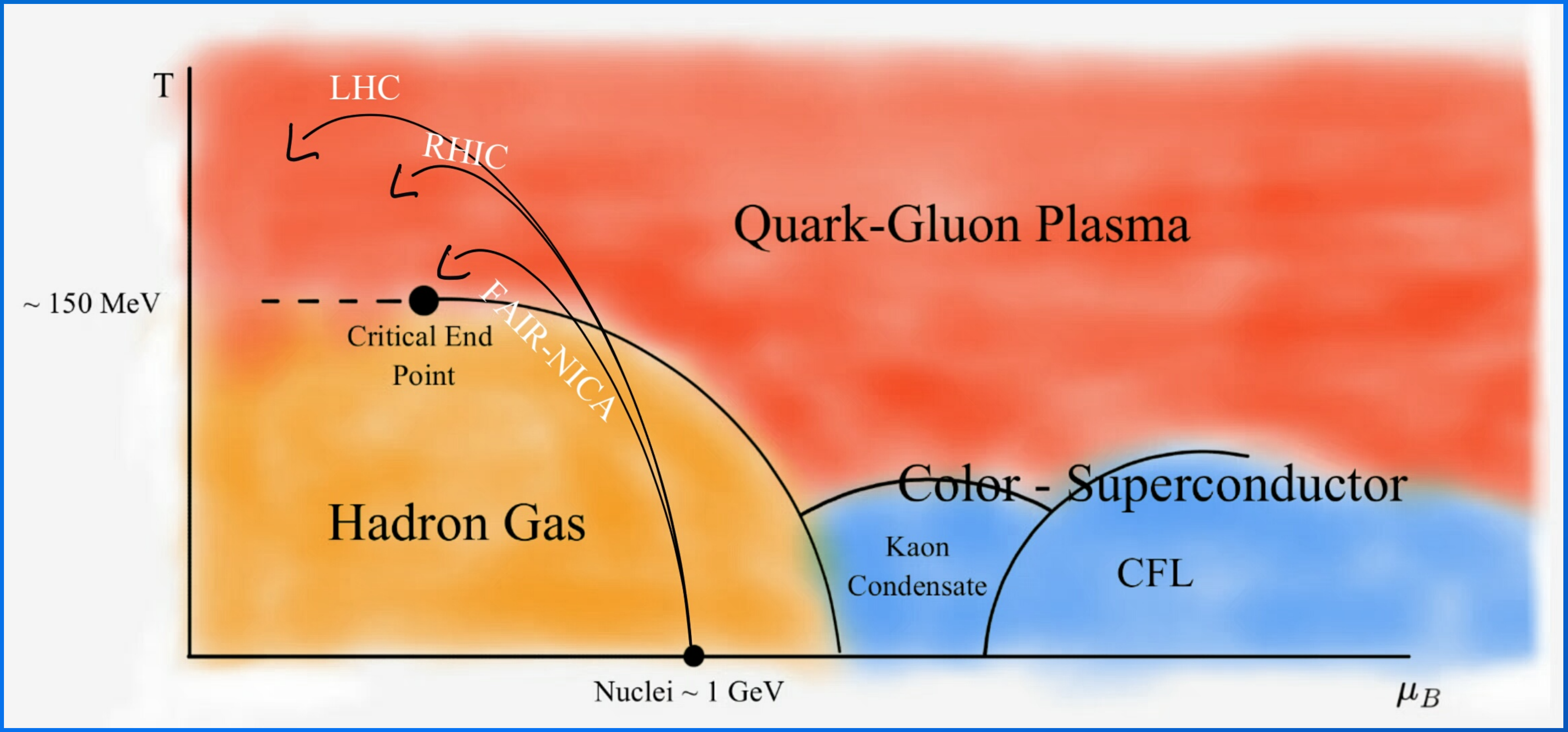
Outline

- What the HRG can do: lattice vs experiment
- Importance of the particle list: PDG vs QM
- Improved HRG: repulsive interactions
- Attractive and repulsive interactions: Results
- Conclusions

A sea of QGP



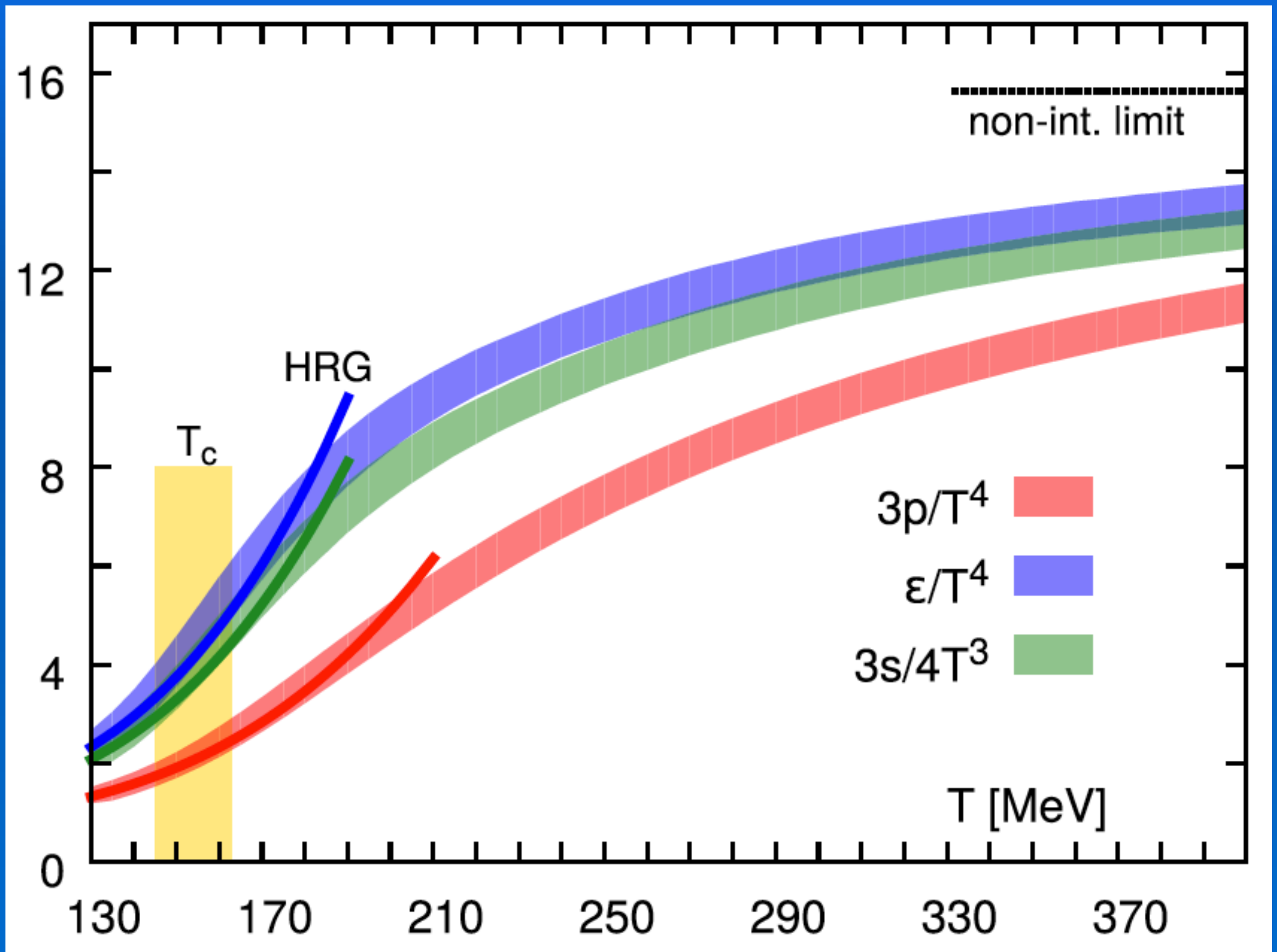
"E naufragar m'è dolce in questo Quark-Gluon Plasma "

(and sweetly I sink in this…)

freely adapted from *L'Infinito* (the Infinity), by the italian poet *Giacomo Leopardi.*

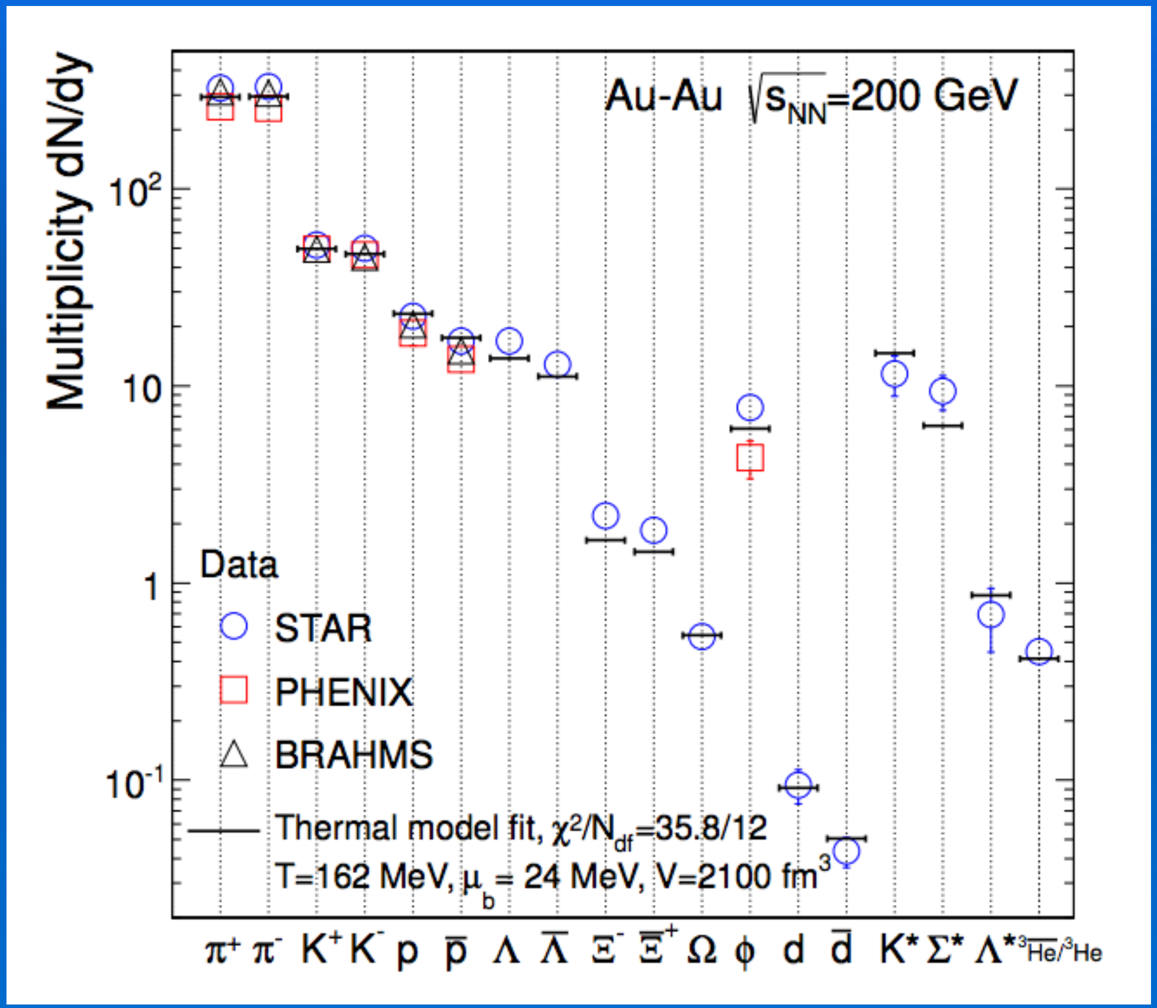
HRG - a useful tool

Lattice



HOTQCD, Phys.Rev. D90 (2014) 094503

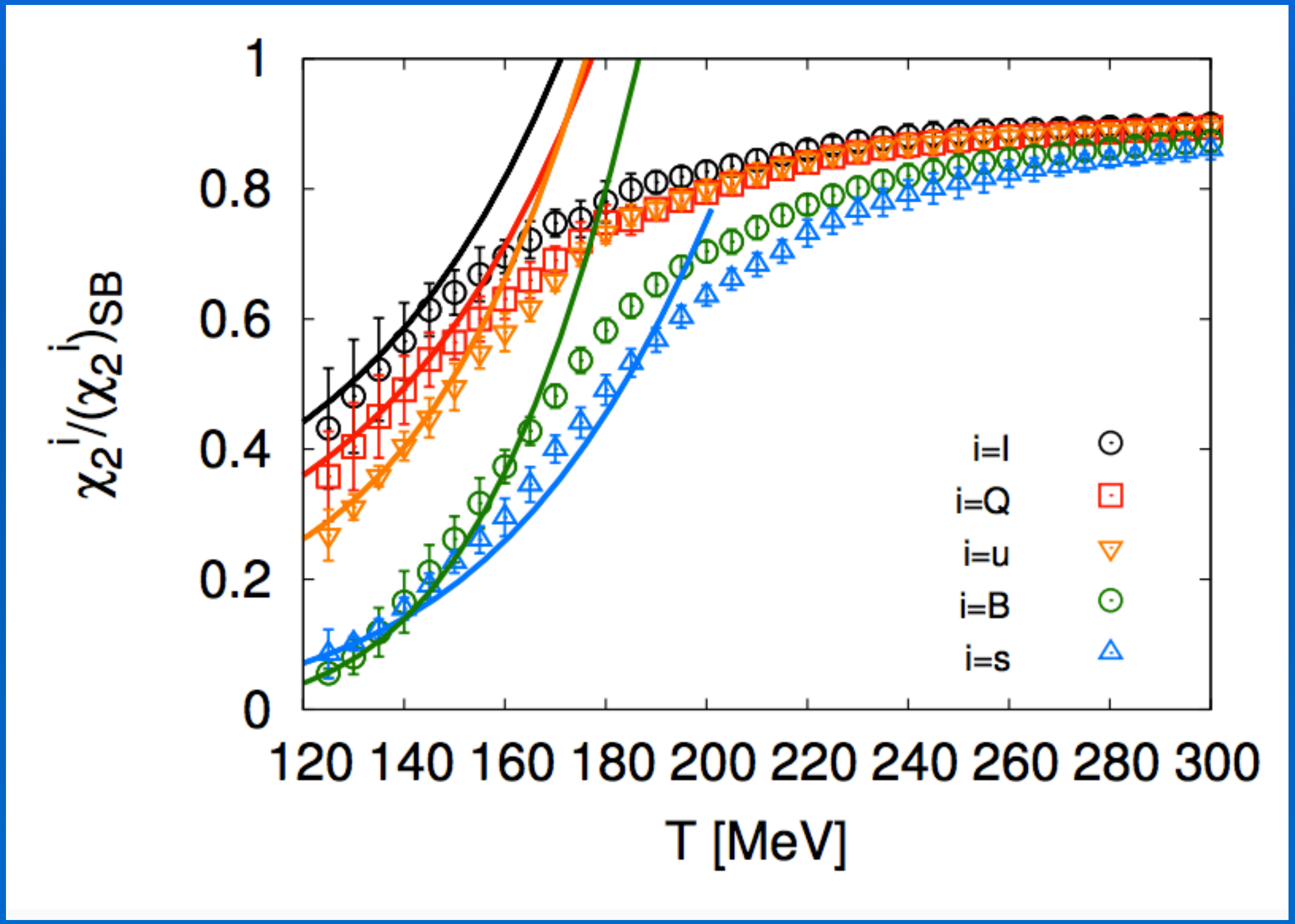
Experiment

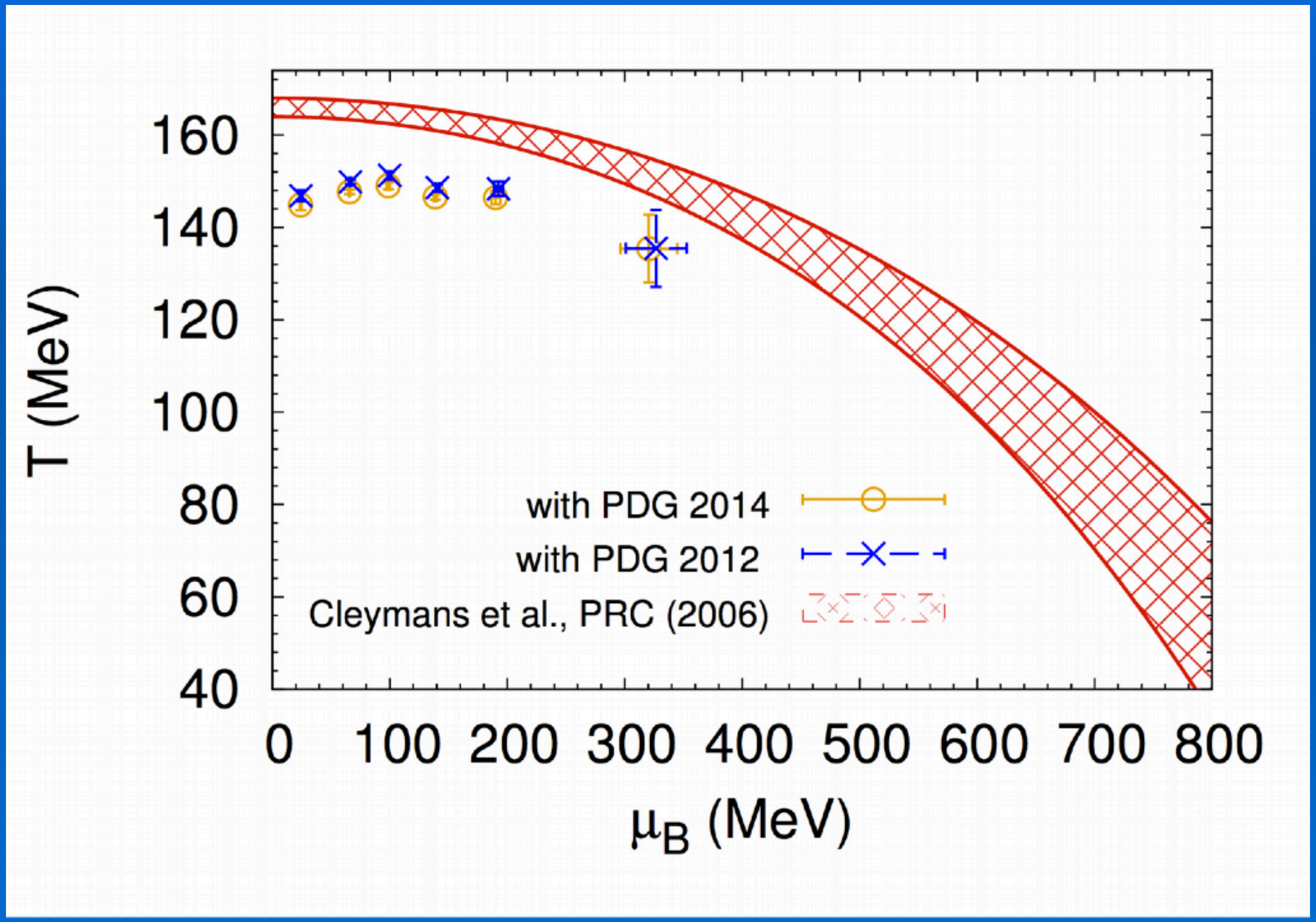


Andronic et al., Nucl.Phys. A904-905 (2013) 535c-538c

HRG - a useful tool

Lattice   Higher-Orders!!!   Experiment





Borsanyi et al., JHEP 1201 (2012) 138

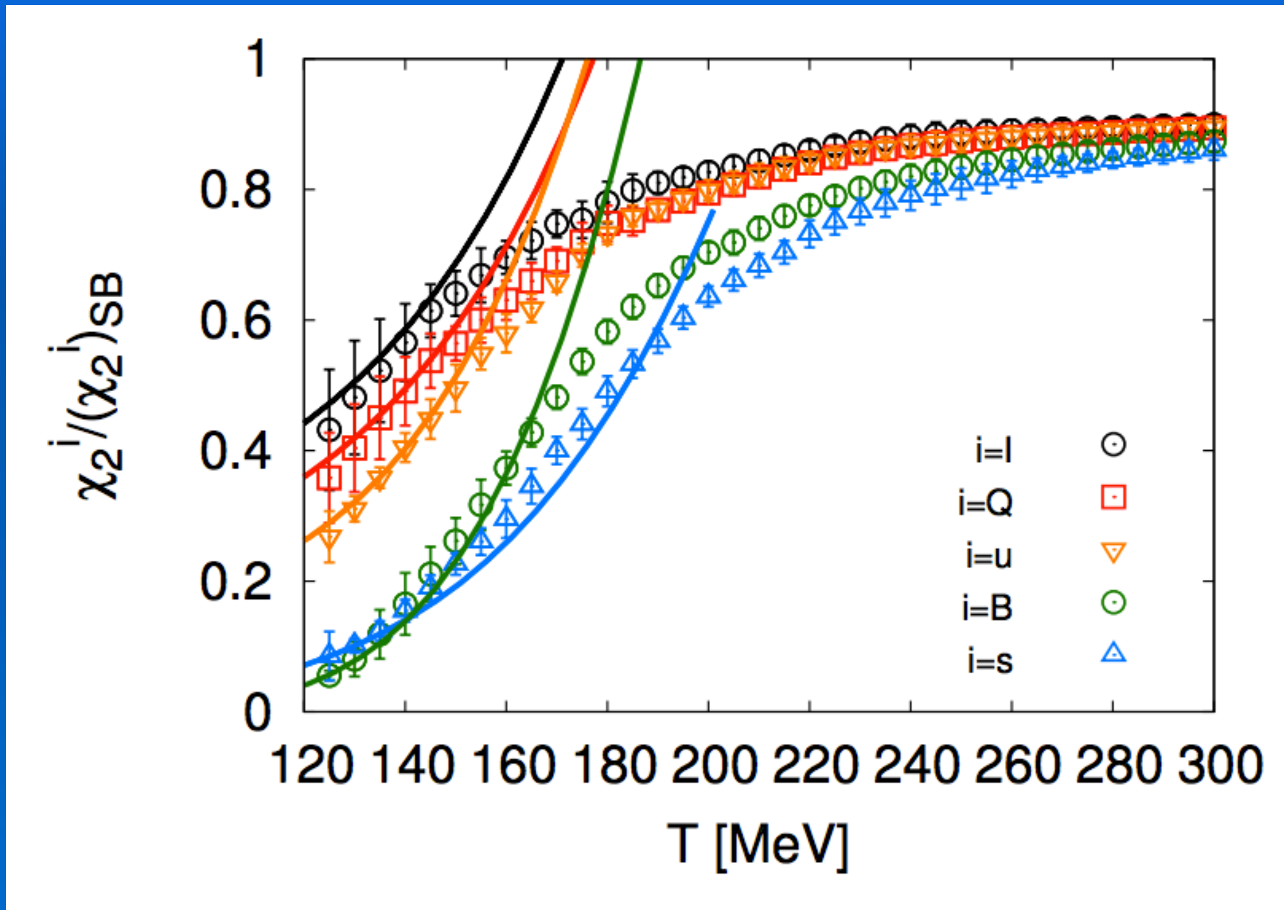
P.A. et al., Phys.Lett. B738 (2014) 305-310

HRG - a useful tool

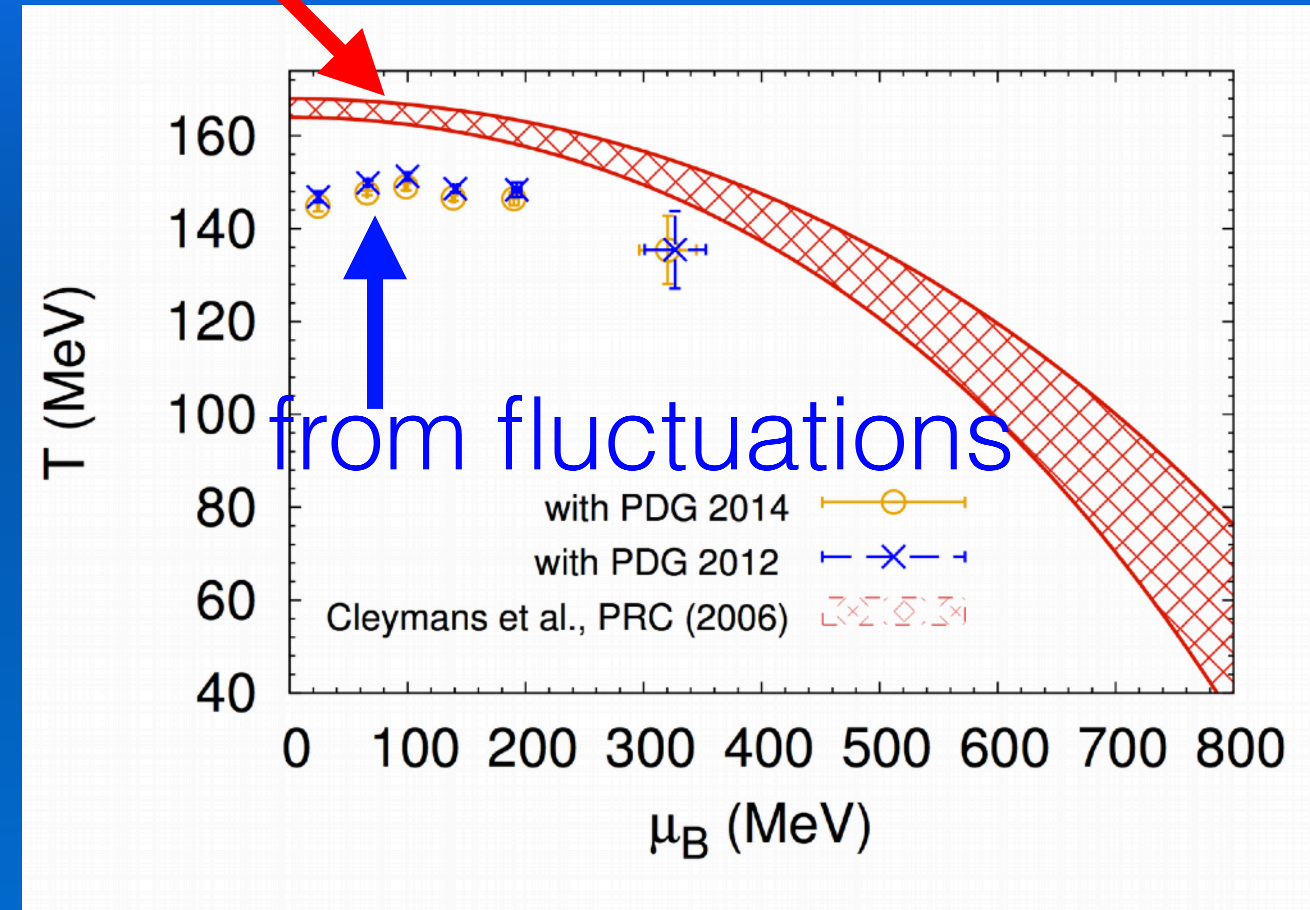
Lattice

from yields

Experiment



Borsanyi et al., JHEP 1201 (2012) 138



P.A. et al., Phys.Lett. B738 (2014) 305-310

The HRG model

A system of non-interacting resonances can describe most of the *attractive* interactions among hadrons.

$$\ln \mathcal{Z}(T,\{\mu_i\}) = \sum_{i \in Particles} (-1)^{B_i+1} \frac{d_i}{(2\pi^3)} \int d^3\vec{p}\, \ln\left[1 + (-1)^{B_i+1} e^{-(\sqrt{\vec{p}^2 + {m_i}^2} - \mu_i)/T}\right]$$

$$\mu_i = chemical\ potential$$

$$B_i = Baryon\ number$$

⋮

Here particles are assumed to be pointlike, with an infinite life-time, masses in vacuum, etc.

The HRG model

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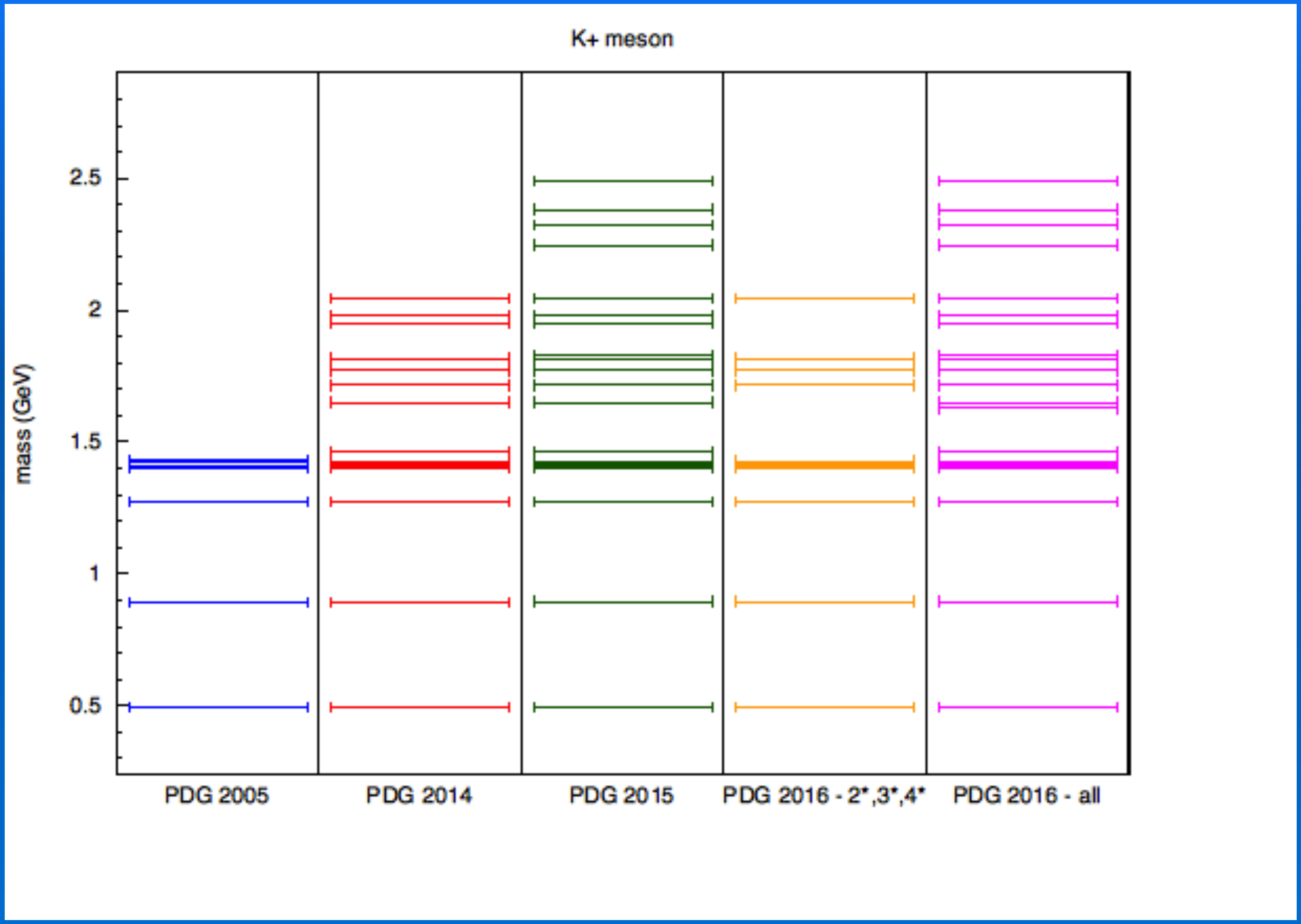
$$\ln \mathcal{Z}(T, \{\mu_i\}) = \sum_{i \in Particles} (-1)^{B_i+1} \frac{d_i}{(2\pi^3)} \int d^3\vec{p} \, \ln \left[1 + (-1)^{B_i+1} e^{-(\sqrt{\vec{p}^2 + {m_i}^2} - \mu_i)/T} \right]$$

$$n_i = \frac{d_i}{(2\pi)^3} \int d^3\vec{p} \frac{1}{e^{(\omega_i - \mu_i)/T} \pm 1}$$

Multiverse of HRG

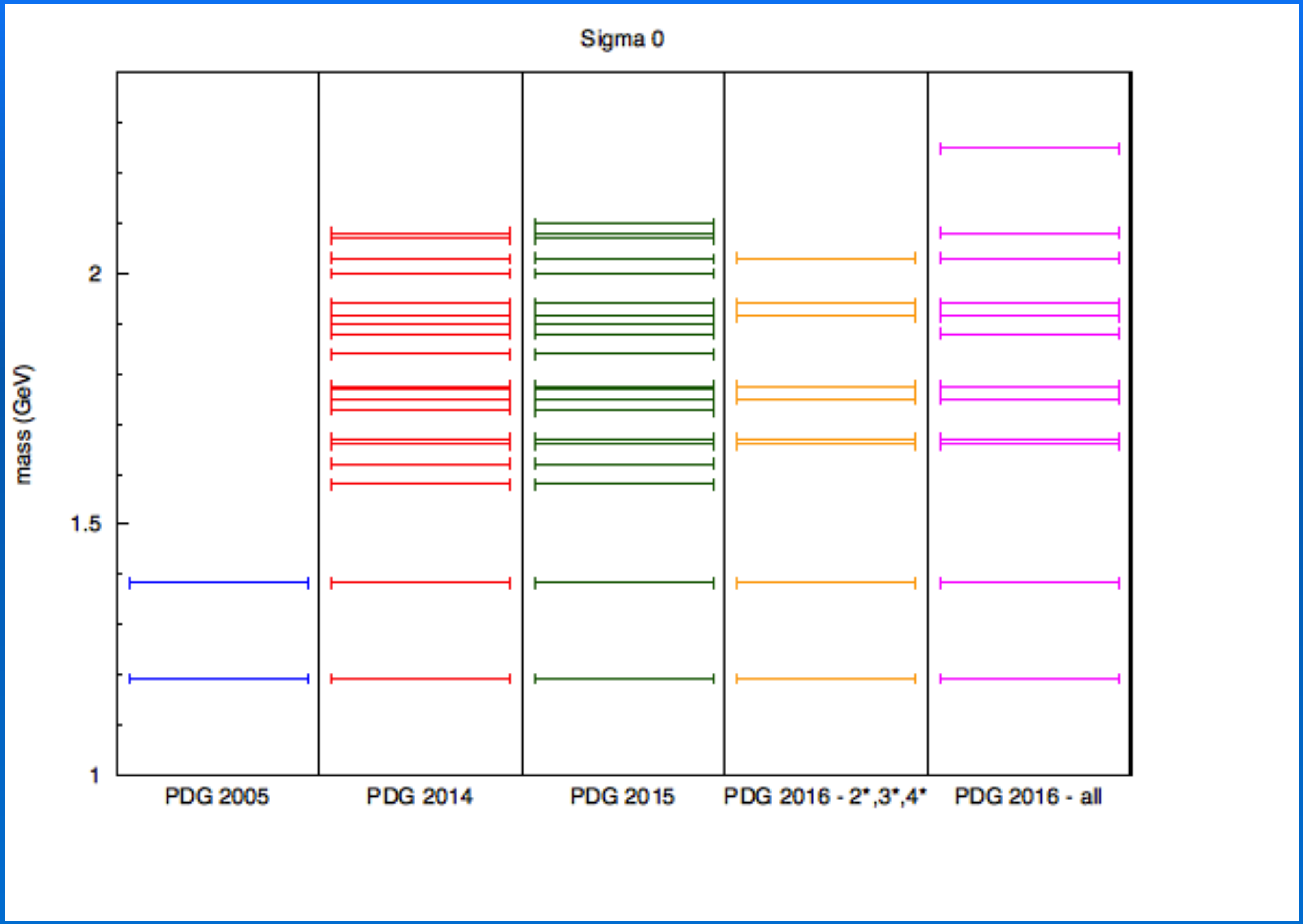
- Particle list: density of states in the hadronic spectrum
- Canonical vs Gran-Canonical (ensemble)
- Kinematic cuts: exp. particle multiplicities
- Multiple freeze-out scheme: flavour hierarchy — Bellwied et al., Phys.Rev.Lett. 111 (2013); Chatterjee et al., Phys.Lett. B727 (2013)
- Equilibrium vs non-equilibrium — Petran et al., Phys.Rev. C88 (2013)
- Initial system size fluctuations — Skokov et al., Phys.Rev. C88 (2013)
- Repulsive interactions among hadrons
- In medium effects: effective masses — Aarts et al., JHEP 1706 (2017)

Hadronic spectrum



The strange sector is the one which got the largest and most relevant changes in recent years.

Hadronic spectrum



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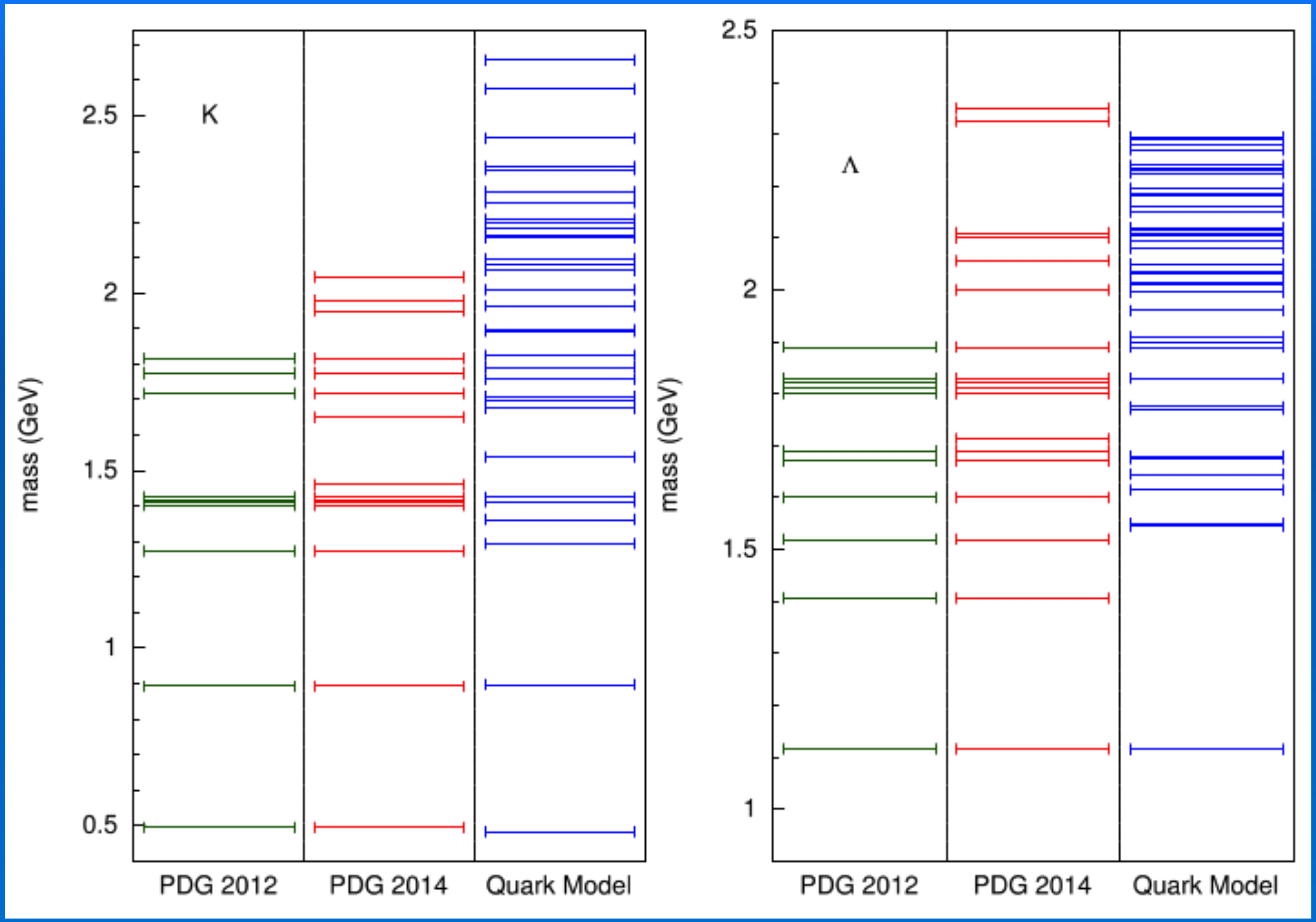


P.A. et al., Phys.Rev. D96 (2017) no.3, 034517

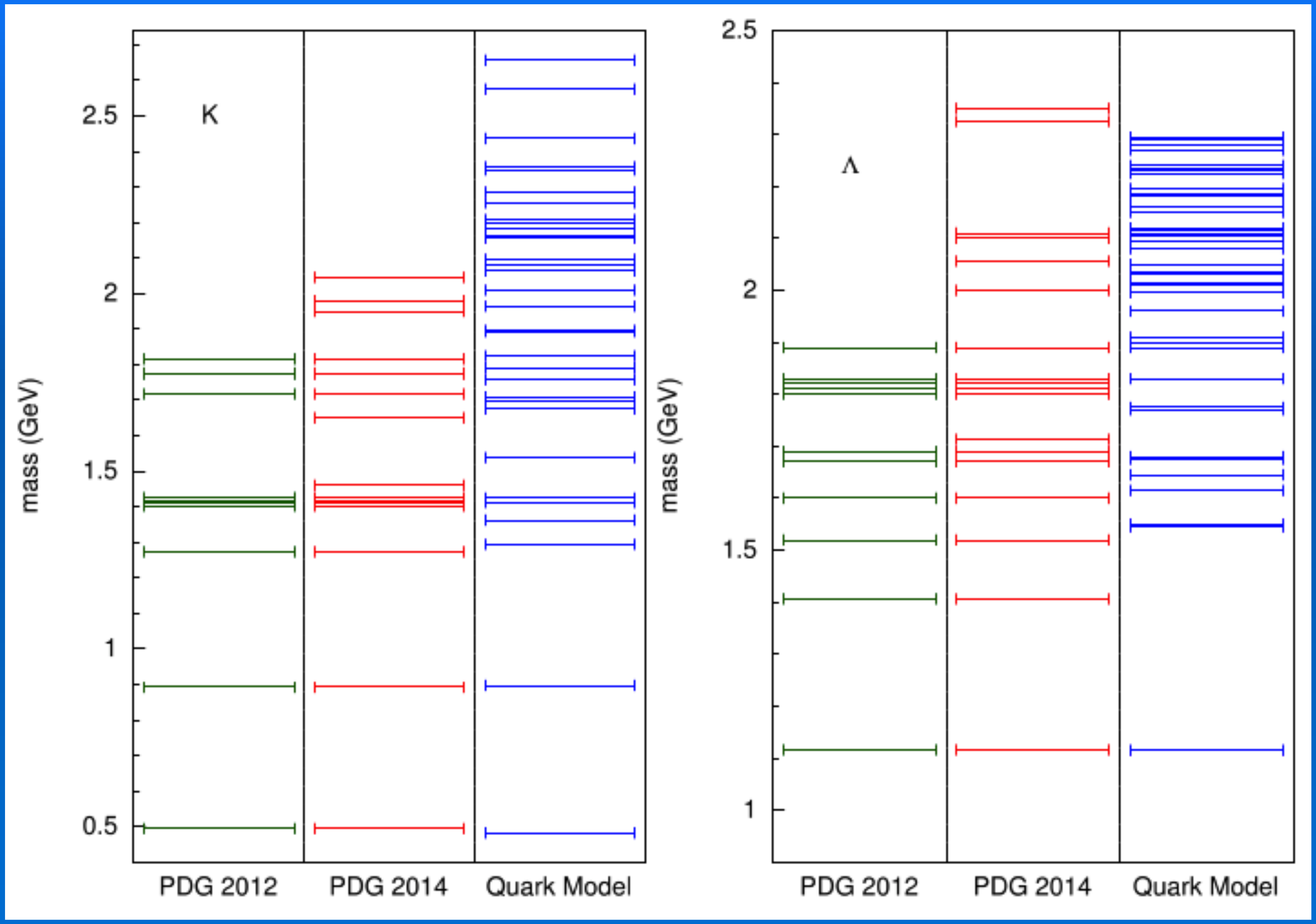
$\text{PDG} \simeq 600$

$\text{QM} \simeq 1500$

The *Quark Model* predicts a larger number of states with respect to the ones actually measured.

D. Ebert et al., Phys.Rev. D79 (2009) 114029
S. Capstick et al., Phys.Rev. D34 (1986) 2809

Hadronic spectrum



P.A. et al., Phys.Rev. D96 (2017) no.3, 034517

$\mathrm{PDG} \simeq 600$

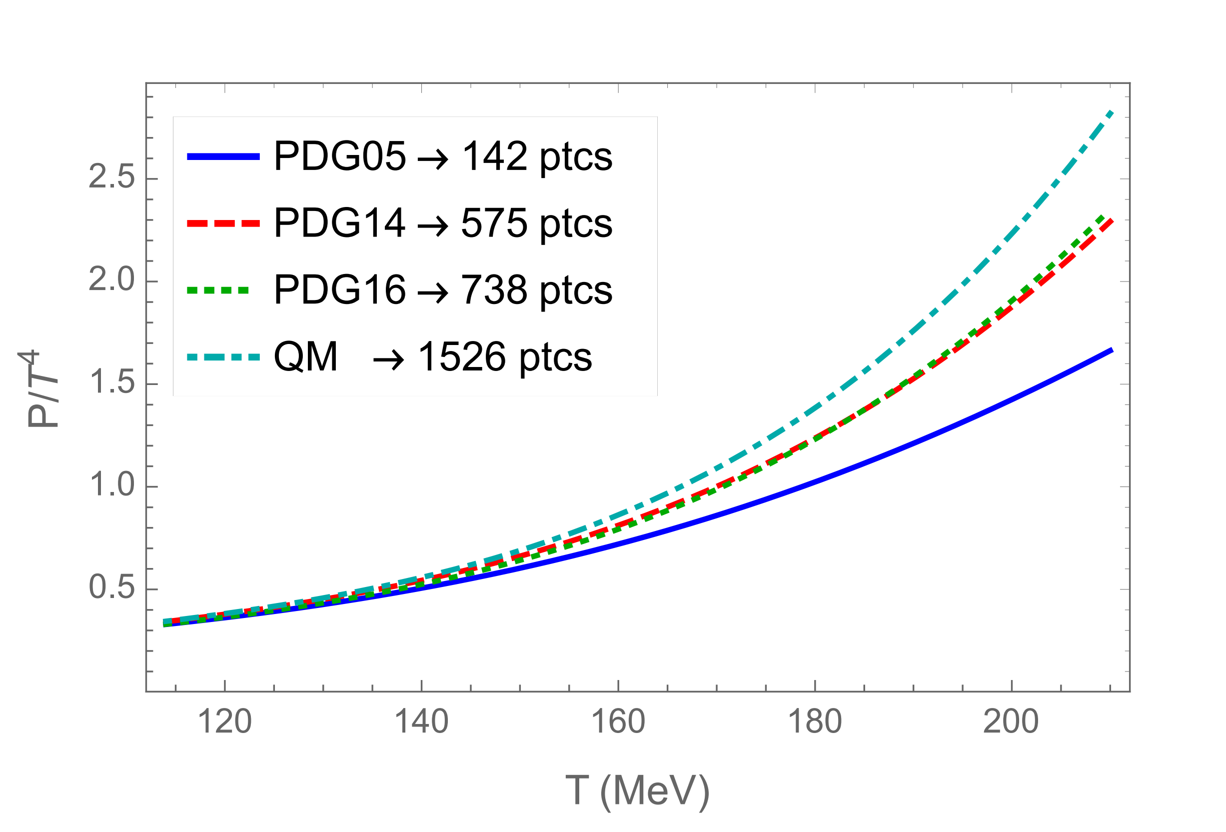
$\mathrm{QM} \simeq 1500$

Above a certain mass threshold, only the number of total states matters for a given set of quantum numbers.

D. Ebert et al., Phys.Rev. D79 (2009) 114029
S. Capstick et al., Phys.Rev. D34 (1986) 2809

Many particle effect

For most observables the effect of additional particles is quite trivial…



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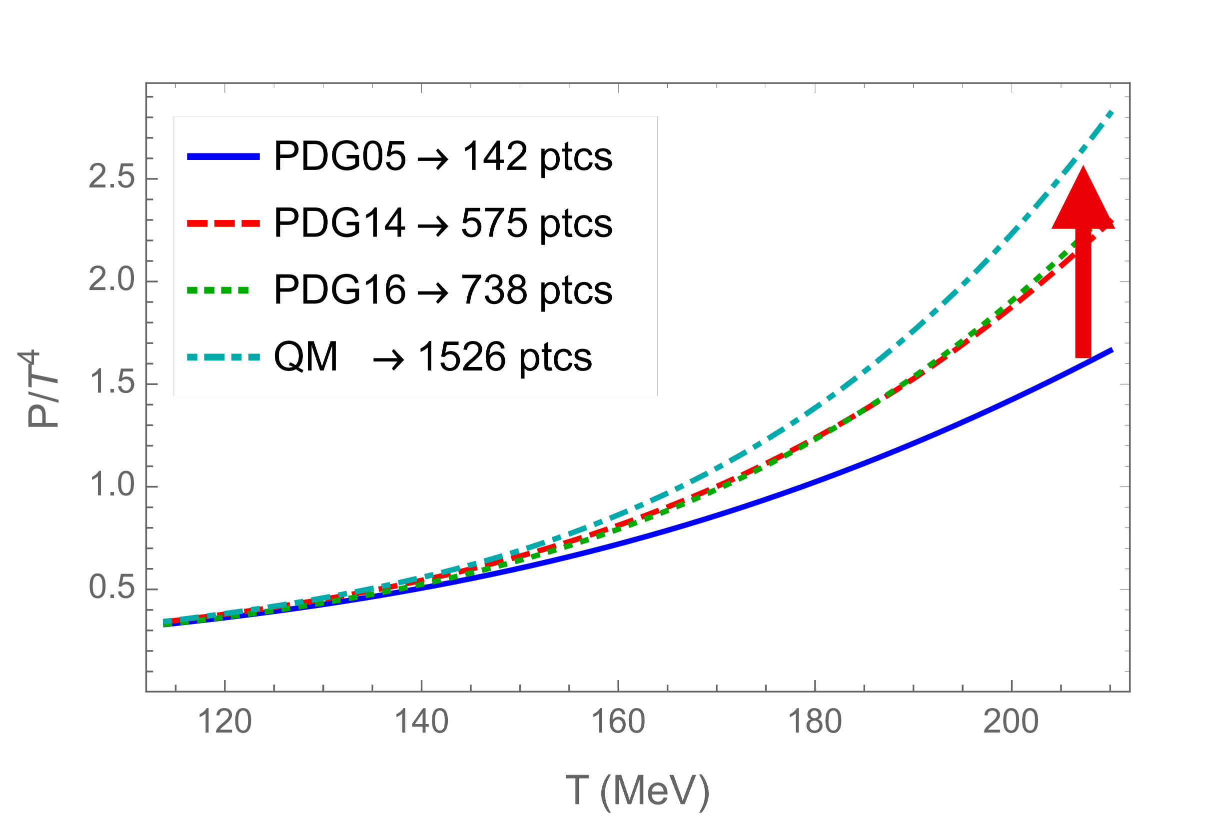
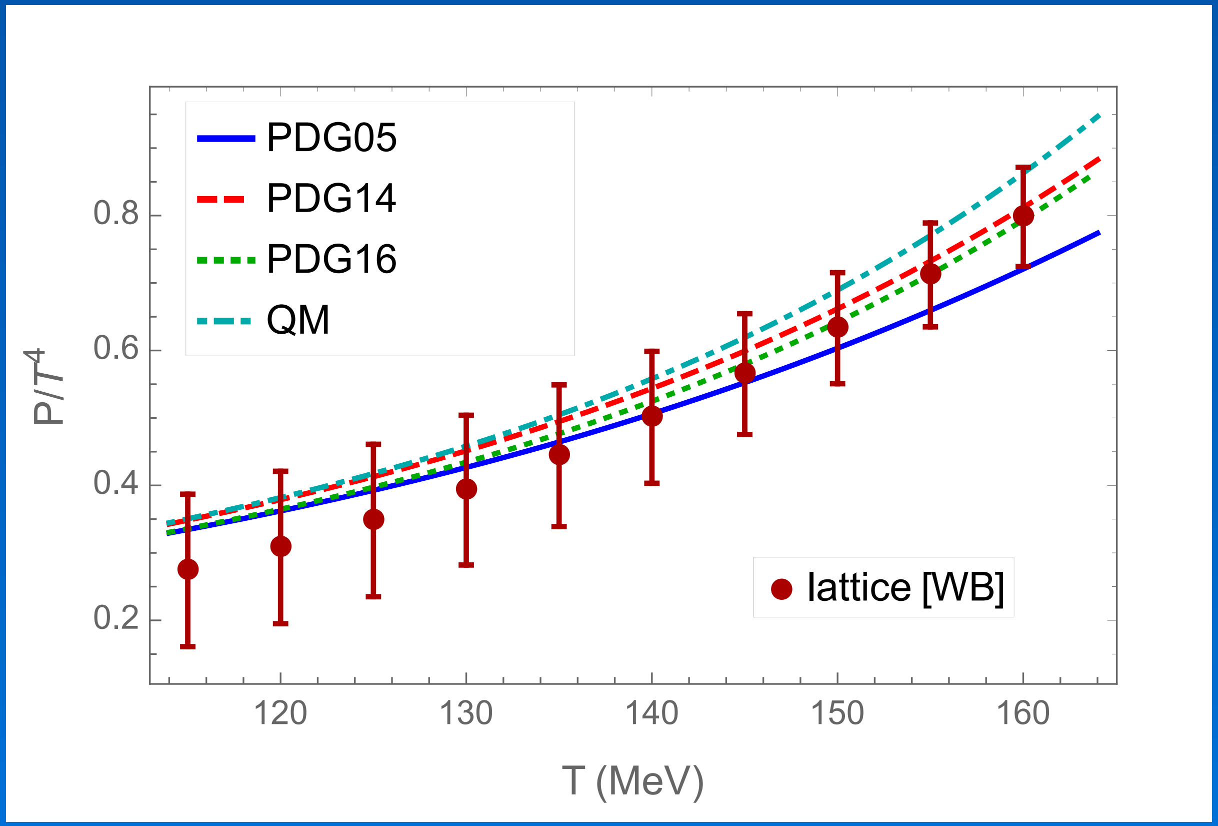


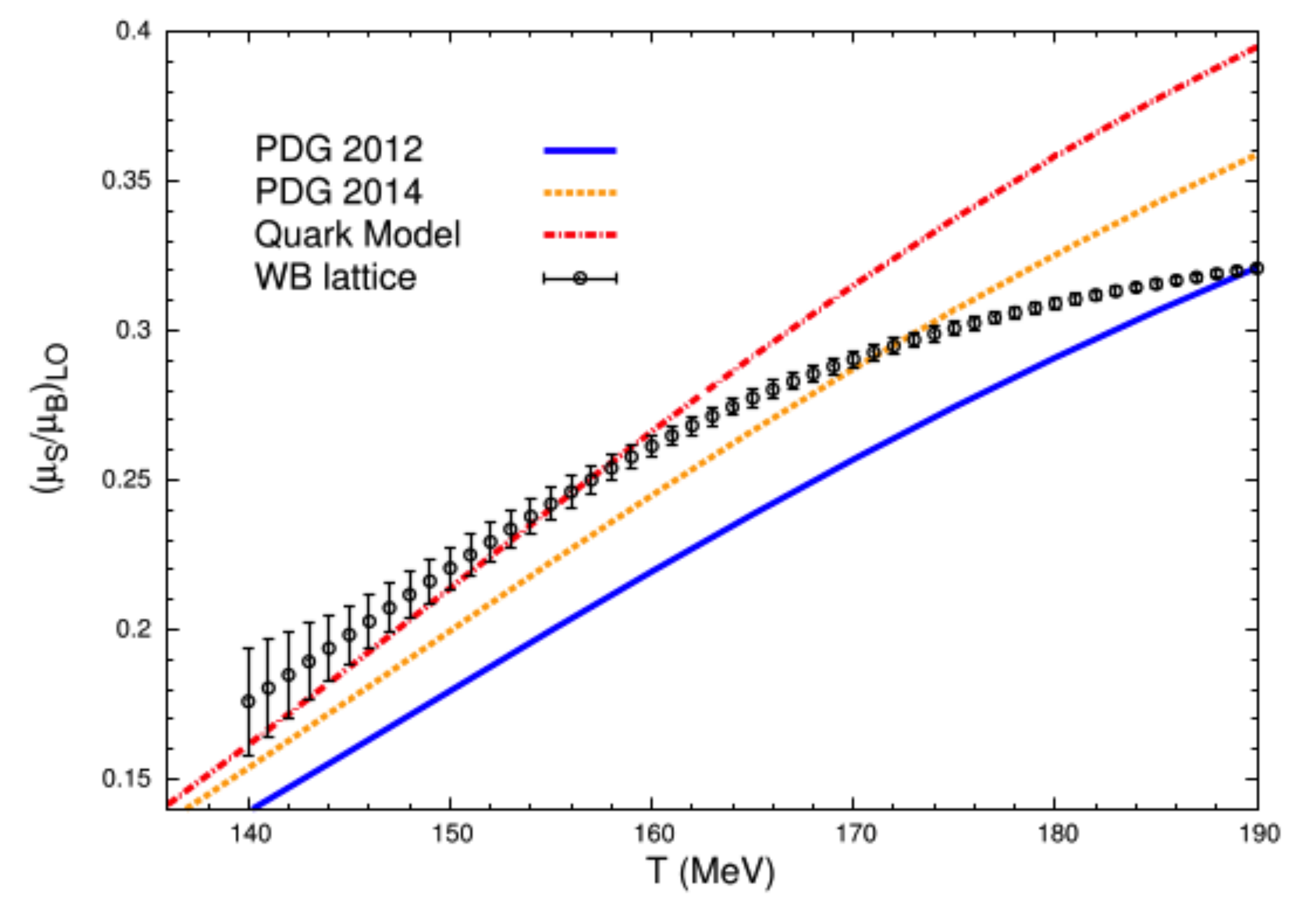
Many particle effect

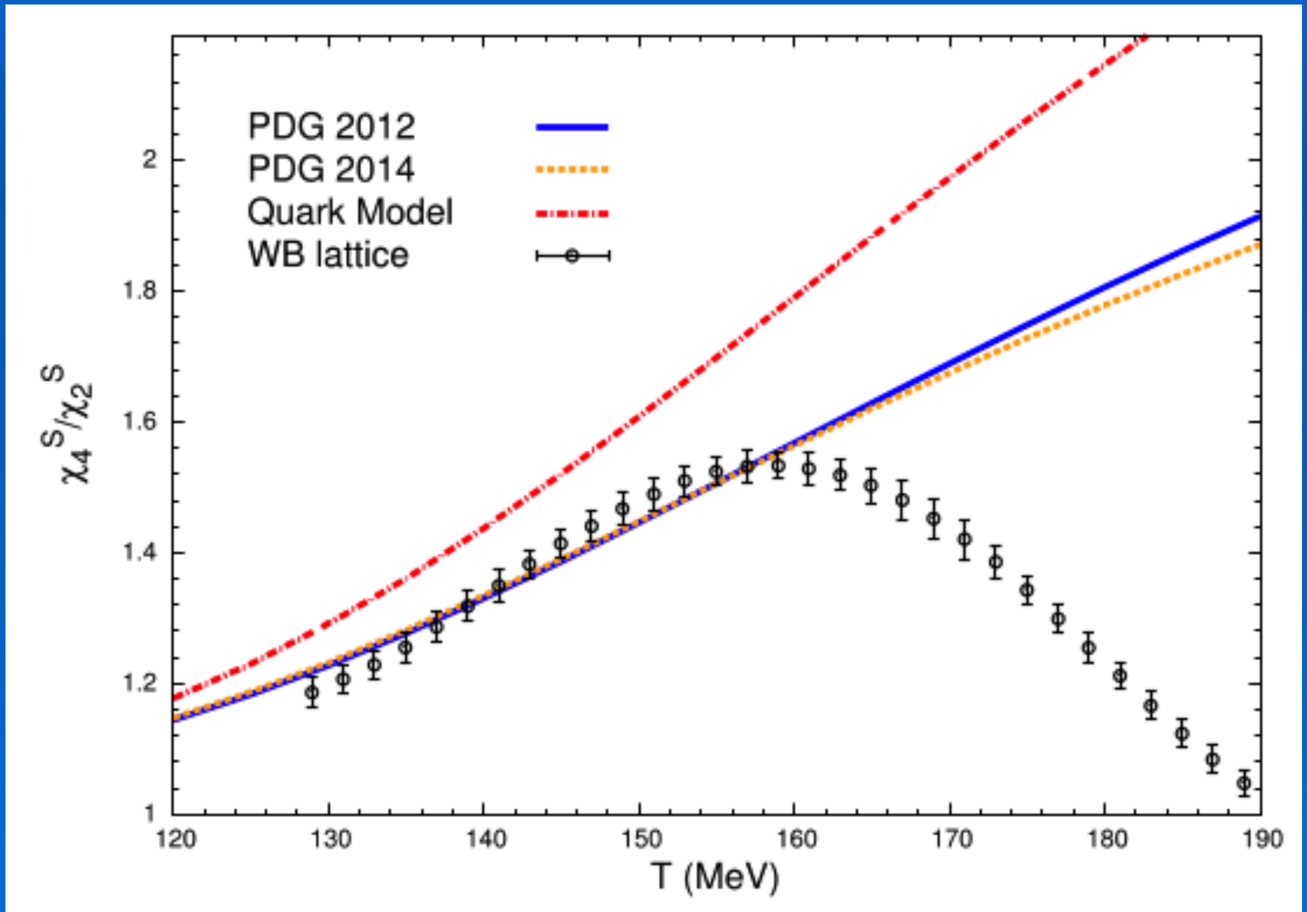
For most observables the effect of additional particles is quite trivial… the important is to have an agreement with lattice data in the relevant temperature range (?)



More strange baryons?

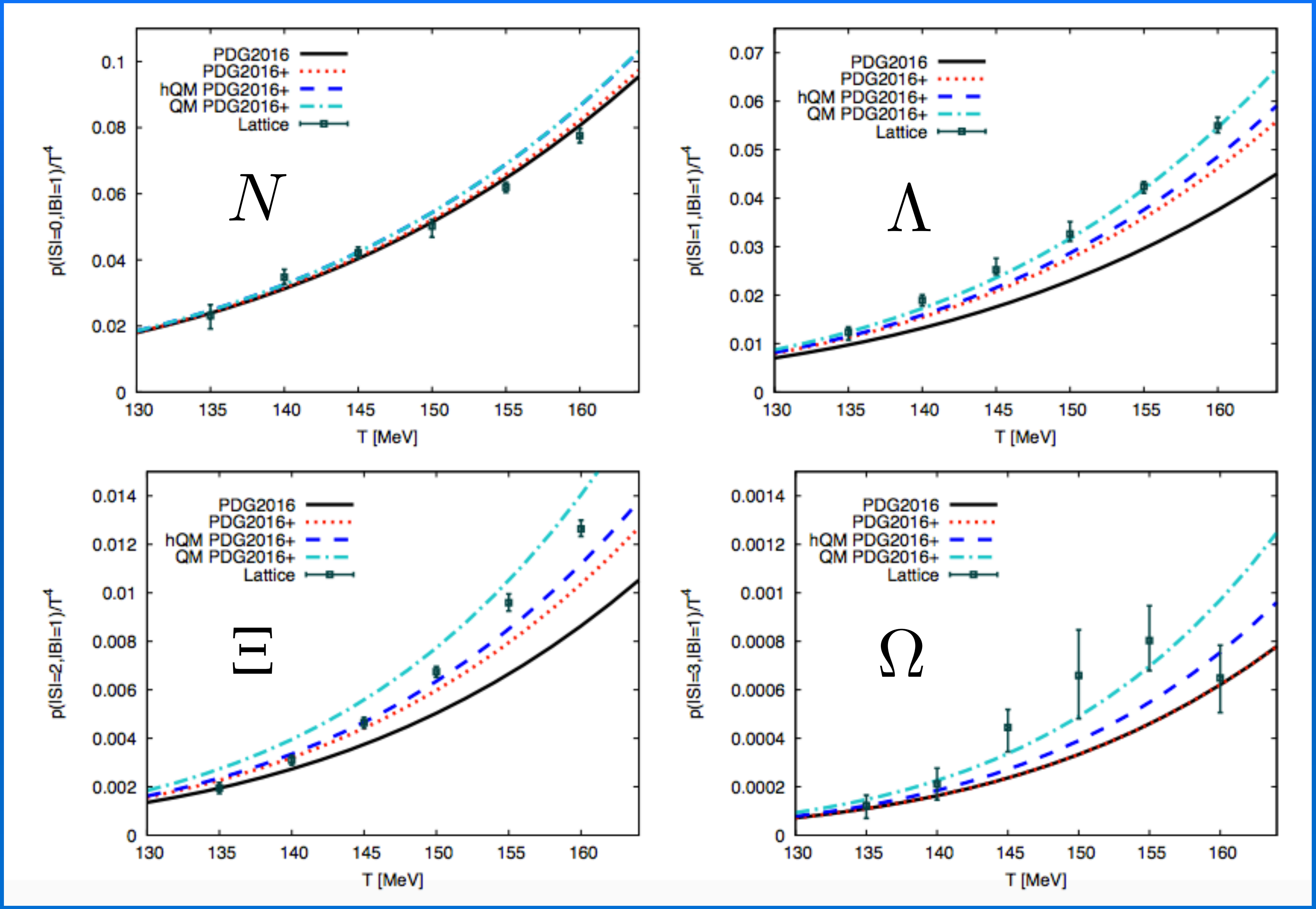
$$\left.\frac{\mu_S}{\mu_B}\right|_{LO} \simeq -\frac{\chi^{11}_{BS}}{\chi^2_S} \qquad\qquad \frac{\chi^4_S}{\chi^2_S} \simeq \langle S^2 \rangle$$





P.A. et al., Phys.Rev. D96 (2017) no.3, 034517
Bazavov et al., Phys.Rev.Lett. 113 (2014) no.7, 072001

Breaking down the spectrum

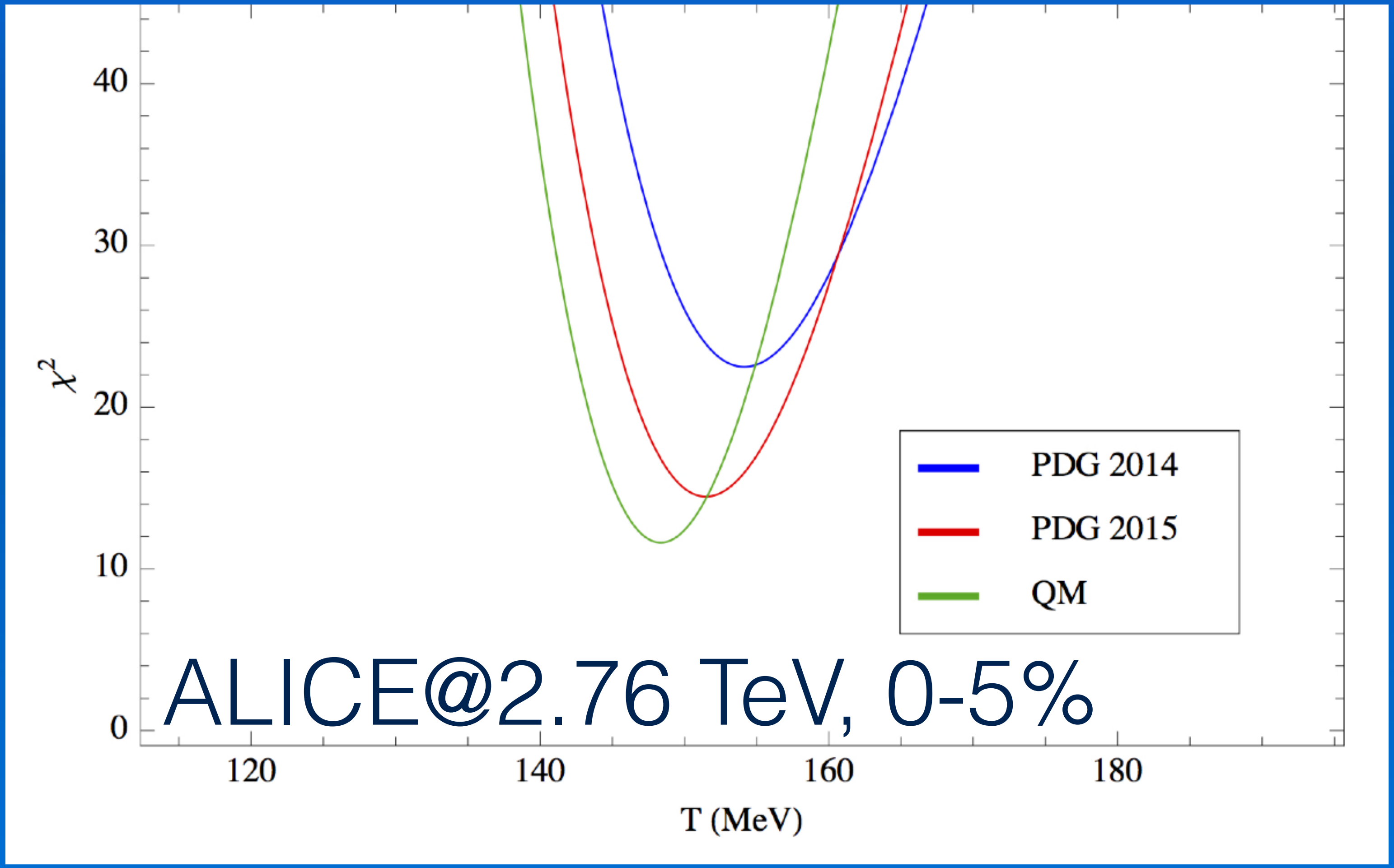


Particular combinations of fluctuations give selective informations on a specific hadronic sector.

P.A. et al., Phys.Rev. D96 (2017) no.3, 034517

Extra resonances on yields

The inclusion of higher mass states systematically improves the description of particle yields.



P.A. et al., in preparation

Extra resonances on yields

ALICE@2.76 TeV

	T (MeV)	μ_B (MeV)	V (fm ^{3})	χ^2/N_{dof}
PDG05	156.2±2.2	5.8±7.2	5224.8±624.8	$14.8/9\simeq1.6$
PDG14	155.2±2.2	3.8±7	4663.1±590.3	$20/9\simeq2.2$
PDG17	147.6±1.8	4.9±6.9	6995.8±792.6	$14.8/9\simeq1.6$
QM	148.3±1.8	6.9±7.2	6182.7±710.4	$11.4/9\simeq1.2$

STAR@200 GeV

	T (MeV)	μ_B (MeV)	V (fm ^{3})	χ^2/N_{dof}
PDG05	160.6±1.9	26.9±9	2208.9±227.1	$43.6/8\simeq5.4$
PDG14	164.1±2.3	29.6±8.4	1492.1±187.8	$14.1/8\simeq1.8$
PDG17	156.5±2.0	25.8±7.9	2234.9±268.7	$14/8\simeq1.8$
QM	157.0±1.9	31.1±8.3	1934.9±232.6	$7.4/8\simeq0.9$

Rapidity cuts: relevant?

ALICE@2.76 TeV

	T (MeV)	μ_B (MeV)	V (fm ^{3})	χ^2/N_{dof}
PDG05	156.2±2.2	5.8±7.2	5224.8±624.8	14.8/9≃1.6
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QM	148.3±1.8	6.9±7.2	6182.7±710.4	11.4/9≃1.2

Baryon chemical potential is stable

Worse chi^2

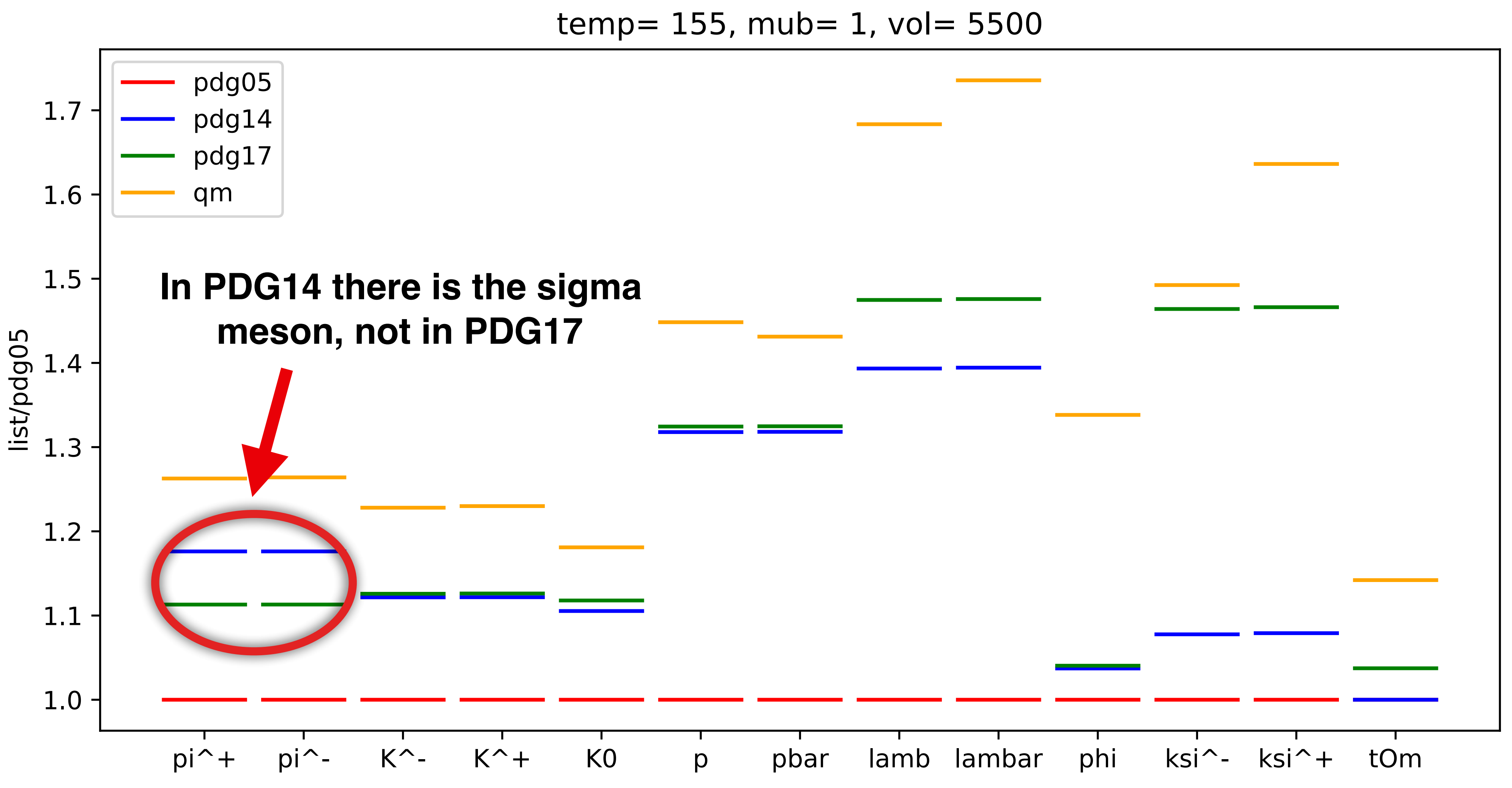
Lower Temperature!

Larger Volume!

Accounting for the rapidity cut, directly in the integral

	T (MeV)	μ_B (MeV)	V (fm ^{3})	χ^2/N_{dof}
PDG05	149.3±2.1	5.1±6.7	9578.7±1156.5	22.2/9≃2.4
PDG14	147.4±2.1	3.2±6.4	9203.7±1195.8	27.7/9≃3.08
PDG17	141.5±1.7	4.4±6.3	12662.9±1450.4	22./9≃2.4
QM	142.3±1.7	6.1±6.7	11176.1±1307.7	17.1/9≃1.9

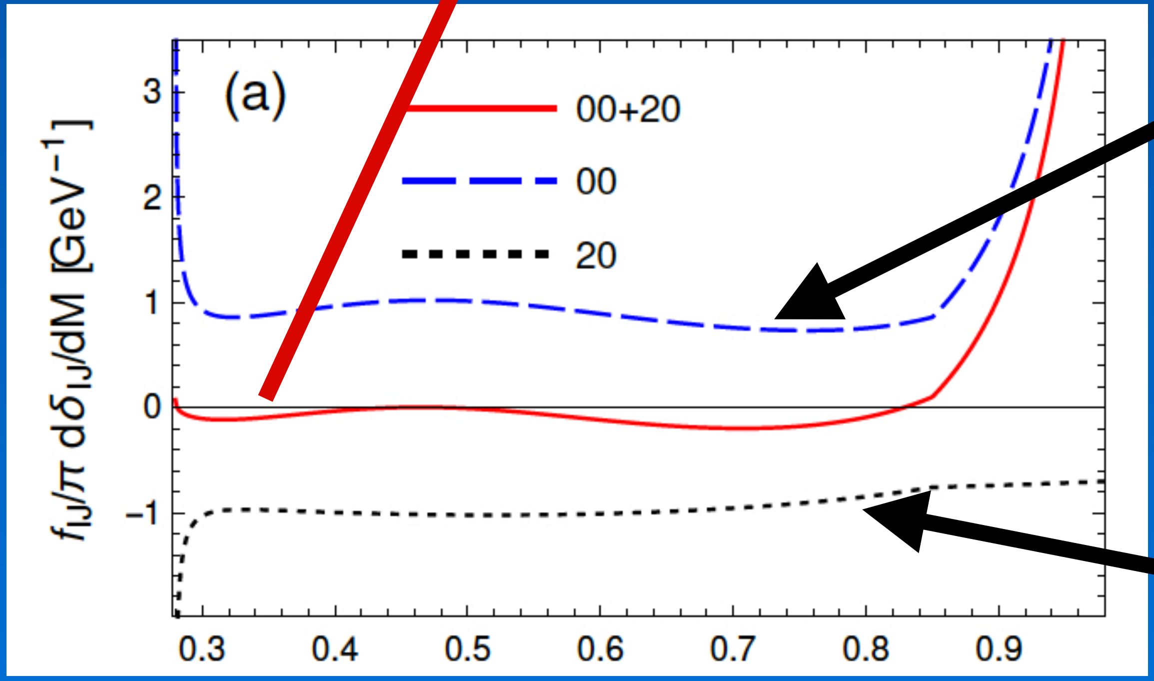
The chemistry of particle lists



Phase shift vs HRG

Repulsive channels counteract interactive ones. In the case of the sigma meson they completely balance each other.

$$\ln \mathcal{Z} = \ln \mathcal{Z}_\pi + f_{IJ} \int_0^\infty dM \frac{d\delta_{IJ}}{\pi dM} \int \frac{d^3\vec{p}}{(2\pi)^3} \ln \left[1 - e^{-E_p/T}\right]^{-1}$$



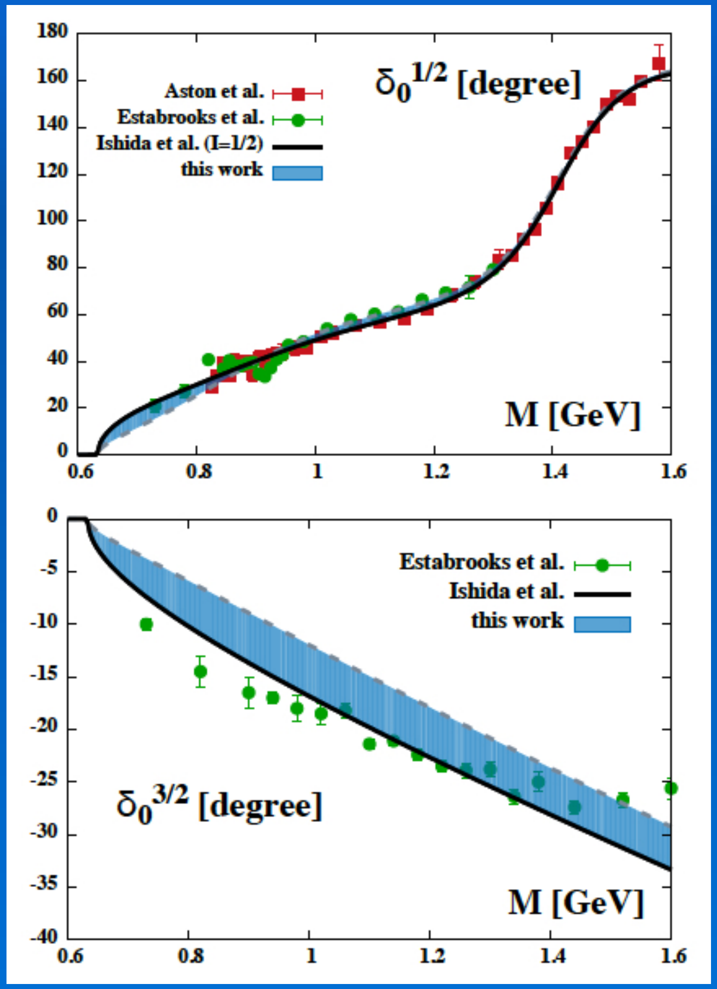
What is usually included in the HRG

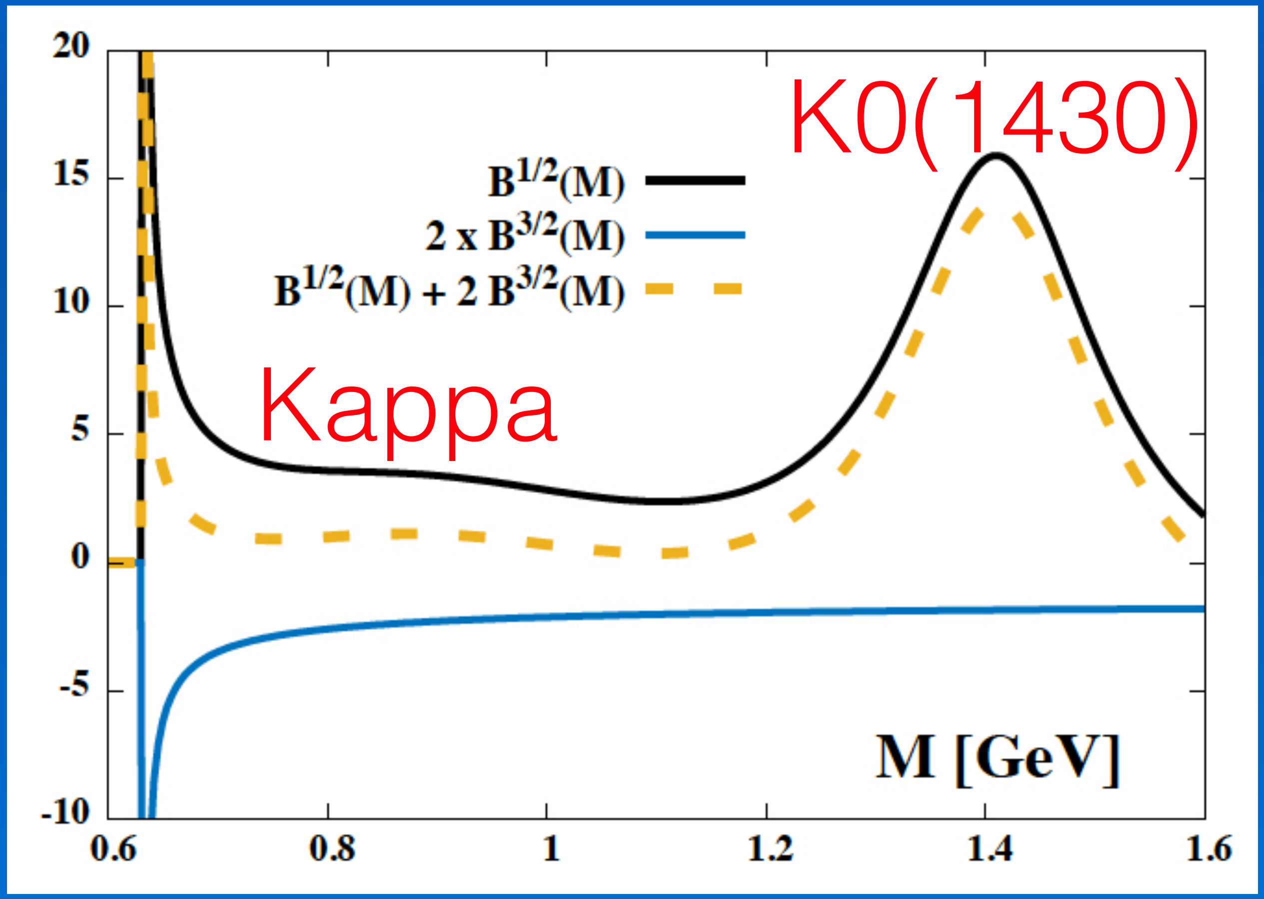
The contribution from repulsive channel

Broniowski et al., Phys.Rev. C92 (2015) no.3, 034905

Phase shift vs HRG

The same applies partially for other states, BUT we need to have experimental data for the corresponding channels.





Pok Man Lo et al., Phys.Rev. D92 (2015) no.7, 074003

Phase shift vs HRG

NN phase shifts have been used to calculate BS susceptibilities.

$$b_2(T) = \frac{2T}{\pi^3}\int_0^\infty dE(\frac{ME}{2}+M^2)K_2\left(2\beta\sqrt{\frac{ME}{2}+M^2}\right)\frac{1}{4i}\mathrm{Tr}\left[S^\dagger\frac{dS}{dE}-\frac{dS^\dagger}{dE}S\right]$$

$$\frac{1}{4i}\mathrm{Tr}\left[S^\dagger\frac{dS}{dE}-\frac{dS^\dagger}{dE}S\right] = \sum_{s=\pm}\sum_J(2J+1)\left(\frac{d\delta_s^{J,I=0}}{dE}+3\frac{d\delta_s^{J,I=1}}{dE}\right)$$

P. Huovinen et al., 1708.00879 [hep-ph]

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NN phase shifts have been used to calculate BS susceptibilities.

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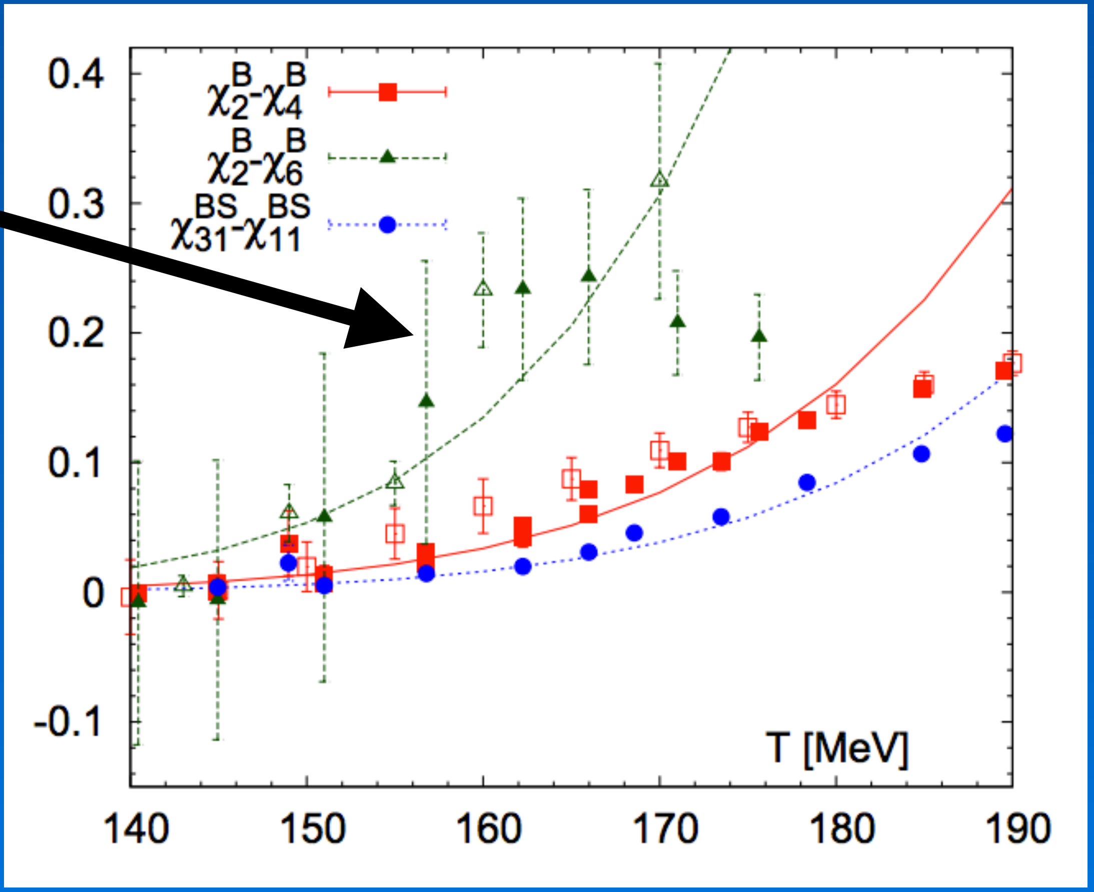
These combinations are zero in the standards HRG!!!!



These effects are entirely due to hadronic interactions!!!

P. Huovinen et al., 1708.00879 [hep-ph]

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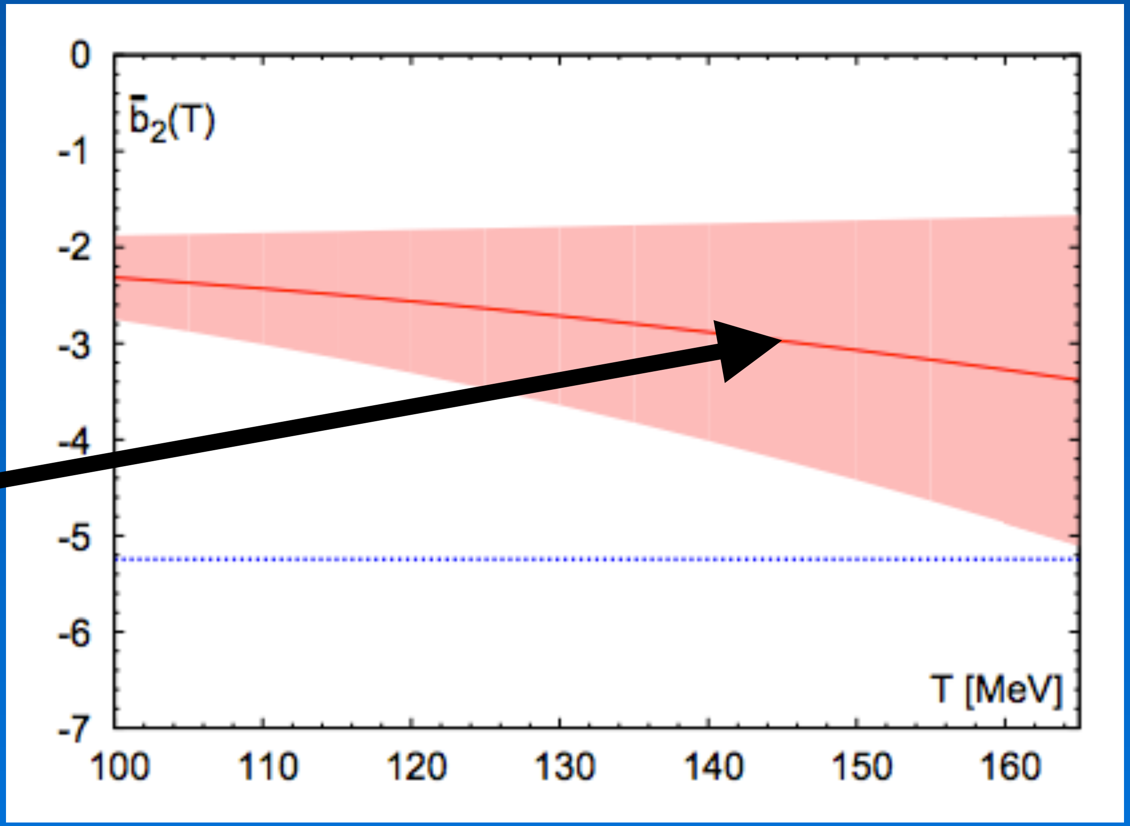
Unfortunately there is a large systematic, due to the lack of experimental data



Possible drawbacks:

- inelastic scattering?
- NN = NL = LL =… ?
- mesonic sector?

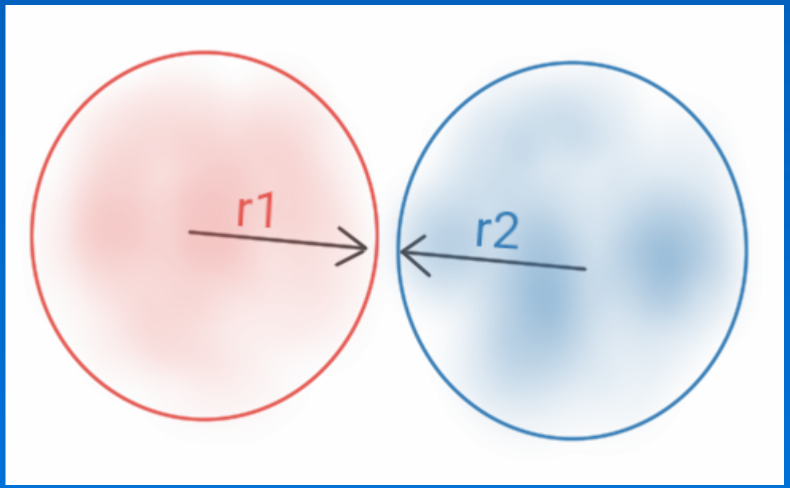
P. Huovinen et al., 1708.00879 [hep-ph]

EV effects into the HRG

Repulsive interactions can be easily implemented in the HRG by means of hard spheres. This results into a shifted chemical potential.

$$p(T,\vec{\mu}) = \sum_j p_j^{id}(T,\mu_j)$$

+



$$p(T,\vec{\mu}) = \sum_j p_j^{id}(T,\mu_j^*)$$

$$\mu_j^* = \mu_j - v_j p(T,\vec{\mu})$$

$$v_j = \frac{16}{3}\pi r_j^3$$

D.H. Rischke et al., Z.Phys. C51 (1991) 485-490

M. Albright et al., Phys.Rev. C90 (2014) no.2, 024915

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Repulsive interactions can be easily implemented in the HRG by means of hard spheres. This results into a shifted chemical potential.

$$n_B(T,\vec{\mu}) = \sum_i \frac{B_i n_i^{id}(T,\mu_i^*)}{1+\sum_j v_j n_j^{id}(T,\mu_j^*)}$$

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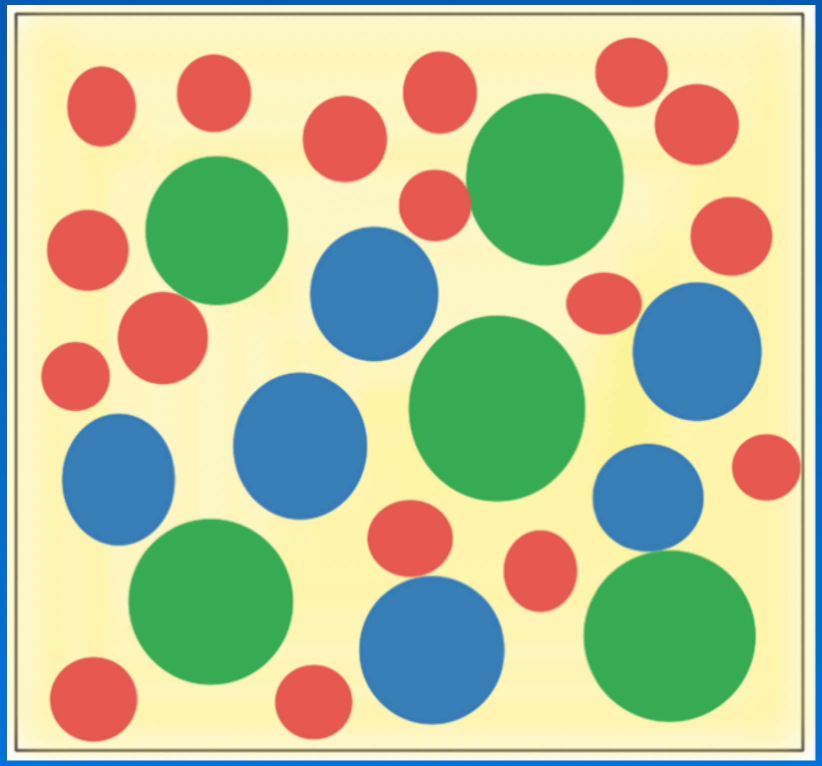

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$$\mu_j^* = \mu_j - v_j p(T,\vec{\mu})$$

$$v_j = \frac{16}{3}\pi r_j^3$$

$$fixed : v_j = v \;\forall\; j$$

$$direct : v_j \propto m_j$$

$$inverse : v_j \propto 1/m_j$$

D.H. Rischke et al., Z.Phys. C51 (1991) 485-490

M. Albright et al., Phys.Rev. C90 (2014) no.2, 024915

EV effects into the HRG

In order to be consistent with the virial expansion of pressure, modifications are needed.

$$n_B(T,\vec{\mu}) = \sum_i \frac{B_i n_i^{id}(T,\mu_i^*)}{1+\sum_j \widetilde{b}_{ij} n_j^{id}(T,\mu_j^*)}$$

$$p(T,\vec{\mu}) = \sum_j p_j^{id}(T,\mu_j^*)$$

$$\mu_i^* = \mu_i - \sum_j \widetilde{b}_{ij} p_j$$

Virial expansion: 2nd order

$$p(T,n_1,...,n_f) = T\sum_i n_i + T\sum_{ij} b_{ij} n_i n_j$$

$$b_{ij} = \frac{2\pi}{3}(r_i+r_j)^3$$

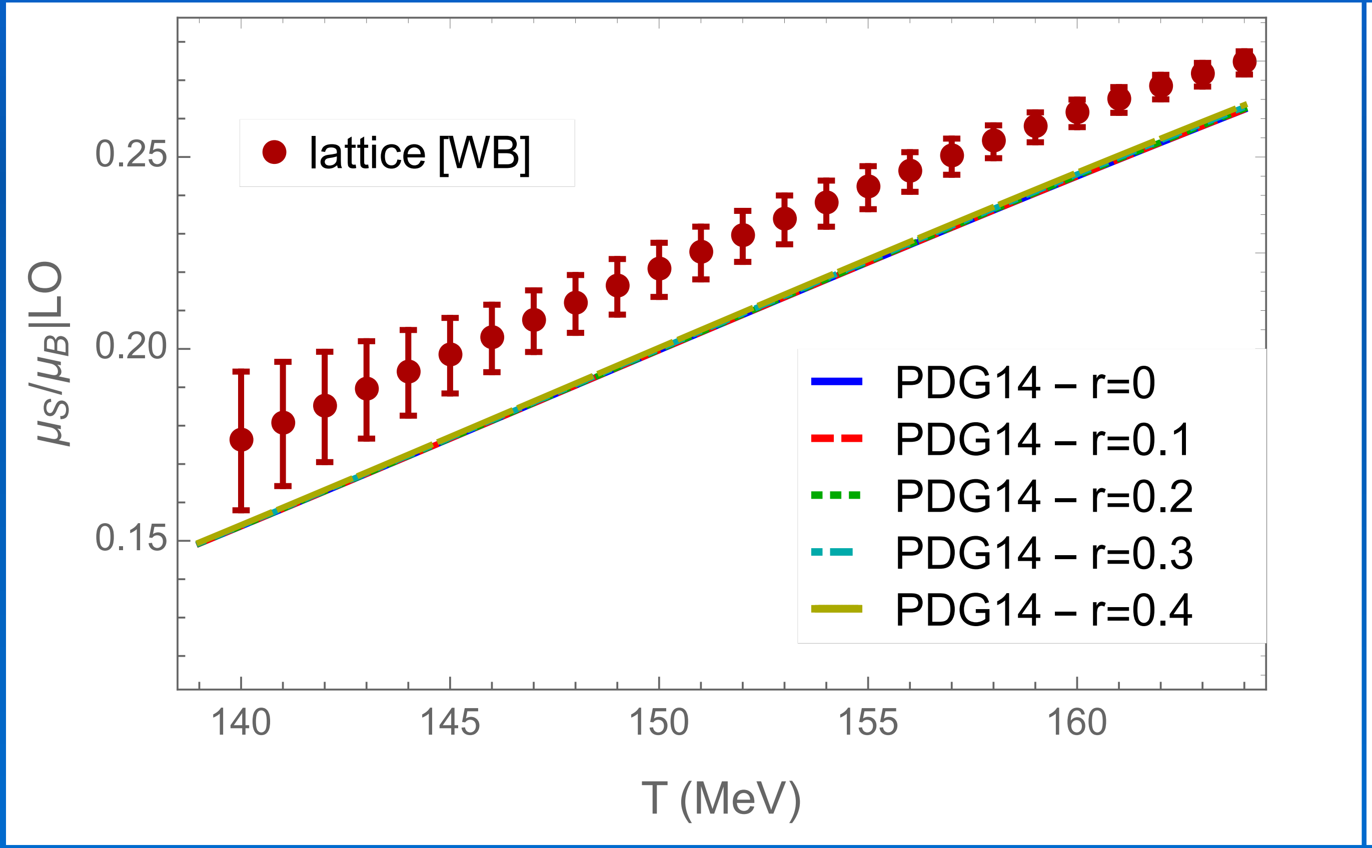
$$\widetilde{b}_{ij} = \frac{2b_{ii}b_{ij}}{b_{ii}+b_{jj}}$$

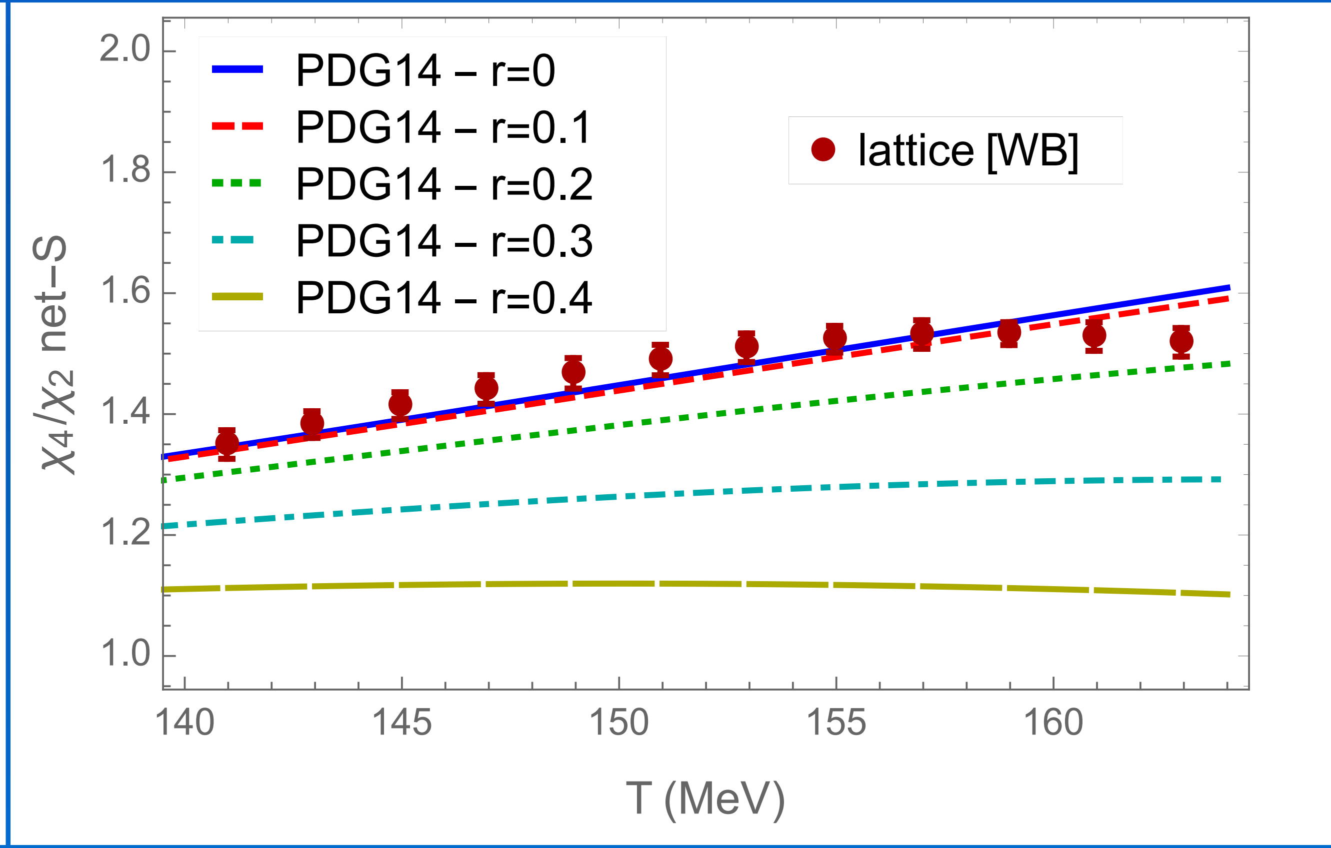
Vovchenko et al., Phys.Rev. C95 (2017) no.4, 044904

EV: lattice QCD

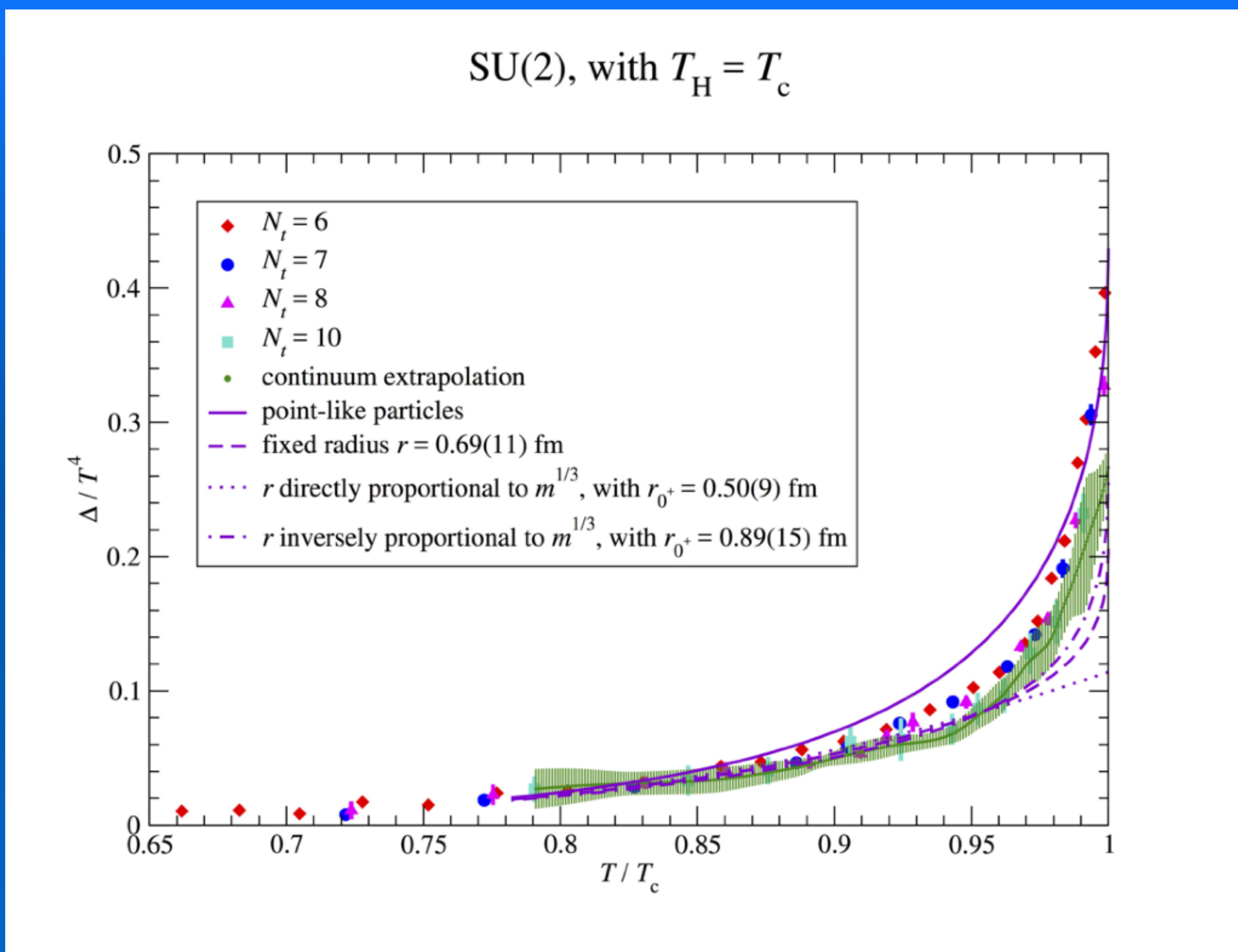
$$\left.\frac{\mu_S}{\mu_B}\right|_{LO} \simeq -\frac{\chi^{11}_{BS}}{\chi^2_S}$$

$$\frac{\chi^4_S}{\chi^2_S} \simeq \langle S^2 \rangle$$

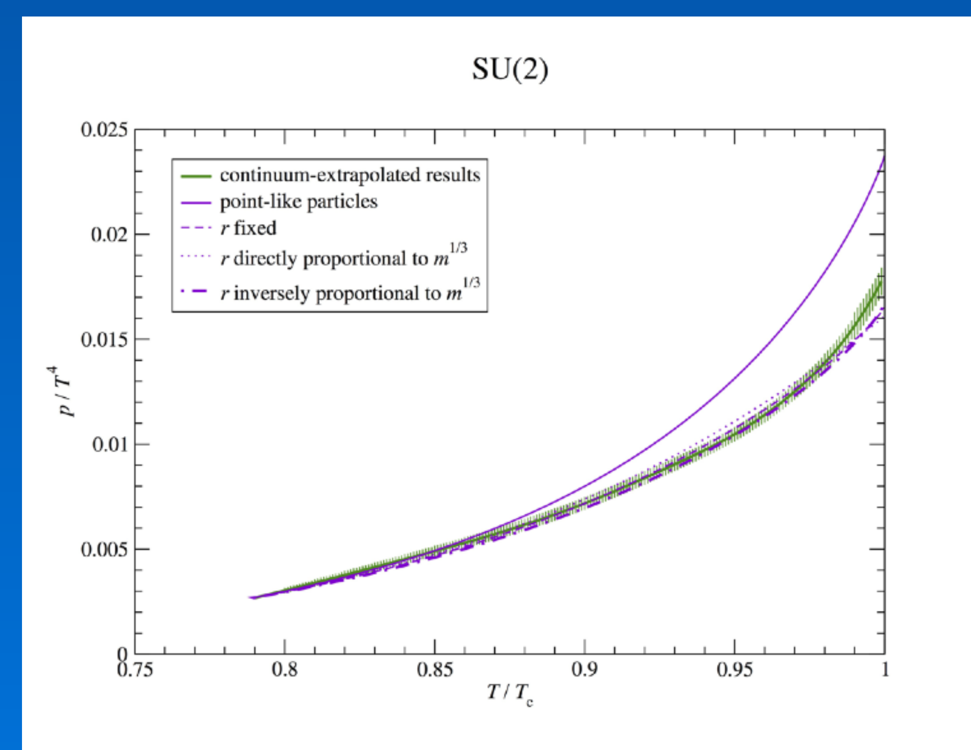




EV: Pure gauge

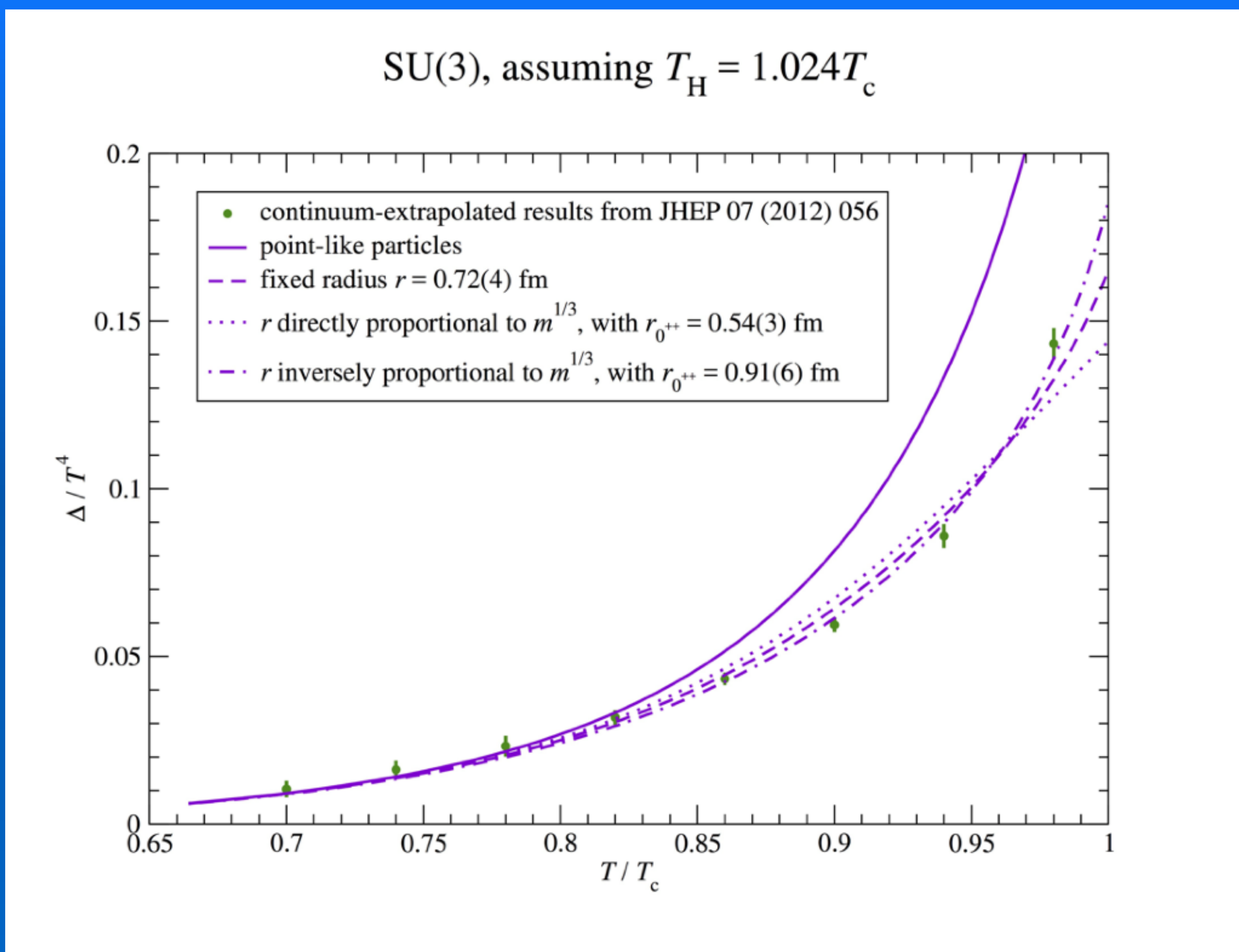


	r (fm)	Δr (fm)	χ^2
point like	0	0	11.25
fixed	0.69	0.114	0.917
direct	0.518	0.095	1.95
inverse	0.861	0.147	0.45

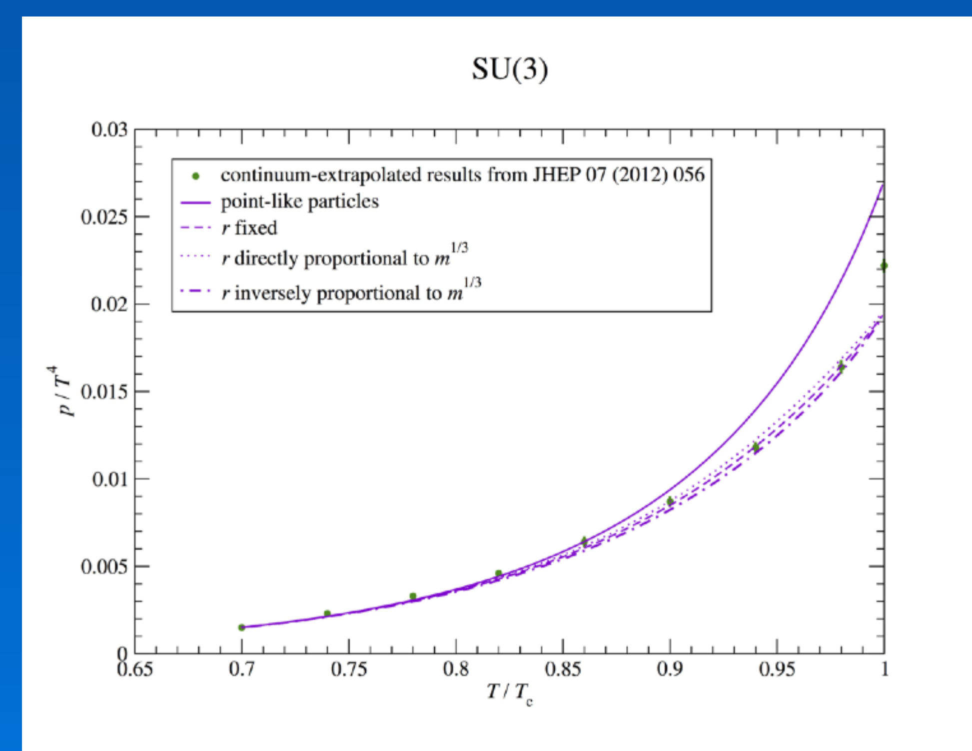


P.A. et al., Phys.Rev. D95 (2017) no.9, 094511

EV: Pure gauge



	r (fm)	Δr (fm)	χ^2
point like	0	0	54.73
fixed	0.717	0.047	2.07
direct	0.526	0.036	3.12
inverse	0.907	0.062	2.05



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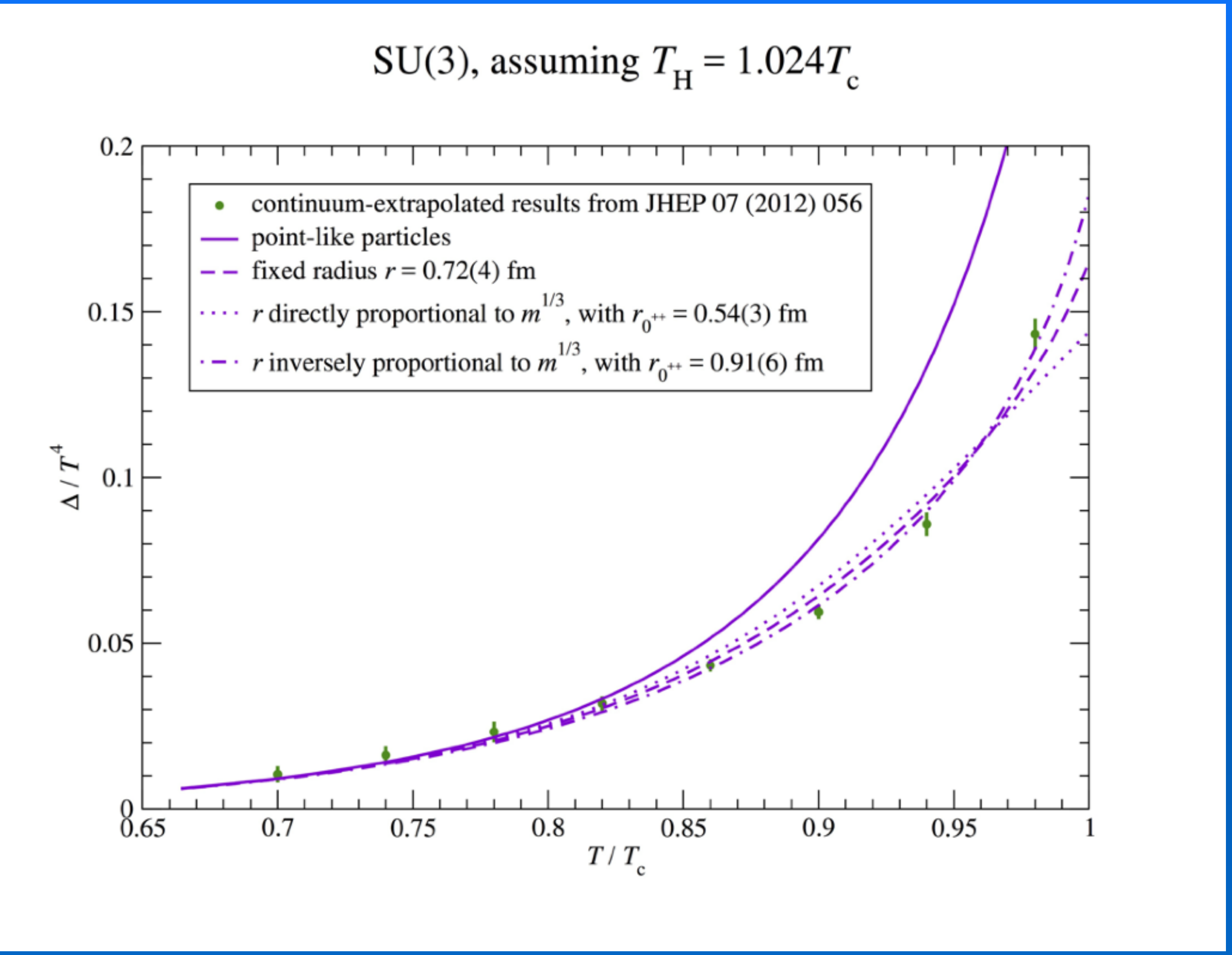

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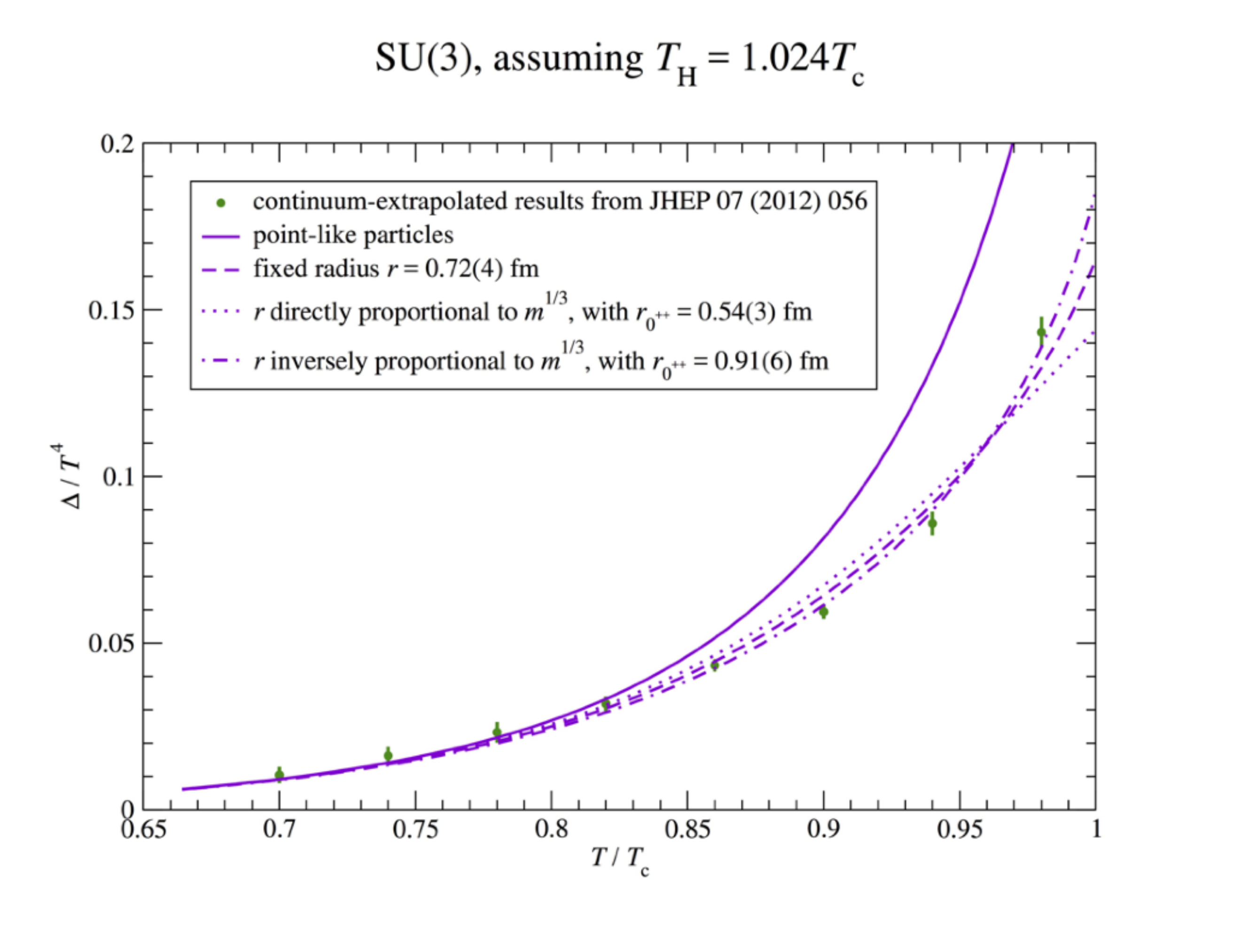
There is consistency between SU2 and SU3 glueball masses!!!!

P.A. et al., Phys.Rev. D95 (2017) no.9, 094511

EV: Pure gauge



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Would effective masses (temperature dependent) have the same effect???

P.A. et al., Phys.Rev. D95 (2017) no.9, 094511

EV + QM: fit to lattice

observables involved in the calculation of χ^2:

- thermodynamic: P/T^2, Δ/T^4;
- light: χ_4/χ_2 net-B, χ_4/χ_2 net-l, χ_{ud};
- strange: χ_4/χ_2 net-S, χ_{us}, μ_S/μ_B LO, χ_2^S .

$$T_{min} = 110 \ \ (\text{MeV})$$
$$T_{max} = 164 \ \ (\text{MeV})$$

number of lattice points = 111

	PDG05	PDG14	PDG17	QM
χ^2/N_{dof}	49.645	10.094	9.331	16.312

EV + QM: fit to lattice

I perform a fit to the lattice data, allowing light and strange particles to have a different behaviour, within the different EV schemes formerly showed.

		Fixed	Fixed
	χ^2/N_{dof}	r_p (fm)	r_Λ (fm)
PDG05	44.3	0.446 ±0.115	0.173 ±0.133
PDG14	5.723	0.389 ±0.101	0.173 ±0.1
PDG17	4.28	0.383 ±0.1	0.217 ±0.066
QM	6.263	0.351 ±0.099	0.274 ±0.044

EV + QM: fit to lattice

I perform a fit to the lattice data, allowing light and strange particles to have a different behaviour, within the different EV schemes formerly showed.

		Direct	Direct
	χ^2/N_{dof}	r_p (fm)	r_Λ (fm)
PDG05	45.48	0.394 ±0.093	0.004 ±0.432
PDG14	4.719	0.375 ±0.081	0.016 ±0.508
PDG17	3.595	0.373 ±0.085	0.172 ±0.073
QM	1.714	0.38 ±0.092	0.266 ±0.034

EV + QM: fit to lattice

I perform a fit to the lattice data, allowing light and strange particles to have a different behaviour, within the different EV schemes formerly showed.

		Direct	Inverse
	χ^2/N_{dof}	r_p (fm)	r_Λ (fm)
PDG05	40.632	0.487 ±0.157	0.249 ±0.052
PDG14	3.717	0.404 ±0.099	0.171 ±0.063
PDG17	2.26	0.391 ±0.092	0.192 ±0.051
QM	8.585	0.353 ±0.078	0.201 ±0.043

EV + QM: fit to lattice

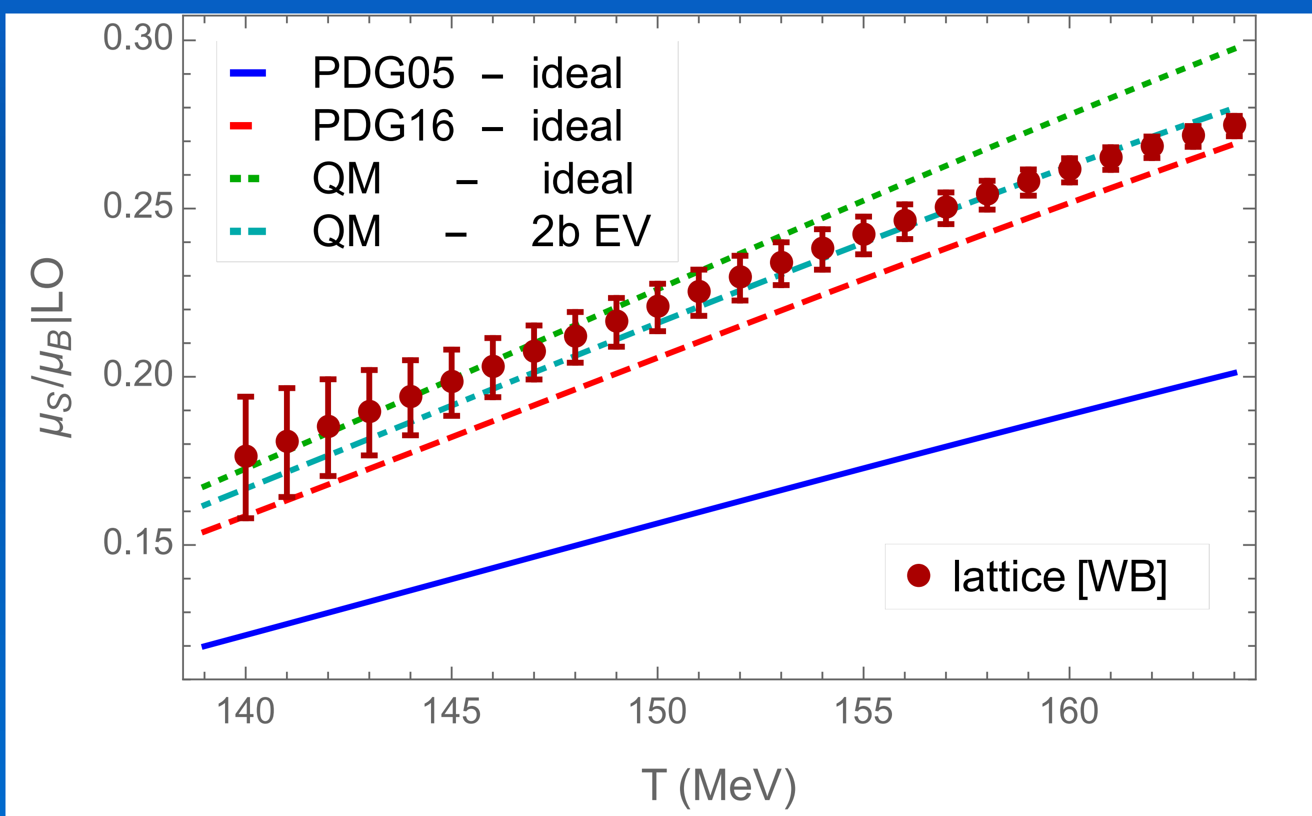
The use of multiple parameters does not drastically improve the quality of the fit, but underlines an interesting systematic difference between PDG lists and the QM one.

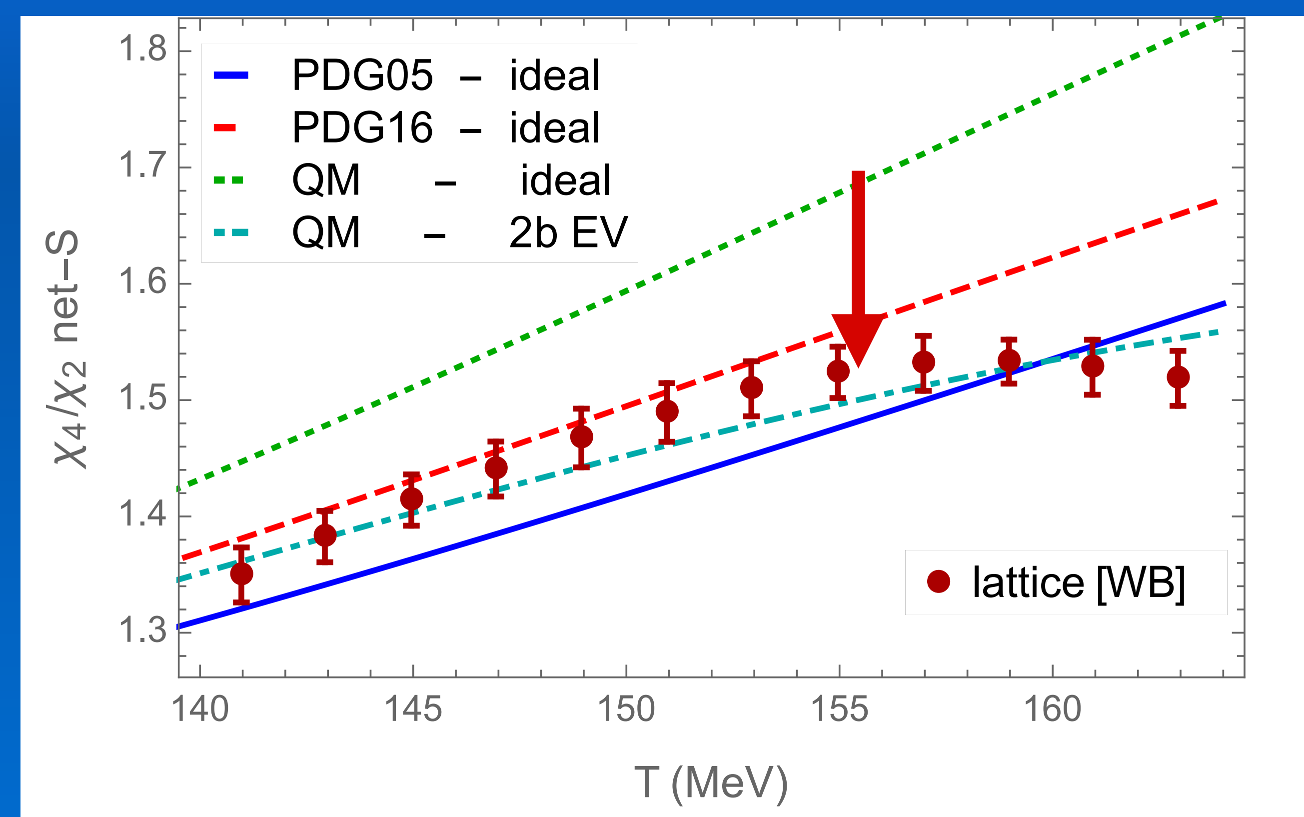
	χ^2/N_{dof}	Fixed r_π (fm)	Fixed r_K (fm)	Fixed r_p (fm)	Fixed r_Λ (fm)
PDG05	15.632	0.757 ±0.093	0.515 ±0.049	0.656 ±0.114	0.006 ±0.73
PDG14	2.611	0.208 ±0.279	0.221 ±0.059	0.446 ±0.102	0.007 ±0.486
PDG17	1.721	0.161 ±0.399	0.224 ±0.058	0.435 ±0.096	0.113 ±0.221
QM	1.257	0.171 ±0.339	0.214 ±0.063	0.42 ±0.095	0.285 ±0.038

EV + QM: strange obs.

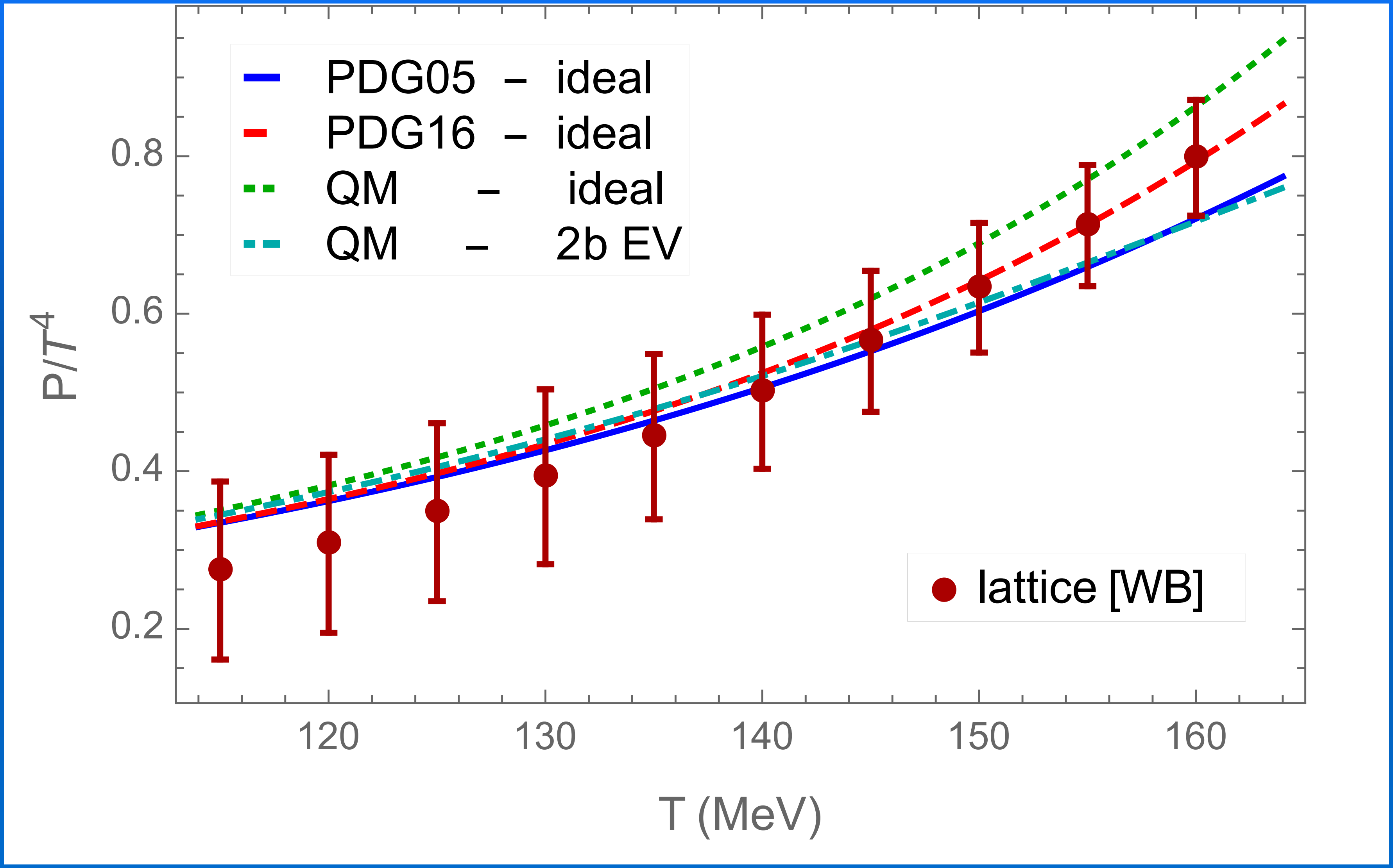
$$\left.\frac{\mu_S}{\mu_B}\right|_{LO} \simeq -\frac{\chi^{11}_{BS}}{\chi^2_S}$$

$$\frac{\chi^4_S}{\chi^2_S} \simeq \langle S^2 \rangle$$



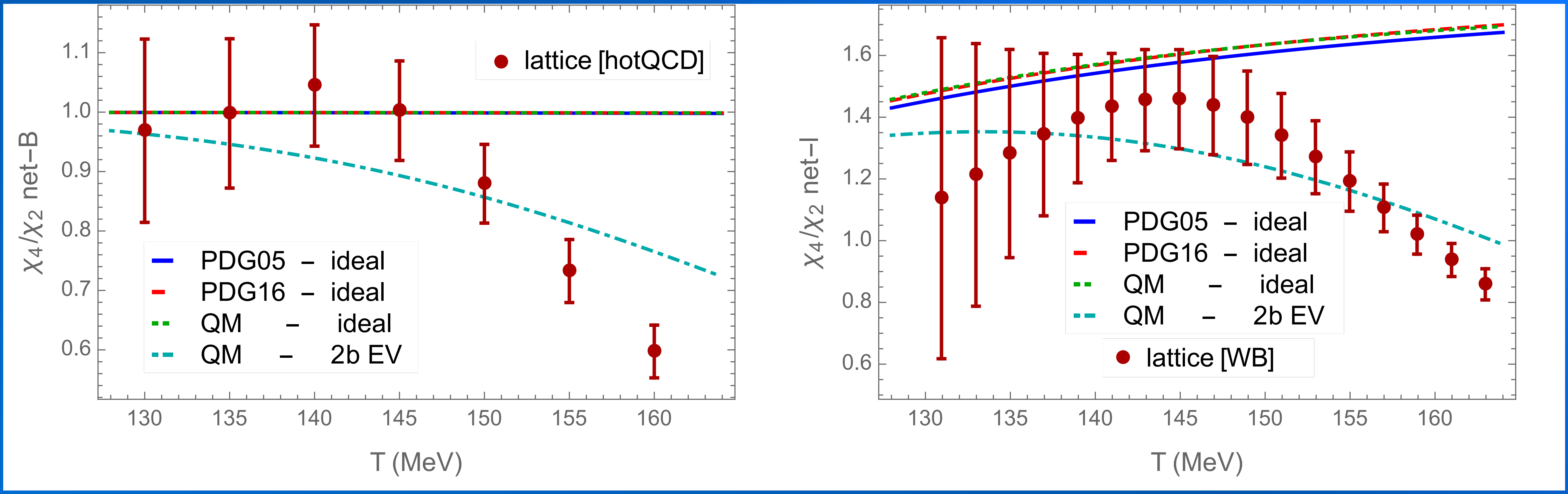


EV + QM: EoS

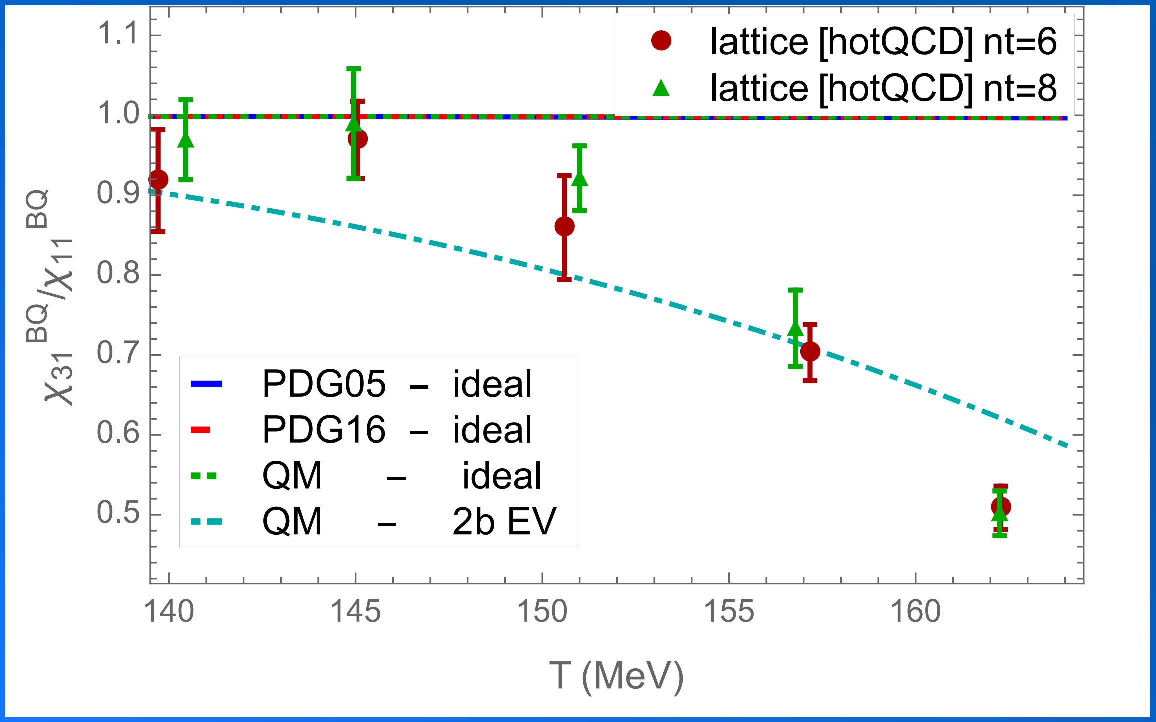


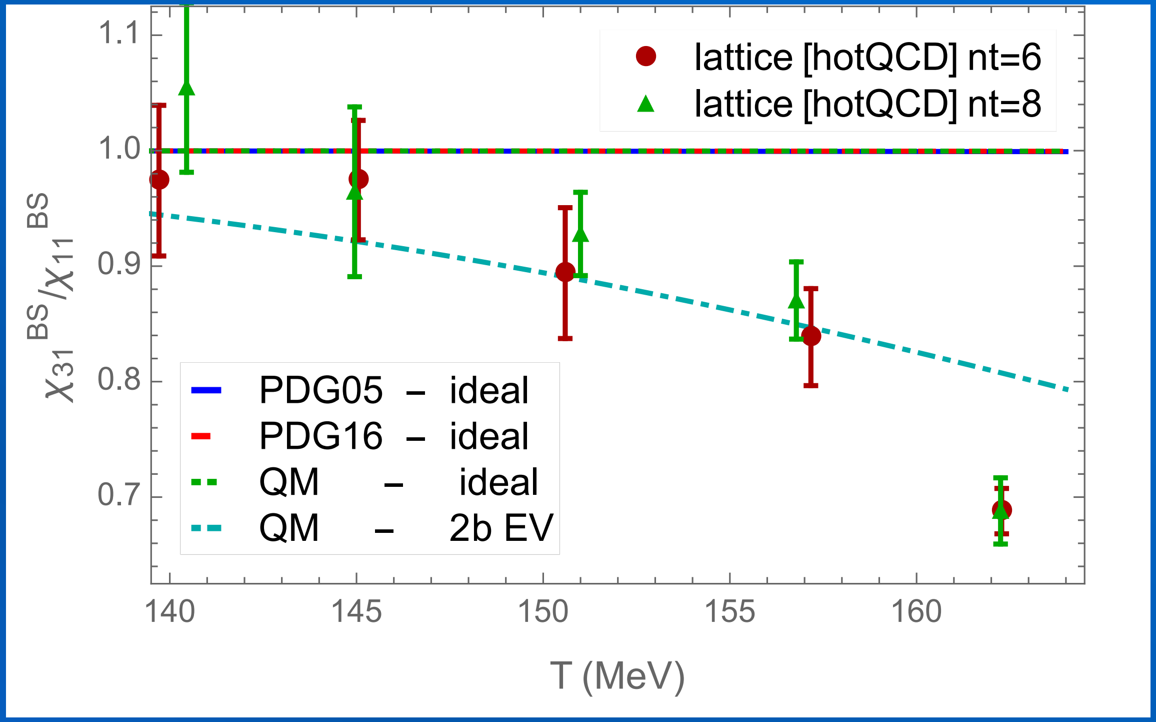
PDG05 – ideal
PDG16 – ideal
QM – ideal
QM – 2b EV
lattice [WB]
P/T^4
0.2
0.4
0.6
0.8
120
130
140
150
160
T (MeV)

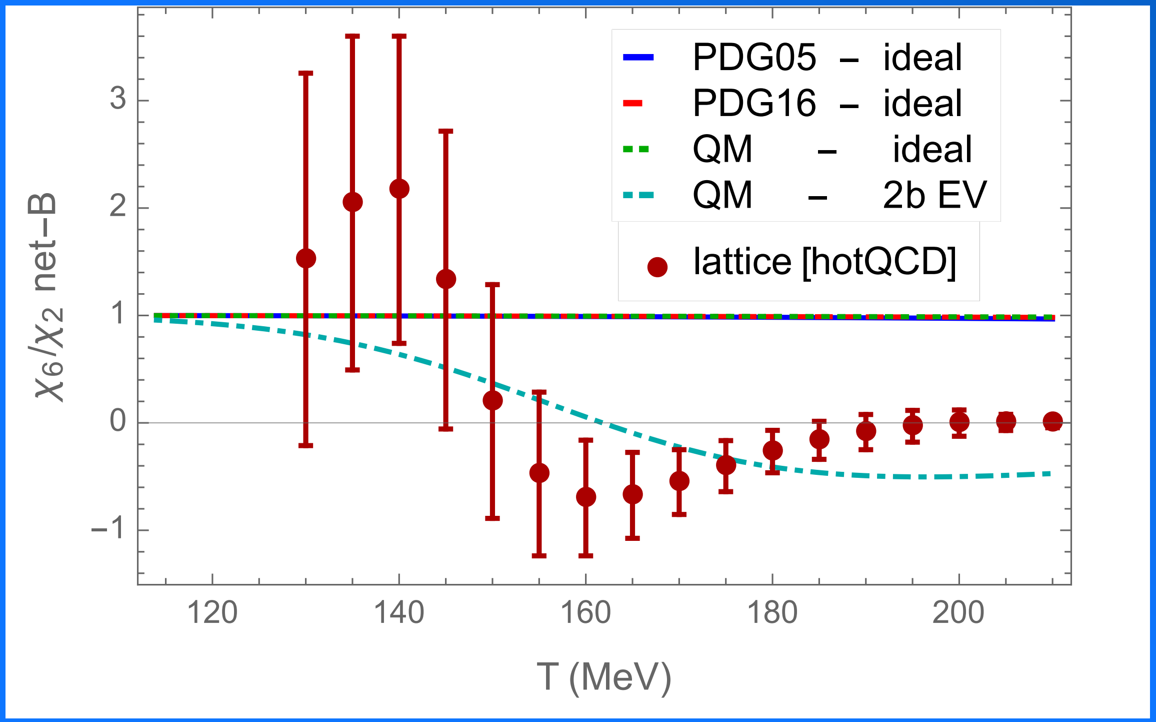
EV + QM: light obs.

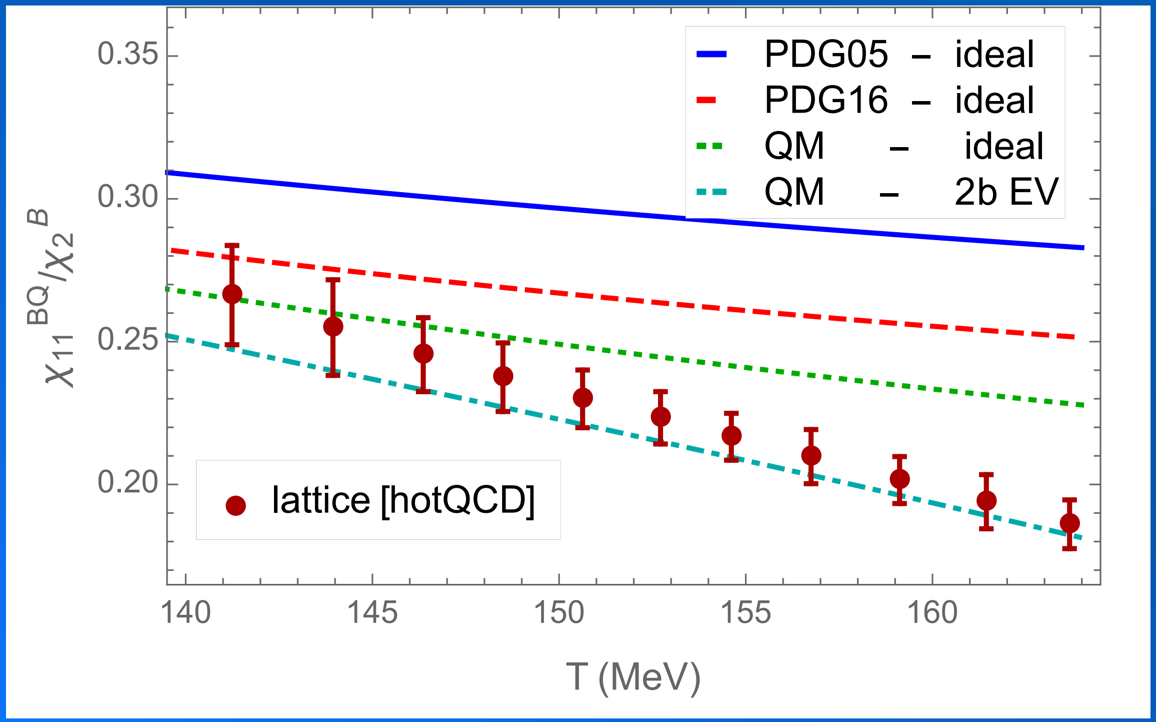


EV + QM: no-fitted obs.

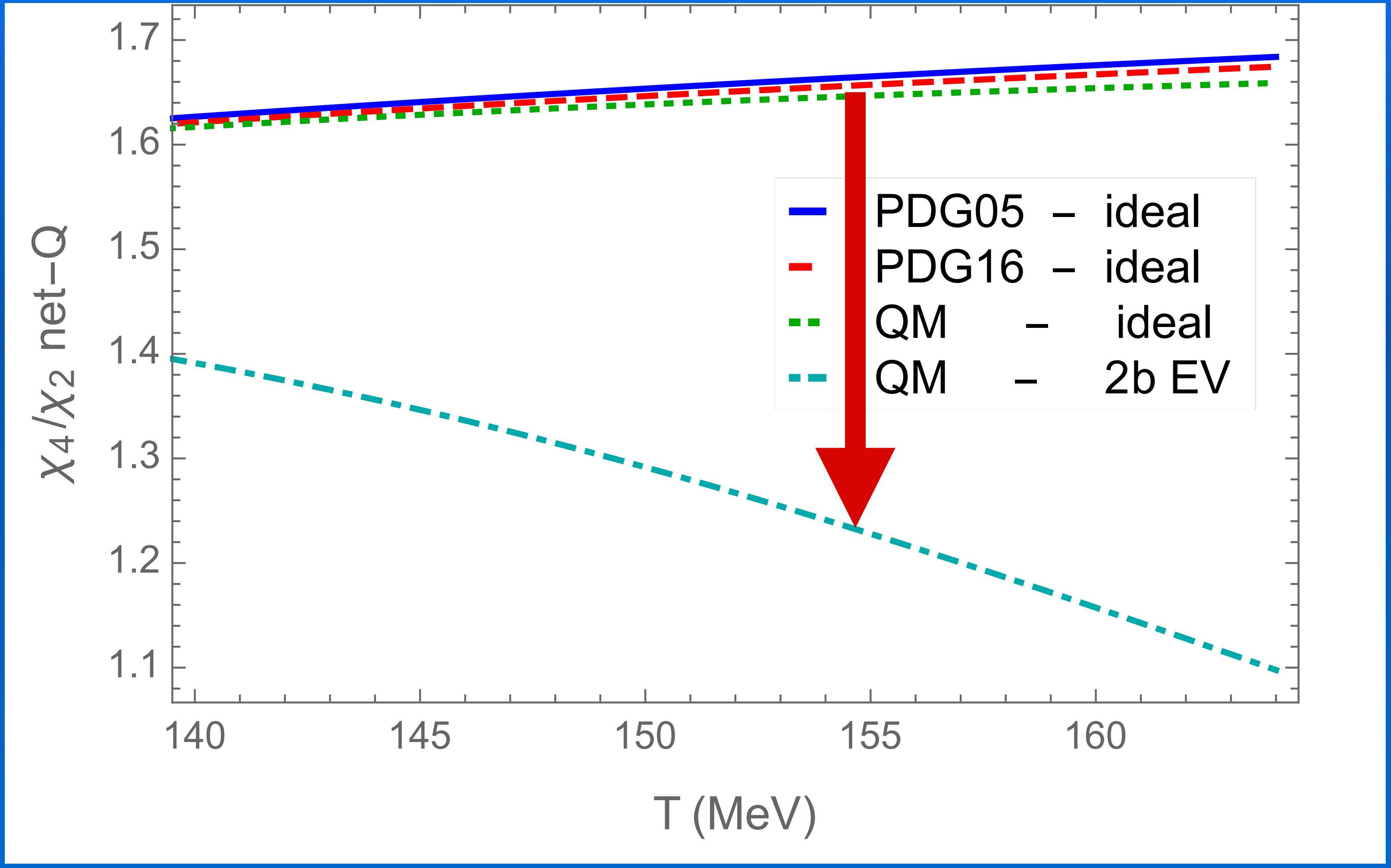








EV + QM: predictions



EV + QM: yields

ALICE@2.76 TeV

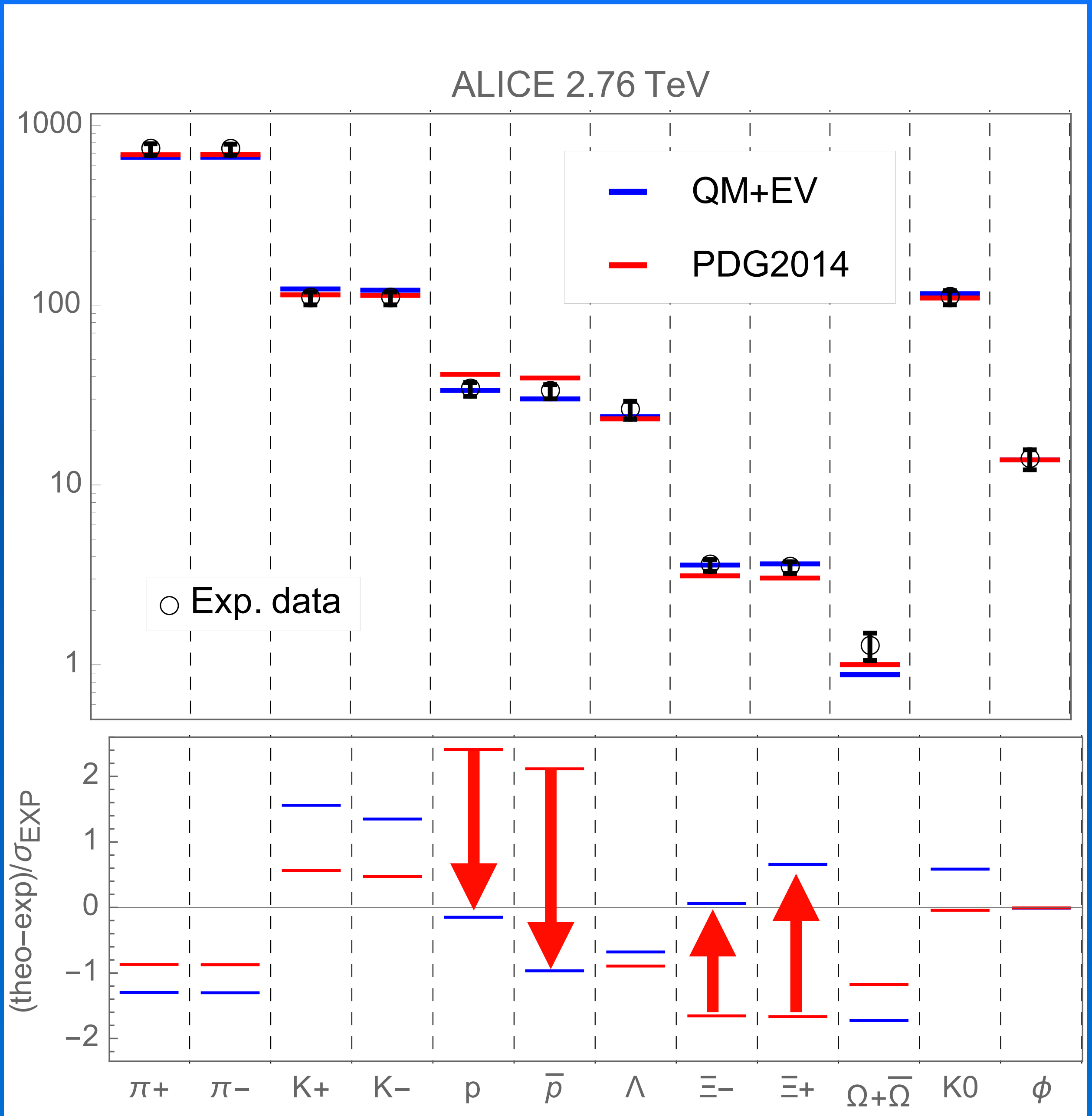
	PDG14	QM
id	χ^2/N_{dof}=20/9≃2.2 T= 155.2±2.2 (MeV) μ_B= 3.8±7 (MeV) V= 4663.1±590.3 (fm ^{3})	χ^2/N_{dof}=11.4/9≃1.2 T= 148.3±1.8 (MeV) μ_B= 6.9±7.2 (MeV) V= 6182.7±710.4 (fm ^{3})
2b		χ^2/N_{dof}=12.8/9≃1.42 T= 149.4±1.78 (MeV) μ_B= 7.6±7.79 (MeV) V= 7323.8±694.6 (fm ^{3})

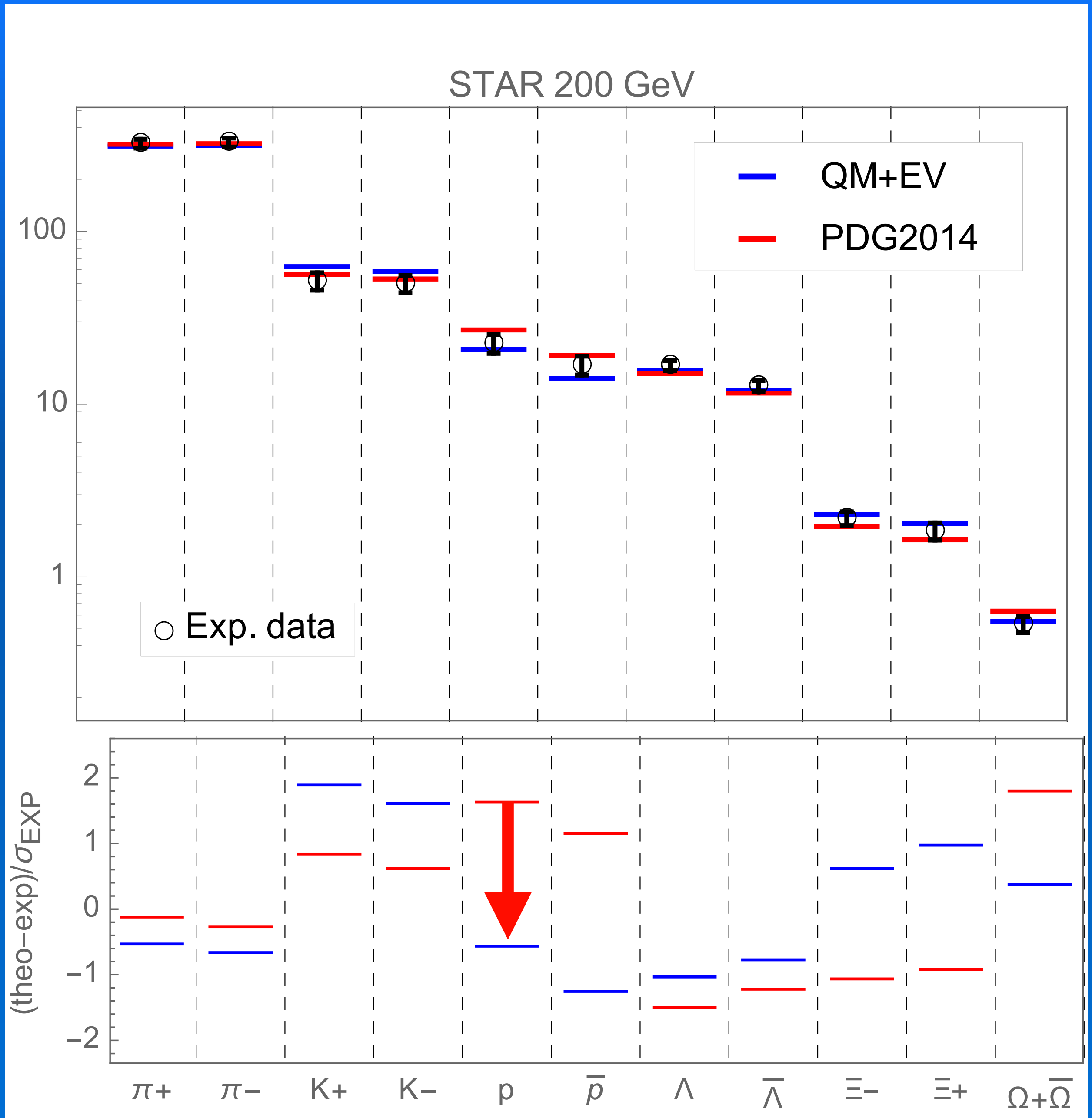
STAR@200 GeV

	PDG14	QM
id	χ^2/N_{dof}=14.1/8≃1.8 T= 164.1±2.3 (MeV) μ_B= 29.6±8.4 (MeV) V= 1492.1±187.8 (fm ^{3})	χ^2/N_{dof}=7.4/8≃0.9 T= 157.0±1.9 (MeV) μ_B= 31.1±8.3 (MeV) V= 1934.9±232.6 (fm ^{3})
2b		χ^2/N_{dof}=11.9/8≃1.48 T= 156.4±1.75 (MeV) μ_B= 30.95±8.6 (MeV) V= 2744.1±239.5 (fm ^{3})

For both energies are used the parameters extracted from the fit to lattice QCD.

EV + QM: yields


ALICE 2.76 TeV
QM+EV
PDG2014
Exp. data
(theo−exp)/σEXP
π+
π−
K+
K−
p
p̄
Λ
Ξ−
Ξ+
Ω+Ω̄
K0
ϕ

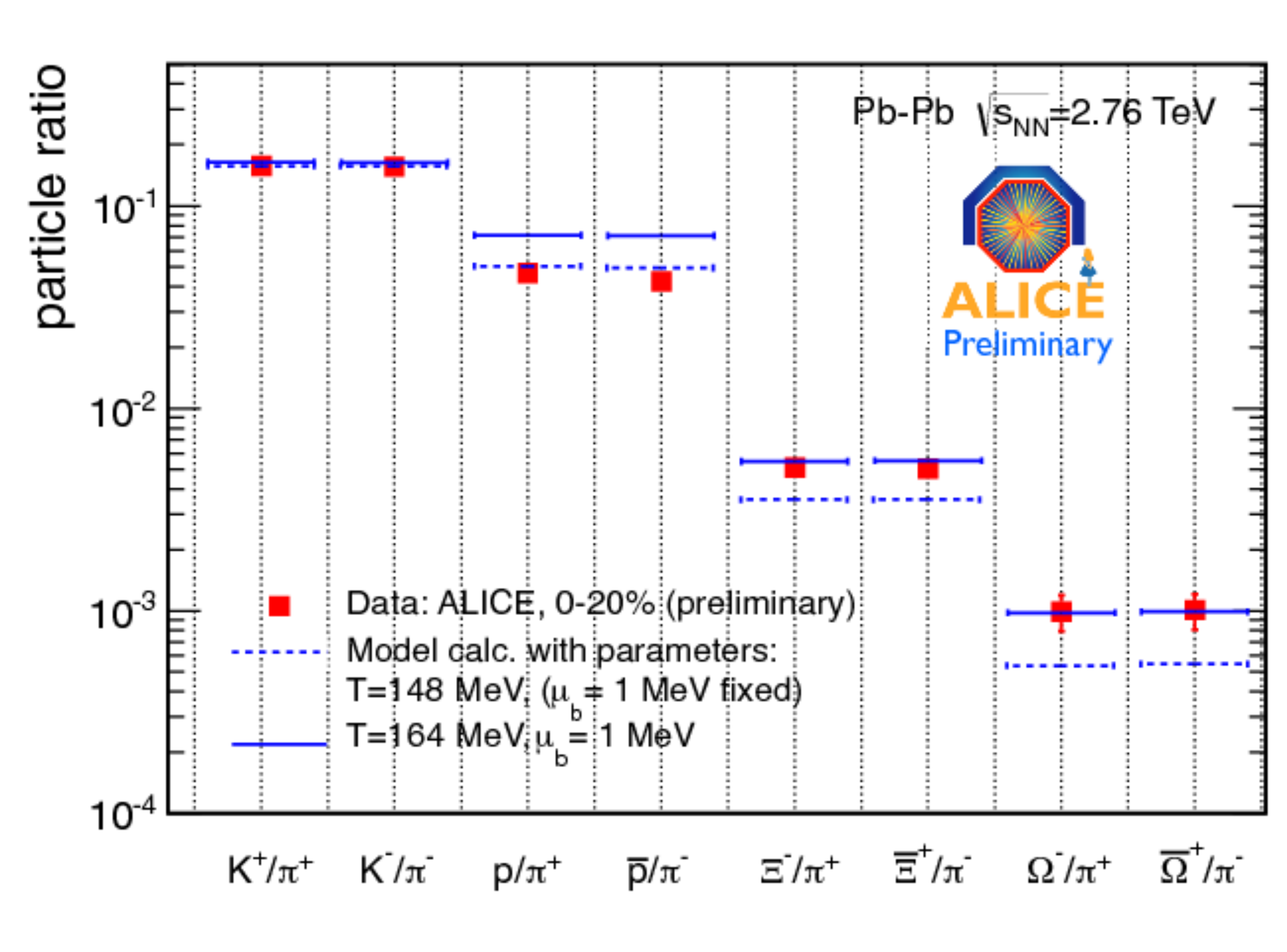

STAR 200 GeV
QM+EV
PDG2014
Exp. data
(theo−exp)/σEXP
π+
π−
K+
K−
p
p̄
Λ
Λ̄
Ξ−
Ξ+
Ω+Ω̄

Conclusions

- The standard implementation of the HRG is unable to describe all the observables/data.
- Missing (strange-)resonances play a relevant role both in lattice calculations and thermal fits, but repulsive interactions MUST be considered in order to restore the agreement with key observables.
- EV effects are an useful tool in order to parametrize effective hadronic interactions
- There are signatures for smaller strange states, both from lattice thermodynamics and particle yields.

Thanks for your attention

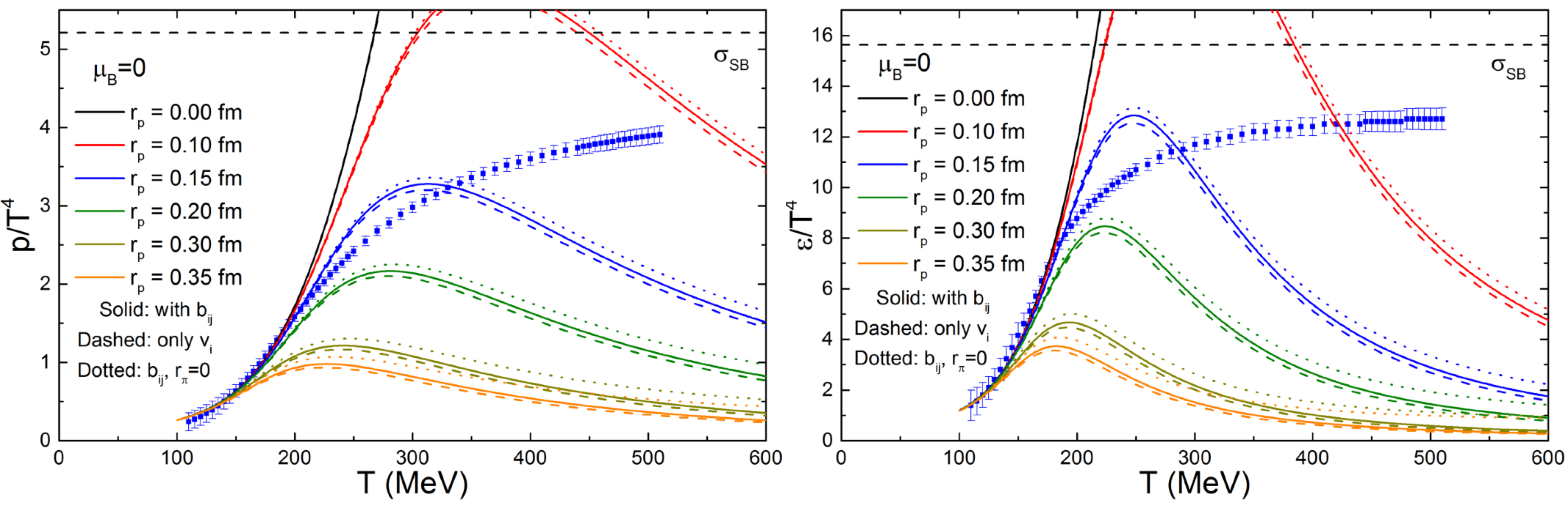
Flavor hierarchy



Preghenella, Acta Phys.Polon. B43 (2012) 555

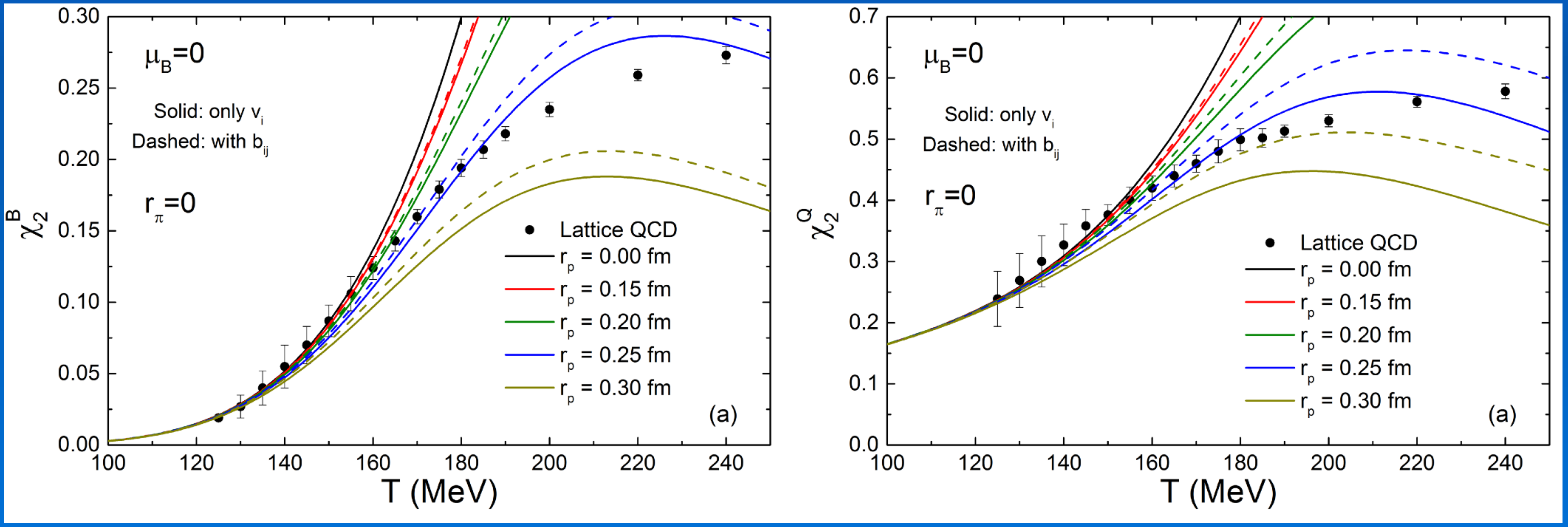
EV: crosstersm

This version of the model is consistent with the 2nd order viral expansion. The number of free parameters does not change.

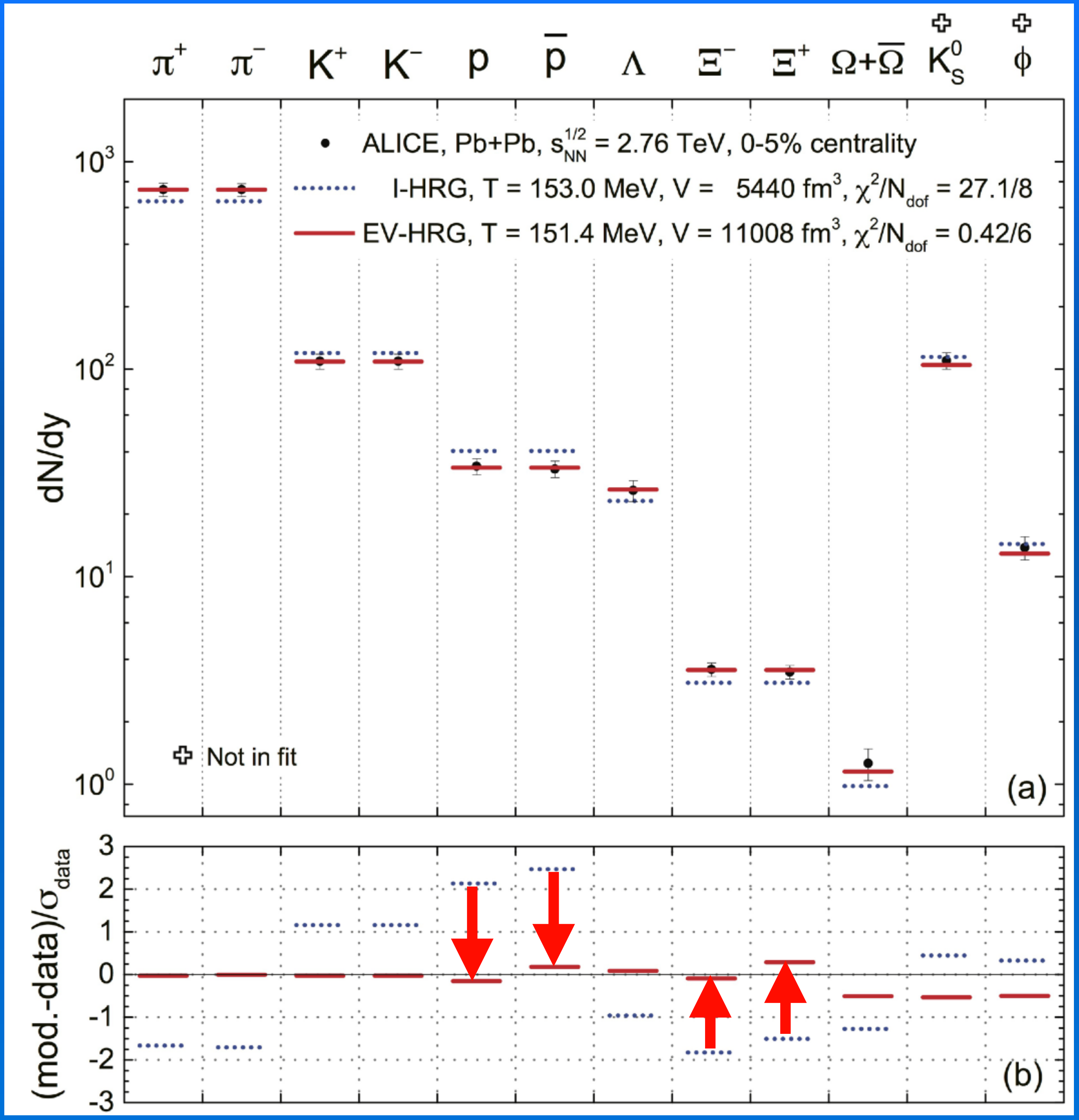


EV: crosstersm

This version of the model is consistent with the 2nd order viral expansion. The number of free parameters does not change.



EV: particle yields



A detailed balance of particle suppression removes the so called proton anomaly.

P.A. et al. arxiv: hep-ph 1606.06542

EV: particle yields

With the parameters extracted from ALICE 0-5%, there is an overall improvement for all centralities and lower energies.

	χ^2/Ndf p.l.	χ^2/Ndf	T (MeV) p.l.	T (MeV)
ALICE 0-5%	2.642537	0.0985746	152.576606	150.270412
ALICE 5-10%	4.038844	0.082681	153.855798	151.702161
ALICE 10-20%	4.831962	0.187238	156.912643	153.761281
ALICE 20-30%	5.779079	0.505264	156.269898	155.342295
ALICE 30-40%	5.290277	0.479082	156.606086	155.778665
ALICE 40-50%	4.320371	0.225175	156.901153	155.046625
ALICE 50-60%	2.528466	0.431904	153.374355	152.640780
ALICE 60-70%	2.522801	0.896884	148.338287	150.736294
ALICE 70-80%	2.480648	0.516741	150.701703	158.829787

	χ^2/Ndf p.l.	χ^2/Ndf	T (MeV) p.l.	T (MeV)
NA49 20GeV	5.868216	3.668726	106.448226	122.919464
NA49 30GeV	7.222598	1.269705	141.555846	136.454728
NA49 40GeV	8.077212	2.292649	139.293714	136.775614
NA49 80GeV	13.783130	4.812104	138.121797	141.917805
NA49 158GeV	5.329034	1.590537	146.535995	142.932057

There are no relevant changes in the freeze-out parameters.