QCD-like theories at finite density

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**Jonas Wilhelm, Lukas Holicki, Philipp Scior,
Dominik Smith, Björn Wellegehausen,
Lorenz von Smekal**

CRC–TR 211
Strong-interaction matter under extreme conditions

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Outline

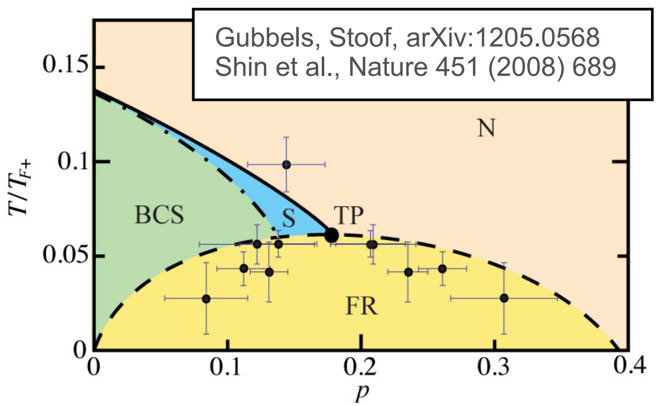
- Intro & Motivation
- G_2-QCD, Two-Color QCD, Isospin Density in QCD
- Goldstone Spectrum in Two-Color QCD with Staggered Quarks
- Lifshitz Transition in Graphene
- Conclusions

QCD-like Theories

- develop functional methods and effective theories

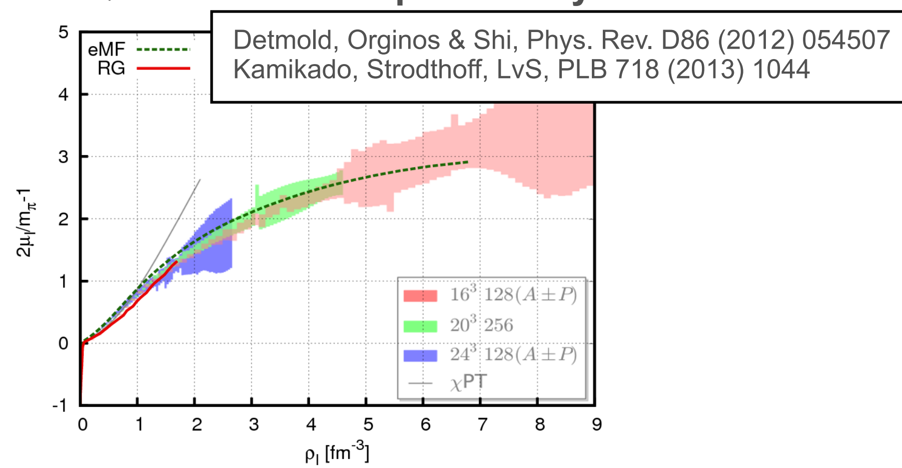
- exploit analogies

polarised fermi gas at unitarity

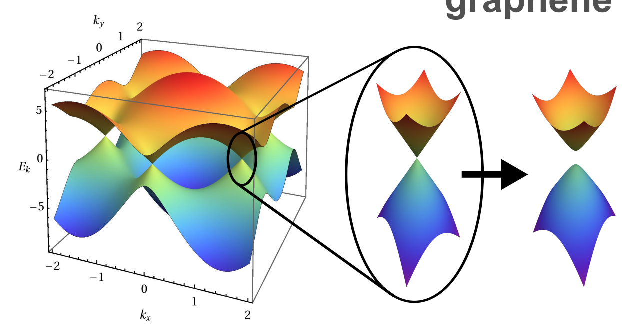


- strongly correlated fermions in condensed matter systems

QCD at finite isospin density



graphene



DFG Deutsche Forschungsgemeinschaft

Fermion-Sign Problem

sign problem:

$$\left(\mathrm{Det}\, D(\mu_f)\right)^* = \mathrm{Det}\, D(-\mu_f)$$

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- **except if:**

  (a) two degenerate flavors with isospin chemical potential

  fermion determinant $\rightsquigarrow \mathrm{Det}\big(D(\mu_I)D(-\mu_I)\big)$

QCD at finite isospin density

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QCD at finite isospin density

(b) anti-unitary symmetry $TD(\mu)T^{-1} = D(\mu)^* \qquad T^2 = \pm 1$

fermion color representation:

(i) pseudo-real $T^2 = 1$ **two-color QCD**

(ii) real $T^2 = -1$ **adjoint QCD, or G_2-QCD**

Fermion-Sign Problem

sign problem: $$\big(\mathrm{Det}\, D(\mu_f)\big)^* = \mathrm{Det}\, D(-\mu_f)$$

- **except if:**

(a) two degenerate flavors with isospin chemical potential — **Dyson index:**

fermion determinant $\rightsquigarrow \mathrm{Det}\big(D(\mu_I)D(-\mu_I)\big)$ — $\beta = 2$

QCD at finite isospin density

(b) anti-unitary symmetry $TD(\mu)T^{-1} = D(\mu)^*$ $\quad T^2 = \pm 1$

fermion color representation:

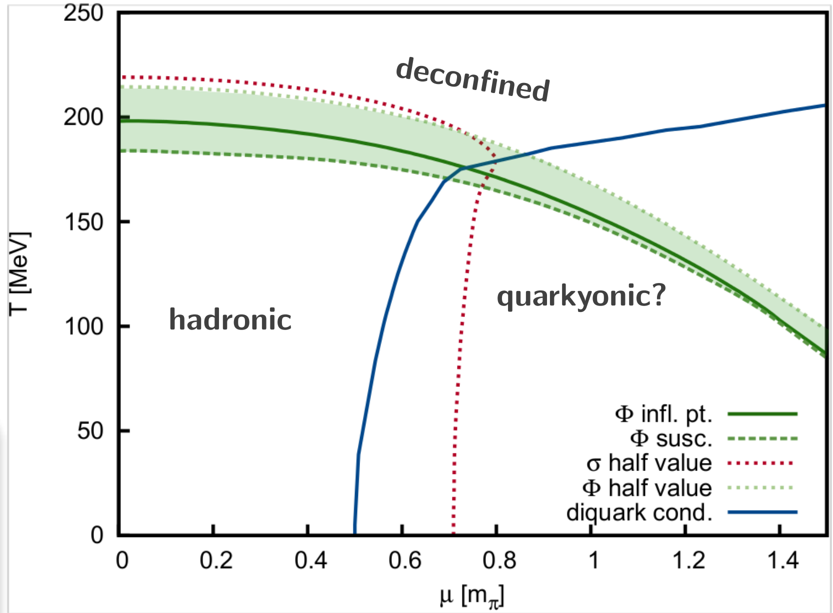
(i) pseudo-real $T^2 = 1$ — **two-color QCD** — $\beta = 1$

(ii) real $T^2 = -1$ — **adjoint QCD, or G_2-QCD** — $\beta = 4$

Phase Diagram of QC$_2$D

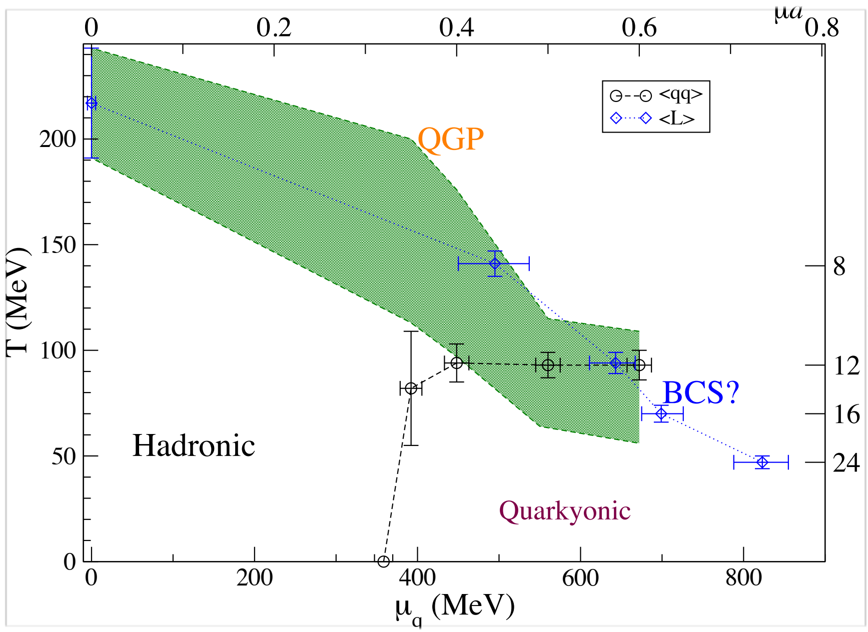
- **Quark-Meson-Diquark model:**

  Strodthoff, Schaefer & LvS,
  Phys. Rev. D85 (2012) 074007



Strodthoff & LvS, PLB 731 (2014) 350

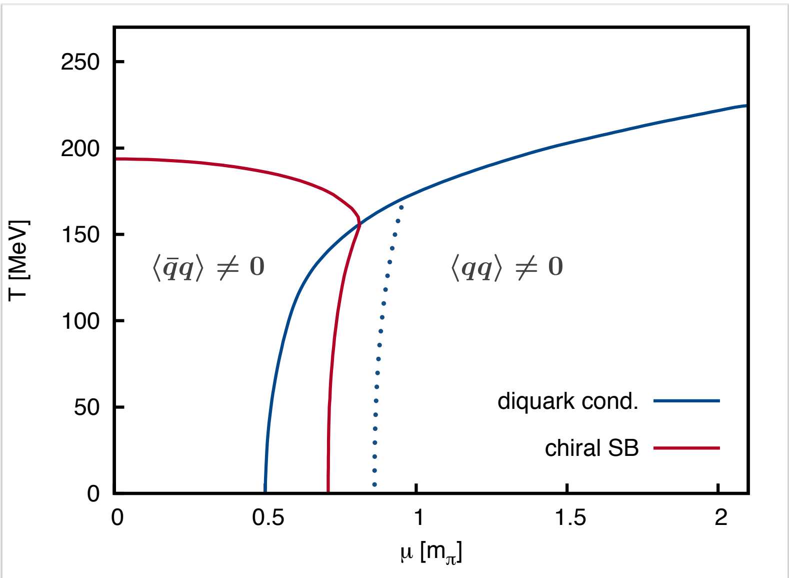
- **Lattice simulations:**



Cotter, Giudice, Hands & Skullerud,
PRD 87 (2013) 034507

Vacuum Realignment

- **QMD model phase diagram**

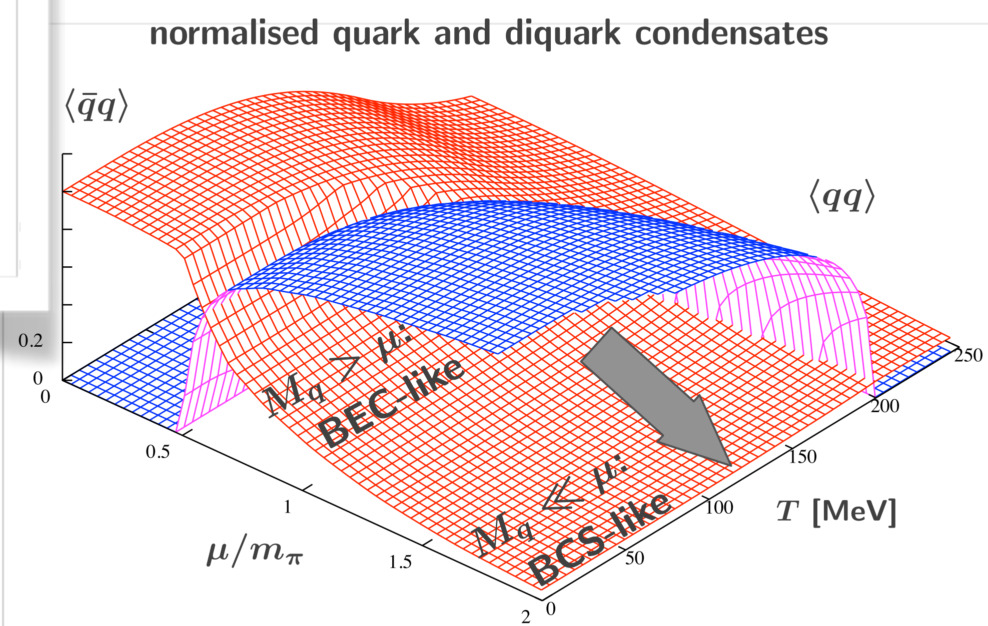


Strodthoff, Schaefer & LvS, PRD 85 (2012) 074007

zero temperature condensates

$$\chi\text{PT:}\quad \langle \bar{q}q \rangle = 2N_f G \cos\alpha$$
$$\langle qq \rangle = 2N_f G \sin\alpha$$
$$n_B = 8N_f F^2 \mu \sin^2\alpha$$

Kogut, Stephanov, Toublan, Verbaarschot & Zhitnitsky, Nucl. Phys. B 582 (2000) 477



Goldstone Spectrum - QC_2D

- **extended flavor symmetry (Pauli-Gürsey), at $\mu = 0$**

$$SU(N_f) \times SU(N_f) \times U(1) \text{ becomes } SU(2N_f)$$

$N_f = 2$: connects pions and σ-meson with scalar (anti)diquarks.

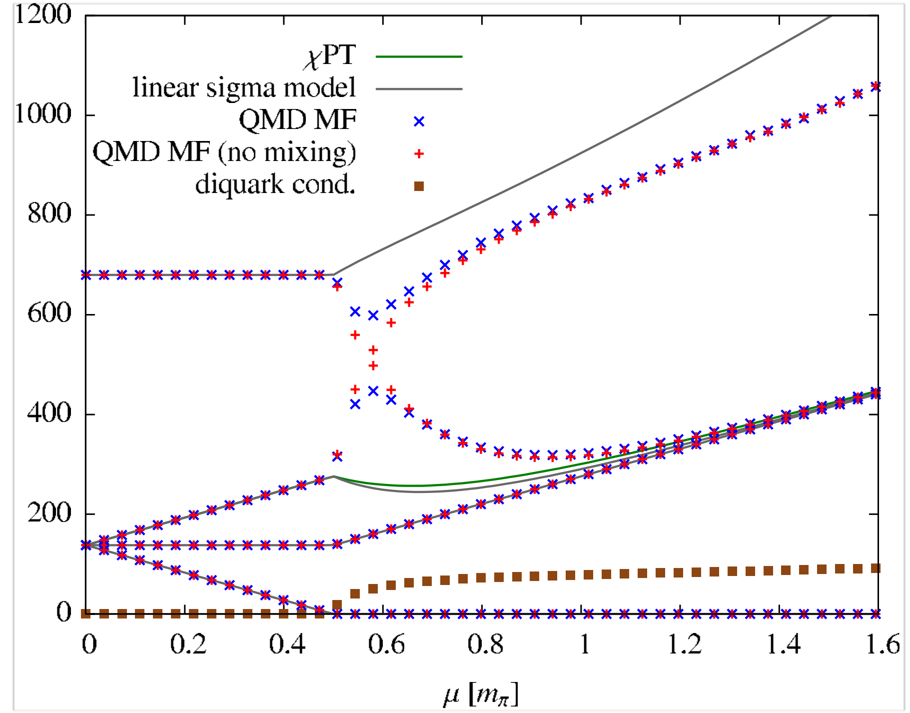
- **Dirac mass (quark condensate)**

$$SU(4) \to Sp(2)$$

or $$SO(6) \to SO(5)$$

Coset: S^5 5 Goldstone bosons: pions and scalar (anti)diquarks

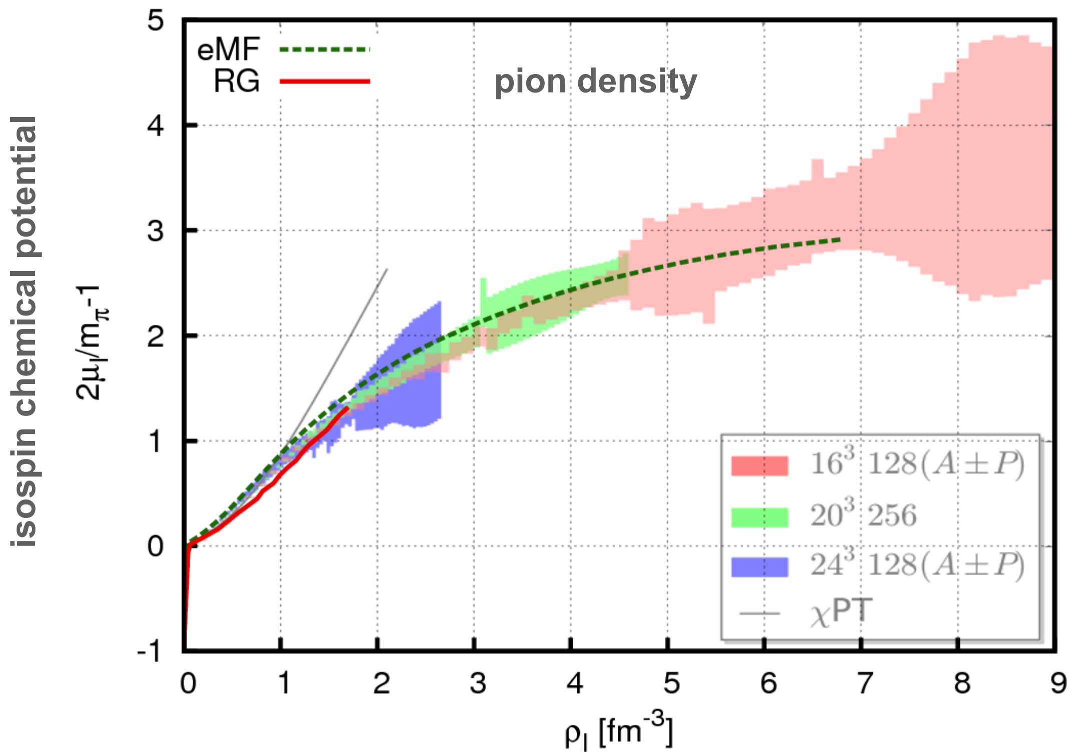
- **color-singlet diquarks (bosonic baryons)**



Strodthoff, Schaefer & LvS, PRD 85 (2012) 074007

QCD with Isospin Chemical Potential

- *T* = 0 isospin density - FRG vs. lattice QCD:



Kamikado, Strodthoff, LvS, PLB 718 (2013) 1044

Detmold, Orginos & Shi, Phys. Rev. D86 (2012) 054507

Lattice MC Simulations

Hands, Montvay, Scorzato & Skullerud,
Eur. Phys. J. C 22 (2001) 451

Hands, Kenny, Kim & Skullerud,
Eur. Phys. J. A 47 (2011) 60, …

Kogut, Toublan, Sinclair,
Phys. Rev. D 68 (2003) 054507

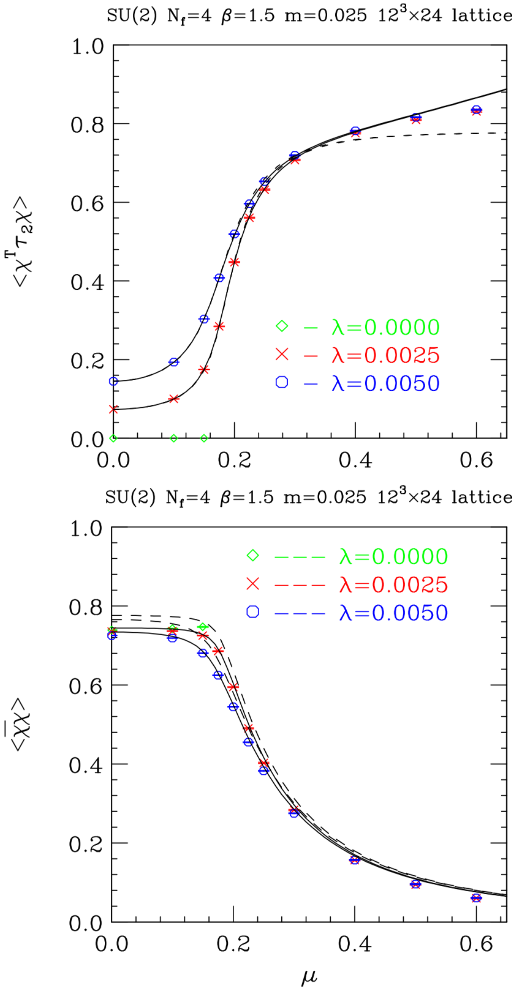
Braguta, Ilgenfritz, Kotov, Molochkov &
Nicolaev, Phys. Rev. D 94 (2016) 114510

Holicki, Wilhelm, Smith, Wellegehausen &
LvS, PoS (Lattice 2016) 070

$N_f = 2$ Flavors of Staggered Quarks

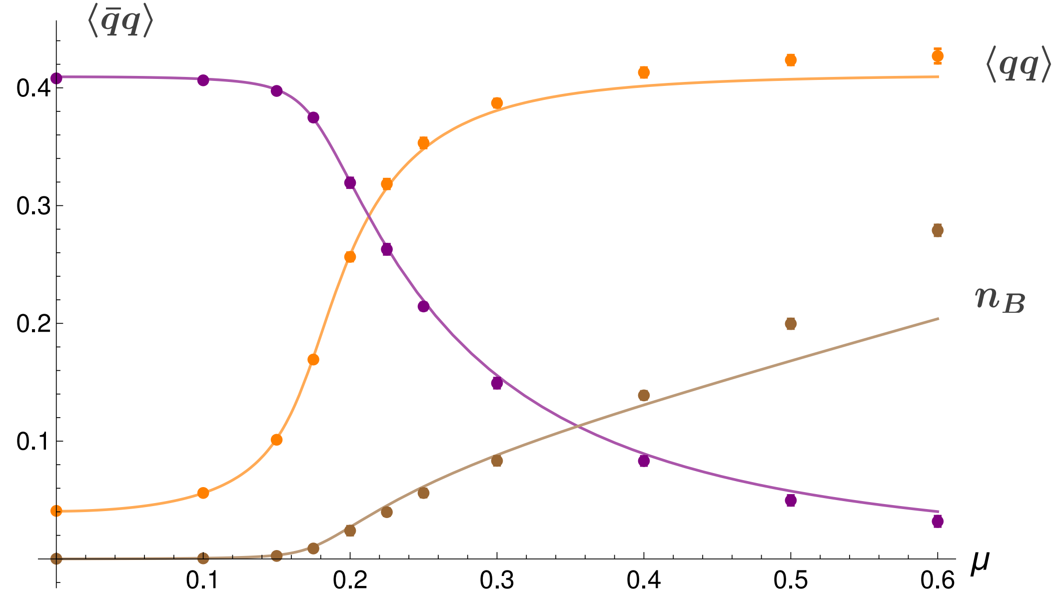
Validate - Previous Results

Kogut, Toublan & Sinclair,
PRD 68 (2003) 054507



- diquark source

$$S_f = \overline{\psi} D(\mu) \psi + \frac{\lambda}{2} \left(\psi^T (C\gamma_5) \tau_2 \psi + \overline{\psi} (C\gamma_5) \tau_2 \overline{\psi}^T \right)$$

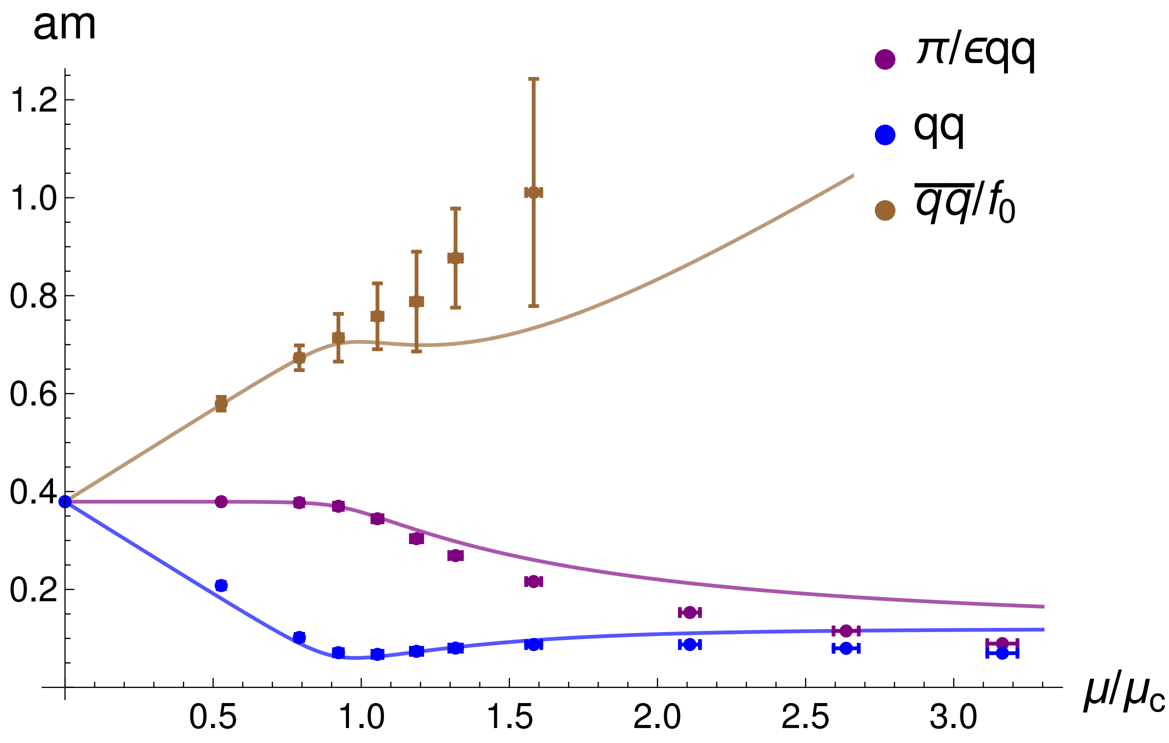


$N_f = 2,\ \beta = 1.5,\ m = 0.025,\ \lambda = 0.0025,\ 12^3 \times 24$ lattice

Goldstone Spectrum - QC_2D

- **mixing at finit density:** f_0/qq: $\frac{1}{2}\left(\chi^T\tau_2\chi + \bar{\chi}\tau_2\bar{\chi}^T\right)\cos\alpha + \bar{\chi}\chi\sin\alpha$

$$\pi/\epsilon qq:\ \bar{\chi}\epsilon\chi\cos\alpha + \frac{1}{2}\left(\chi^T\tau_2\epsilon\chi + \bar{\chi}\tau_2\epsilon\bar{\chi}^T\right)\sin\alpha$$



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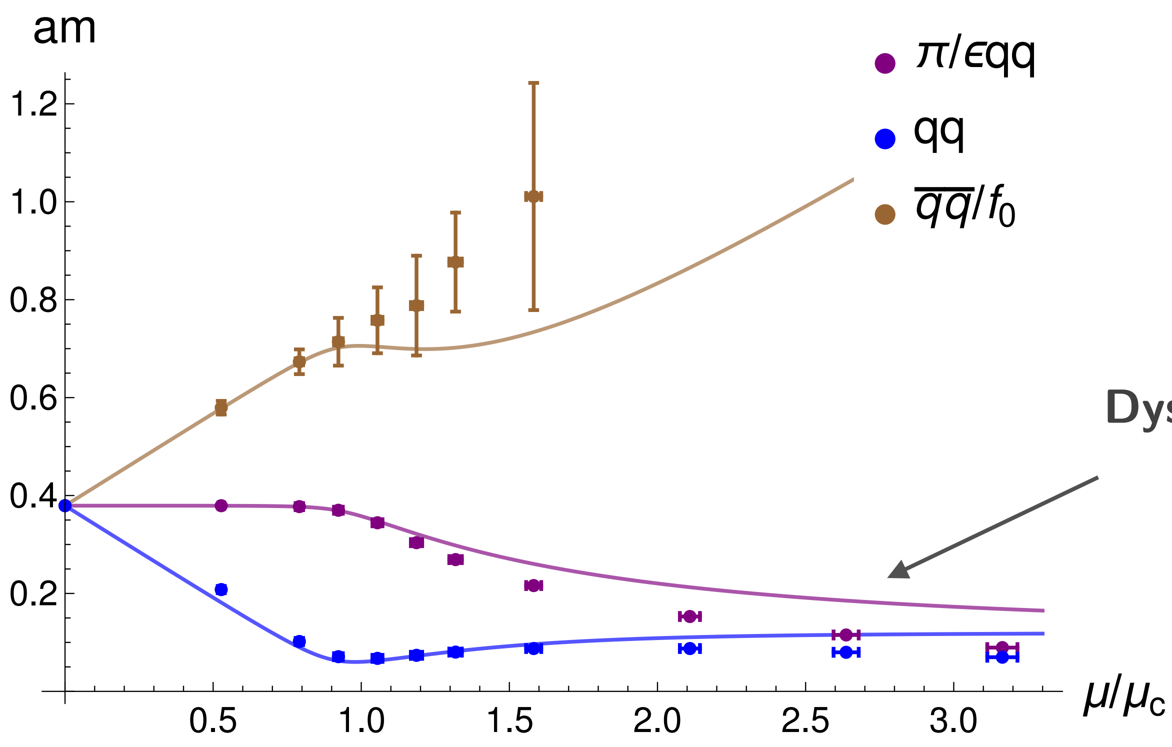
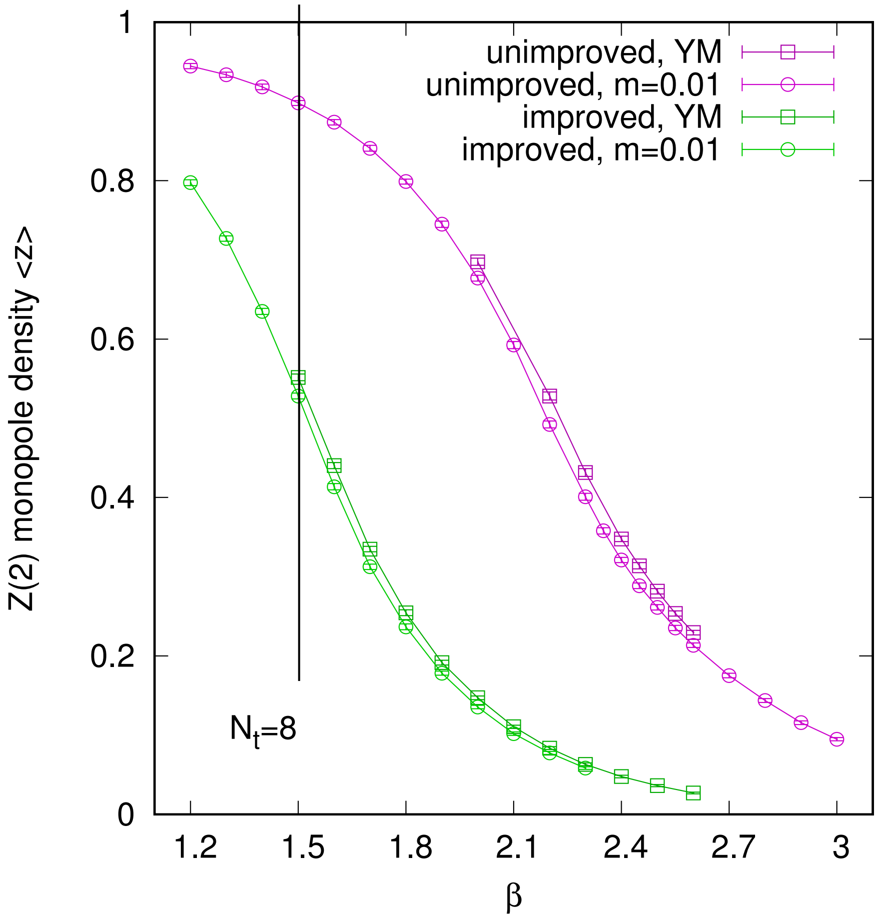


Dyson index:

$\beta = 4$

$N_f = 2,\ \beta = 1.5,\ m = 0.025,\ \lambda = 0.0025,\ 12^3 \times 24$ **lattice**

Bulk Phase of SU(2)



$$\langle z \rangle = 1 - \frac{1}{N_C} \sum_C \prod_{P \in \partial C} \mathrm{sgn}\,\mathrm{tr}\, P$$

- $\beta = 1.5 \to \langle z \rangle \approx 0.95$
- gauge action Symanzik improvement

D. Scheffler, PhD thesis, TU Darmstadt (2015)

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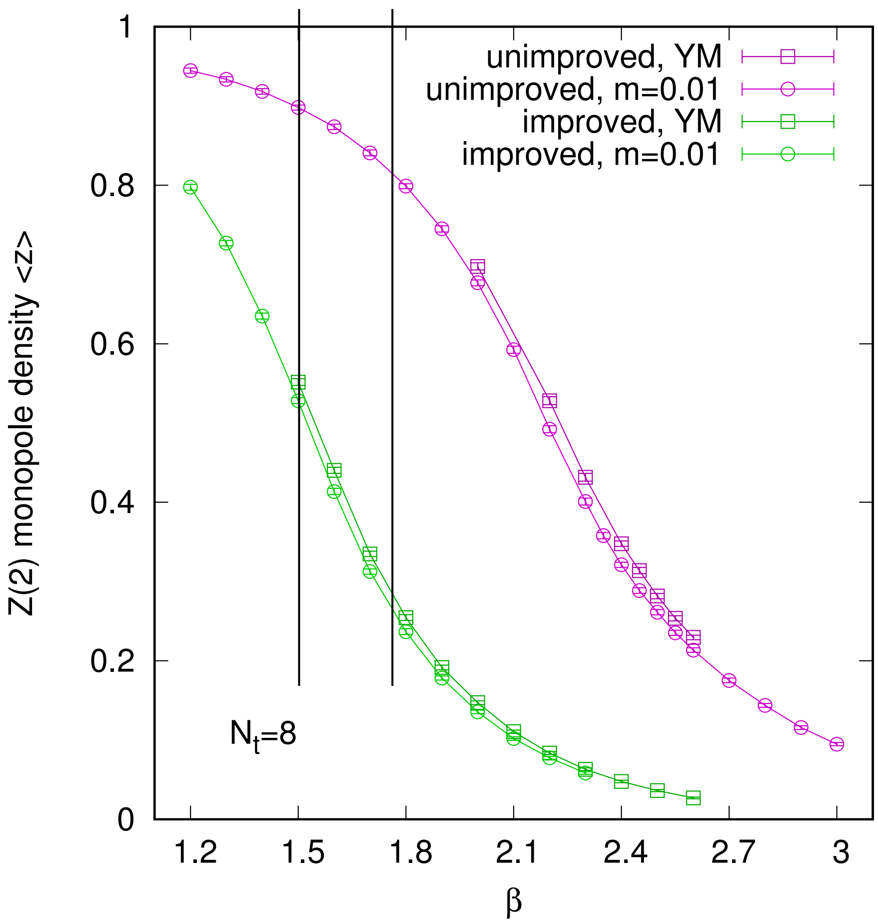

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- $\beta = 1.5 \to \langle z \rangle \approx 0.95$
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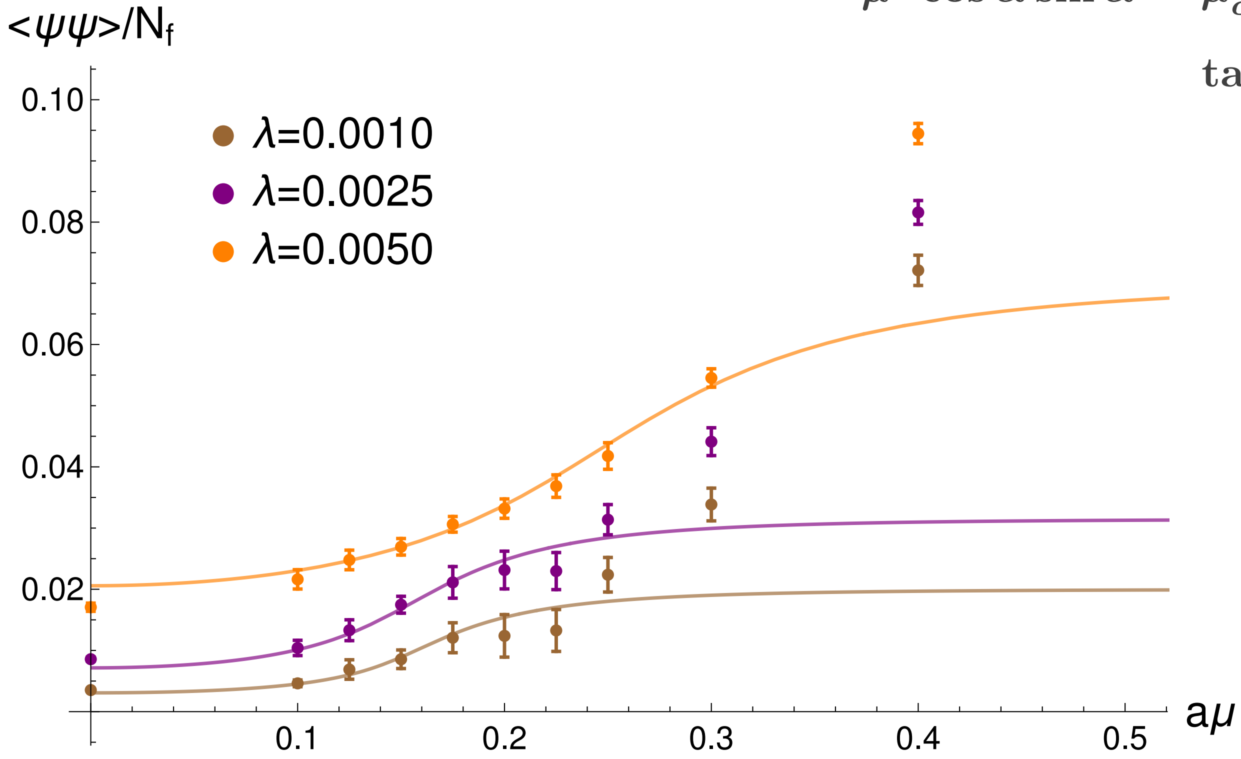
- Compromise: $\beta = 1.7, \; \frac{m_\pi}{m_\rho} = 0.5816(27)$
- $N_s = 16, \; N_t = 32$
- standard rooted staggered quarks ($N_f = 2$), improved gauge action

D. Scheffler, PhD thesis, TU Darmstadt (2015)

Diquark Condensate

$$\mu^2 \cos\alpha \sin\alpha = \mu_c^2 \sin(\alpha - \phi)$$

$$\tan\phi = \lambda/m$$



$$a\mu_c = 0.136(9)$$

$$\langle qq \rangle = 2 N_f G \sin\alpha$$

$$a m_\pi / 2 = 0.143(3)$$

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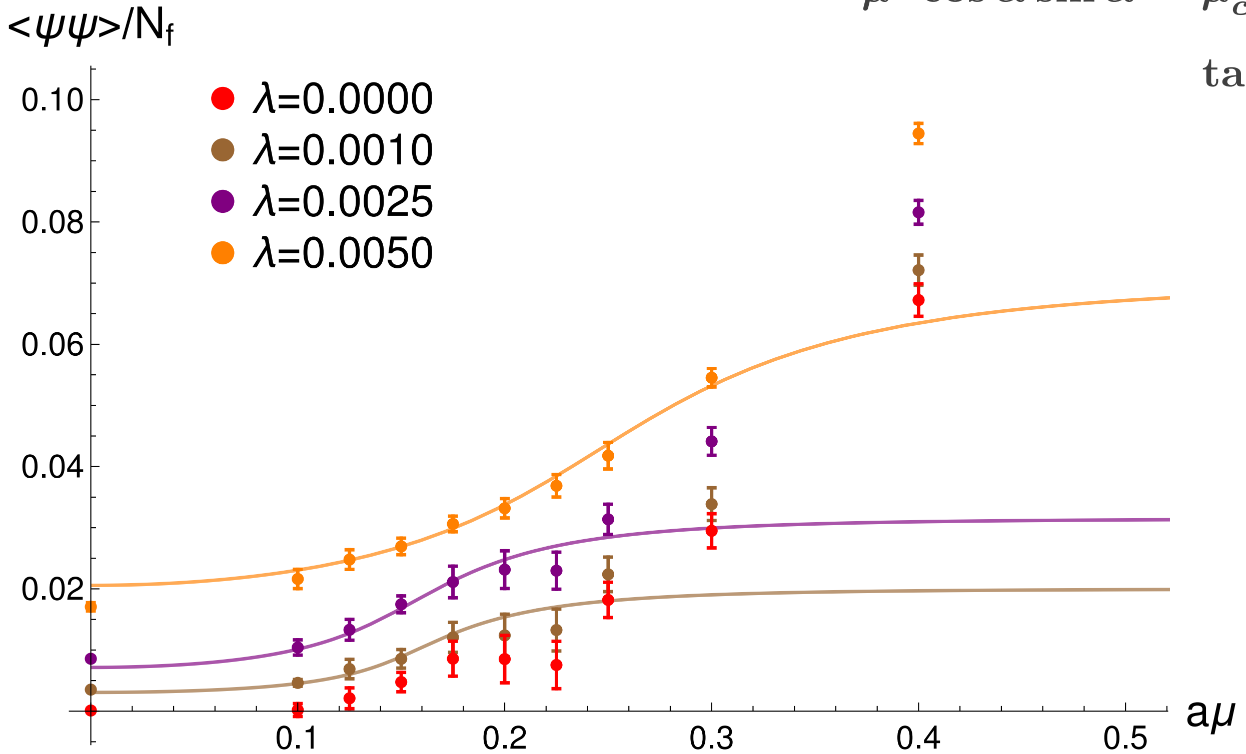

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Baryon Density

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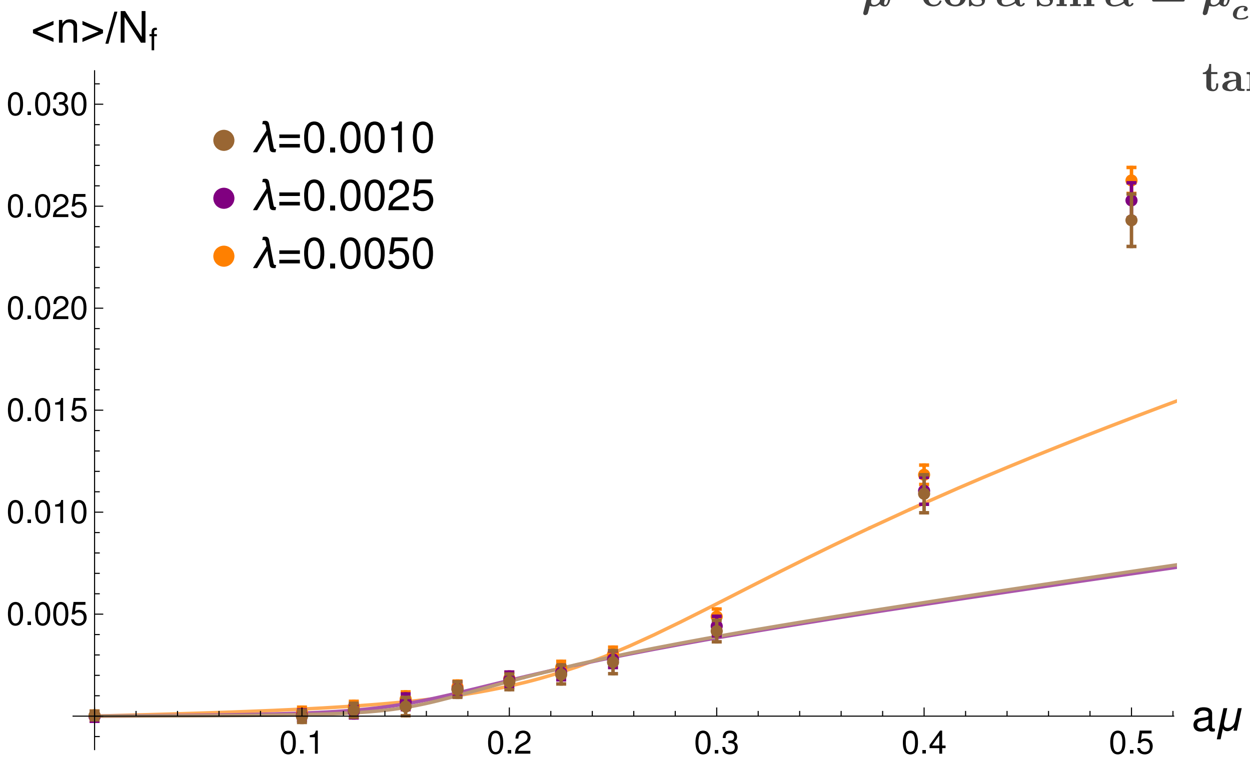

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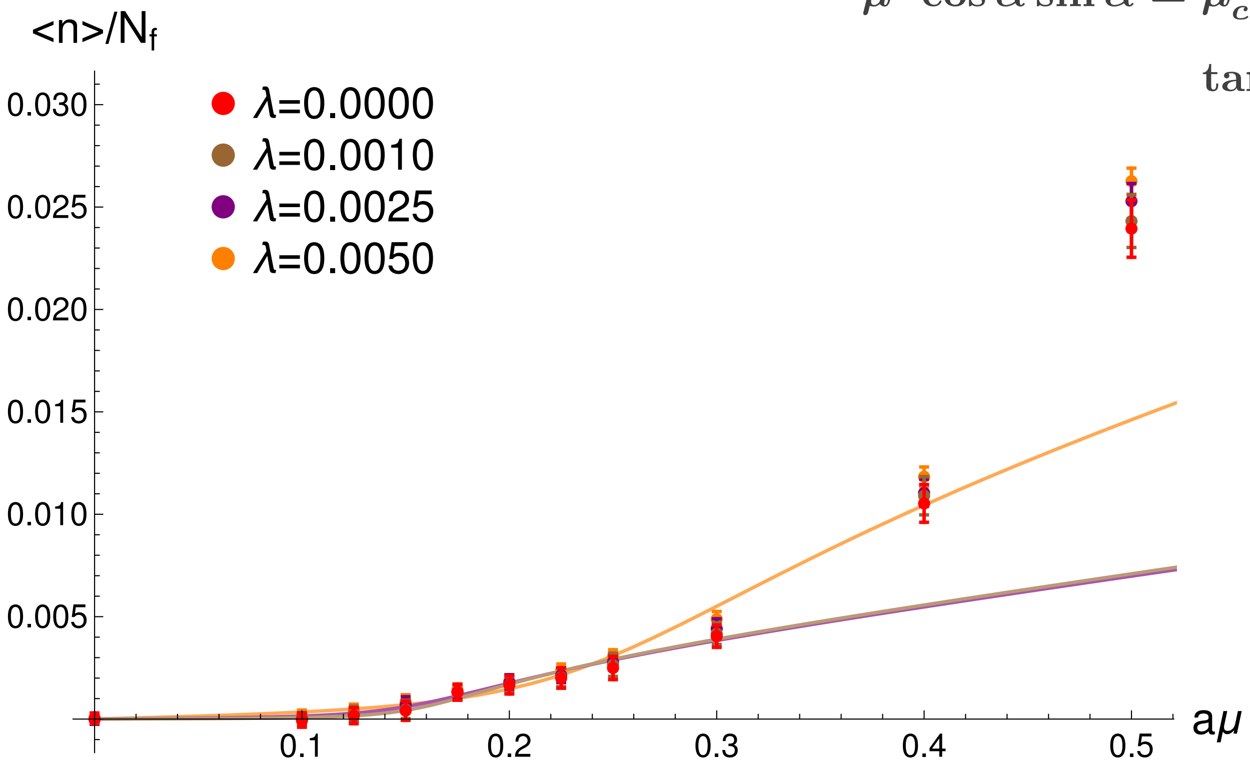

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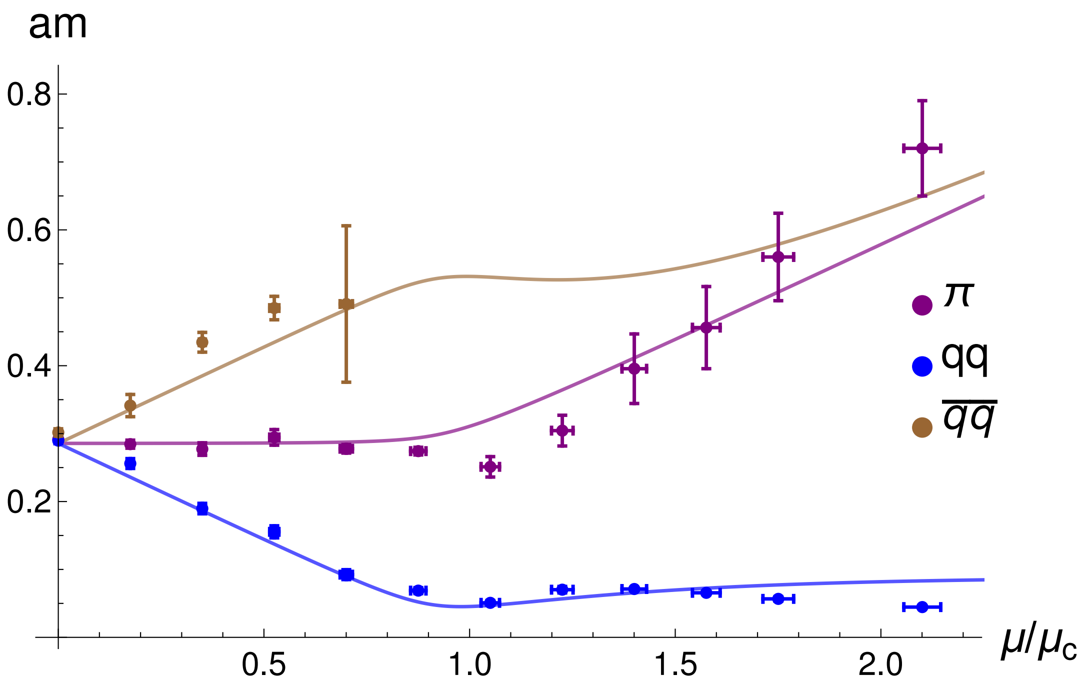
Goldstone Spectrum

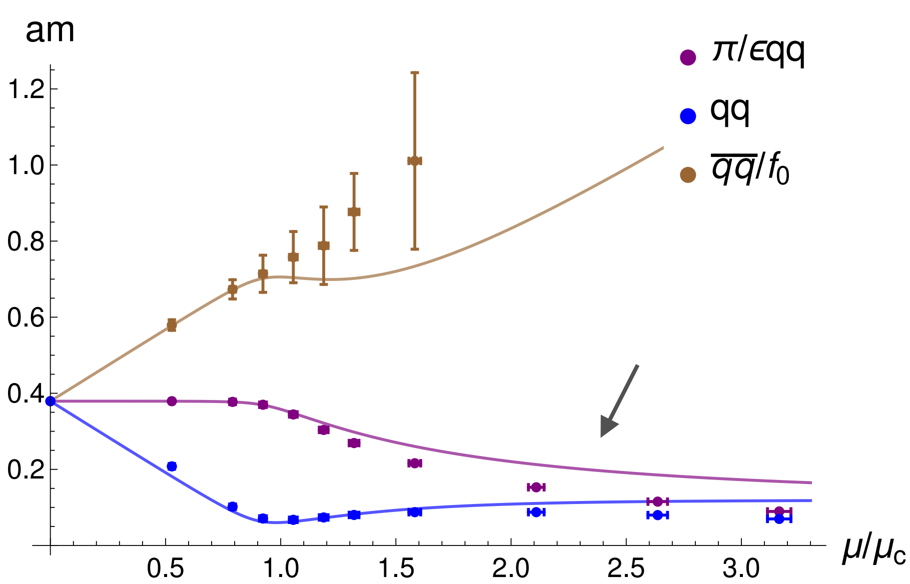
$$a\lambda = 0.001,\ am = 0.01$$



J. Wilhelm, MSc thesis, JLU Giessen (2016)

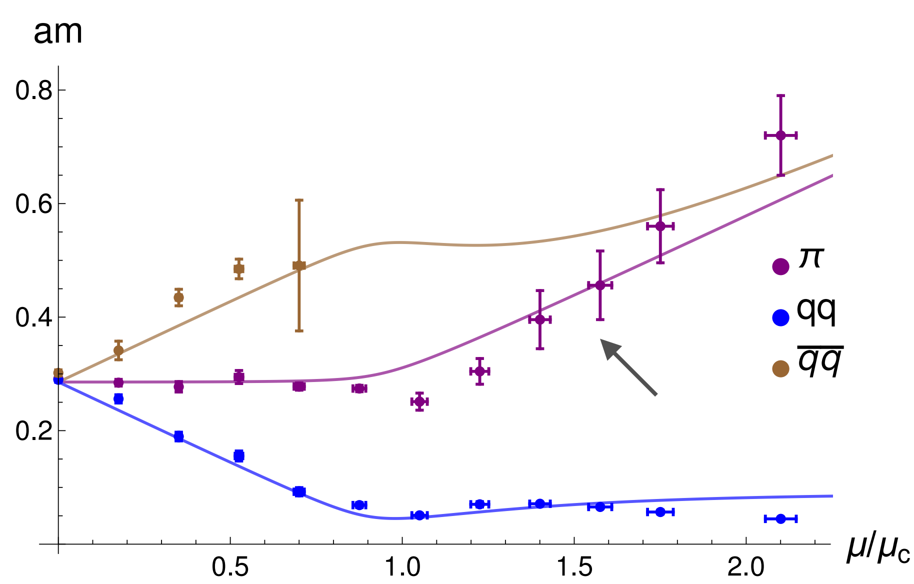
Goldstone Spectrum

unimproved - bulk phase



Dyson: $\beta = 4$

improved - continuum



$\beta = 1$

J. Wilhelm, MSc thesis, JLU Giessen (2016)

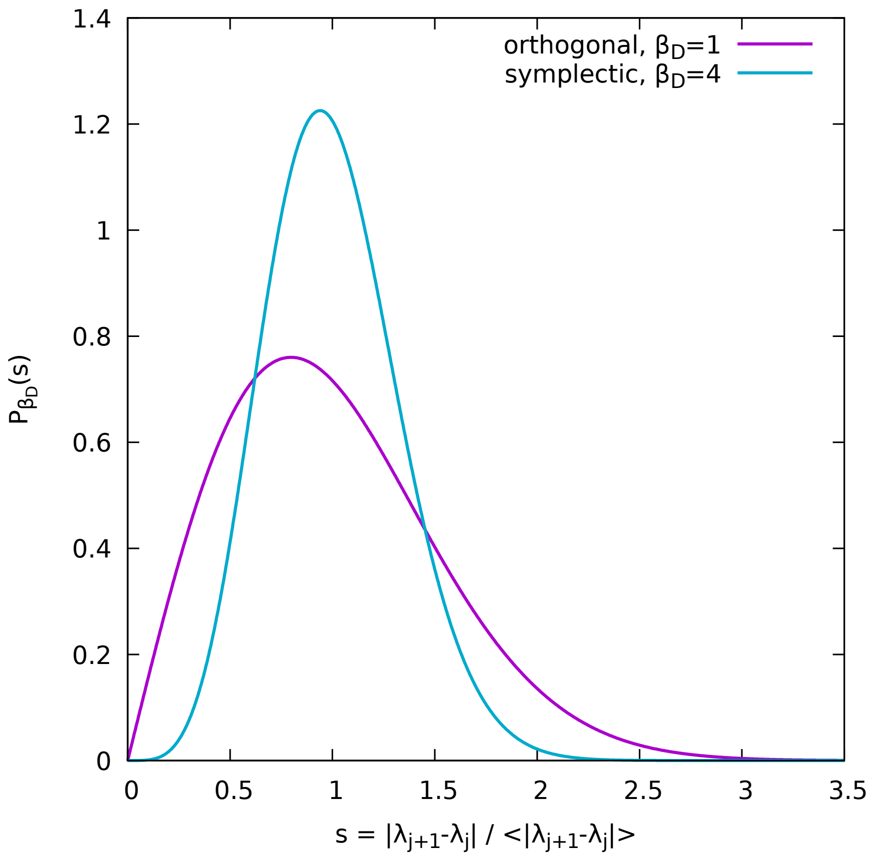
Level-Spacing Statistics

chGOE:

$$P_{\beta=1}(s) = \frac{\pi}{2}\, s\, \exp\left(-\frac{\pi}{4}s^2\right)$$

chGSE:

$$P_{\beta=4}(s) = \left(\frac{8}{3}\right)^6 \frac{s^4}{\pi^3}\, \exp\left(-\left(\frac{8}{3}\right)^2 \frac{s^2}{\pi}\right)$$



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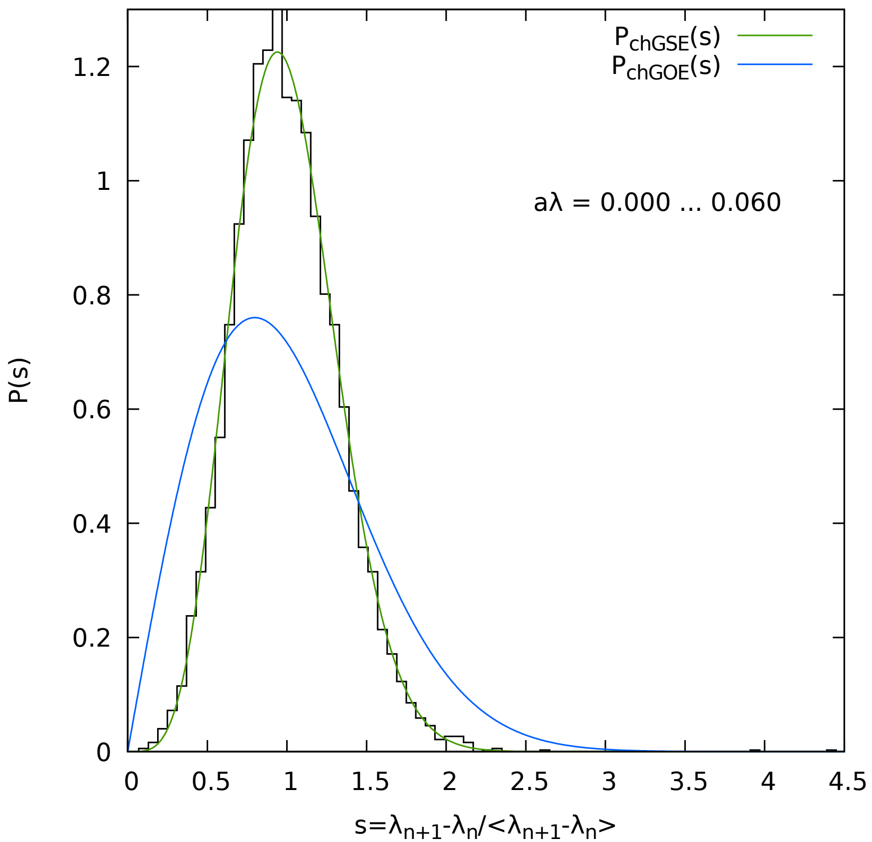

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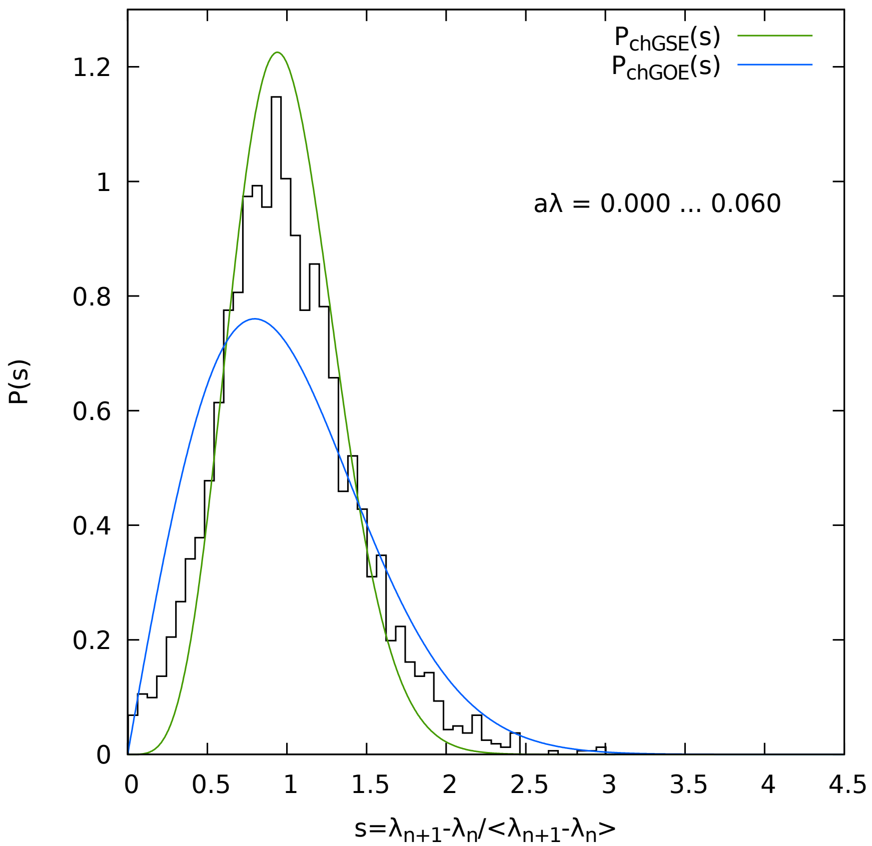

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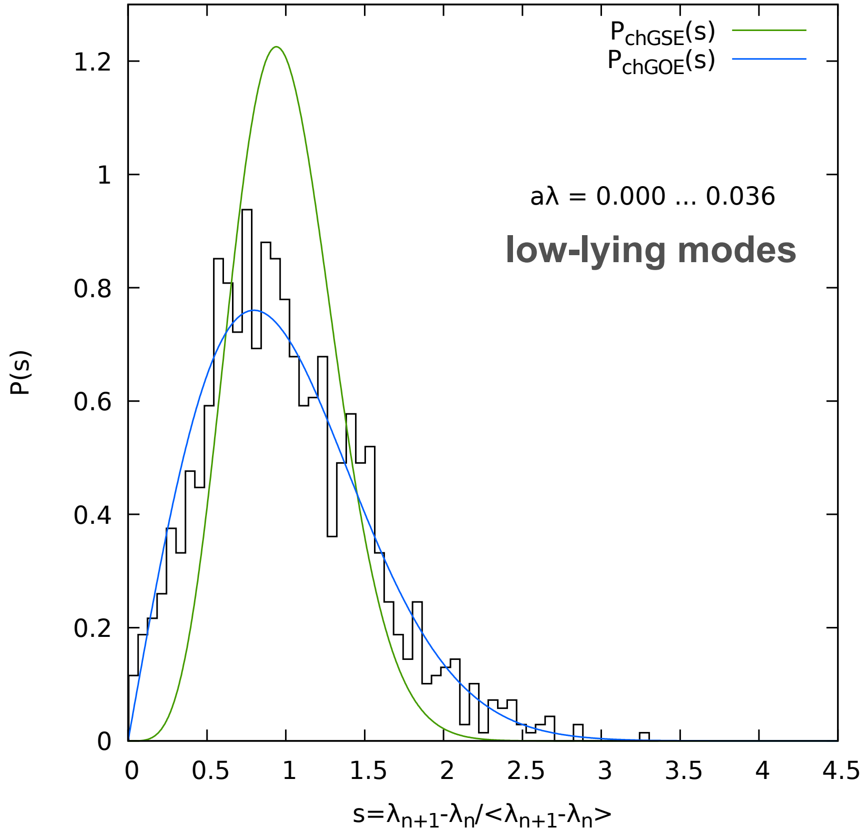

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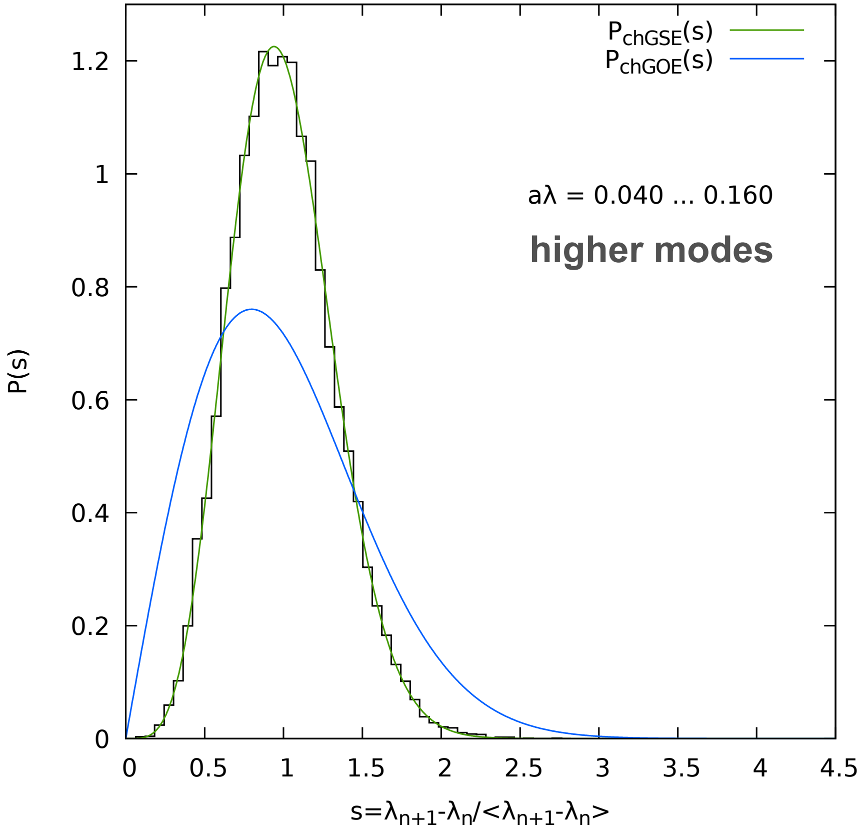
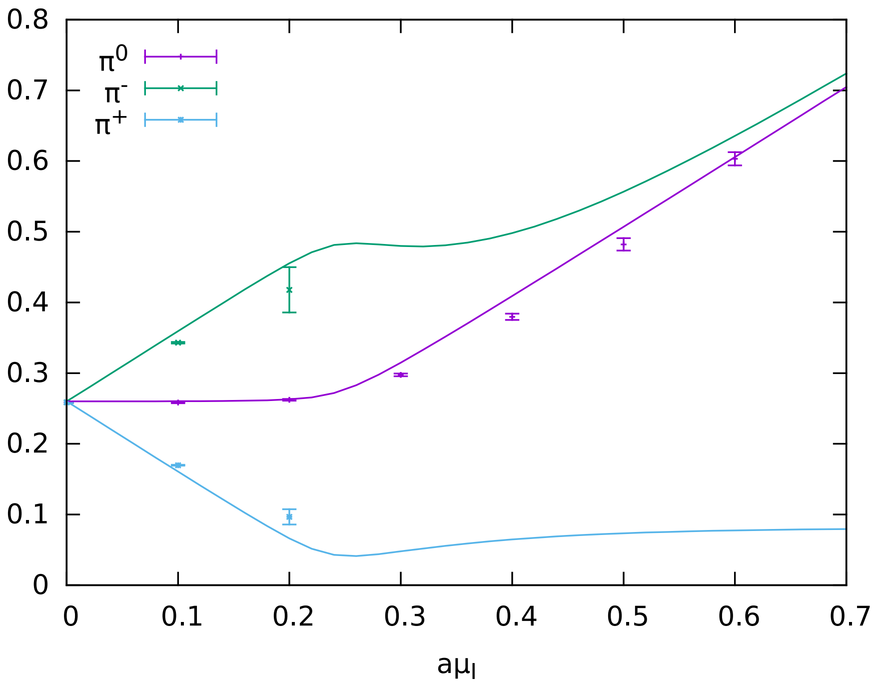
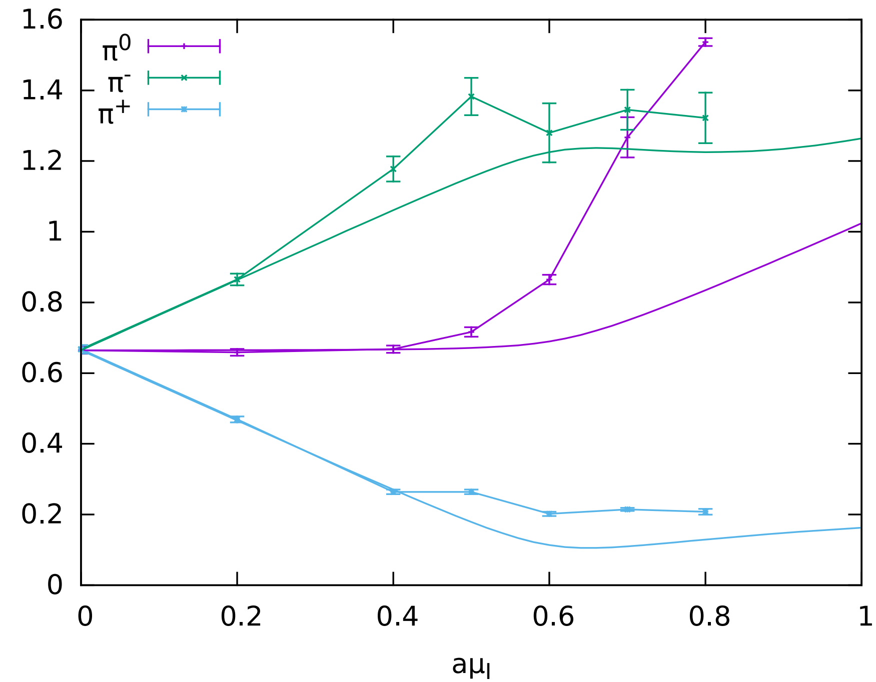


QCD @ finite Isospin Density

- **Goldstone spectrum:**





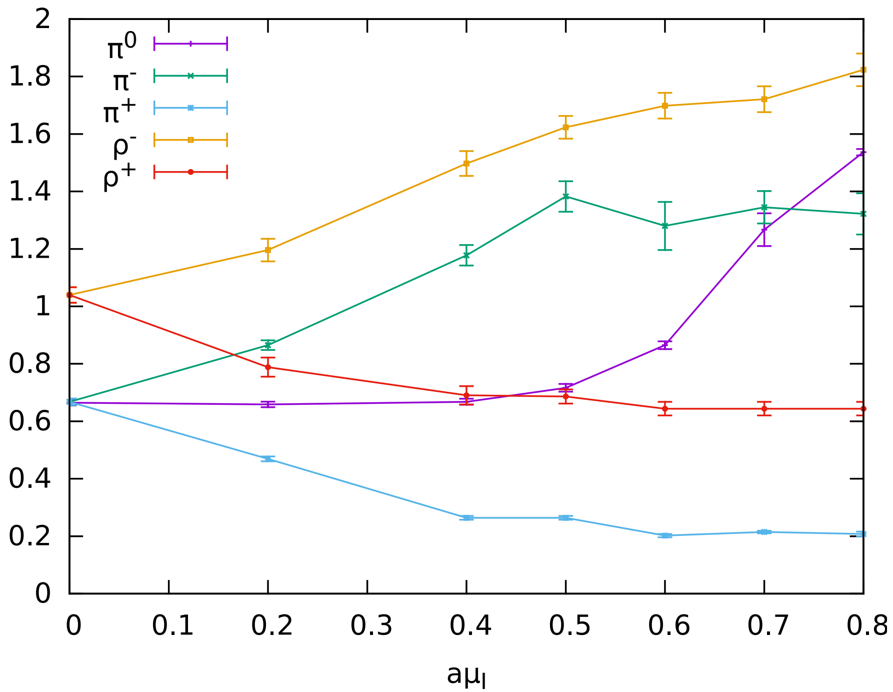
$\beta = 3.5, a\lambda = 0.001, \; am = 0.01$ $\qquad$ $\beta = 4, a\lambda = 0.005, \; am = 0.05$

Ph. Scior, D. Smith, LvS, EPJ 175 (2018) 07042

QCD @ finite Isospin Density

$\beta = 4, a\lambda = 0.005,\ am = 0.05$

- **Vector Mesons & Nucleons:**



$$m_b = m_B - I_3 \mu_I \cos\alpha$$

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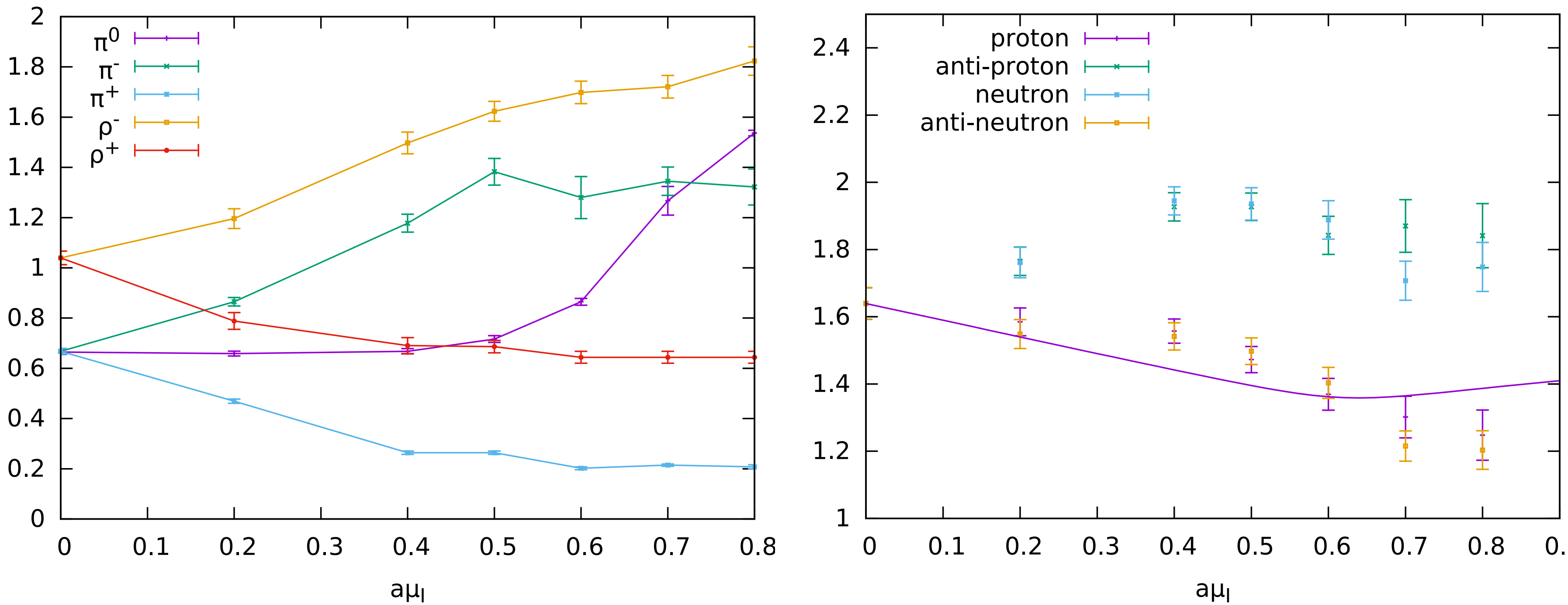

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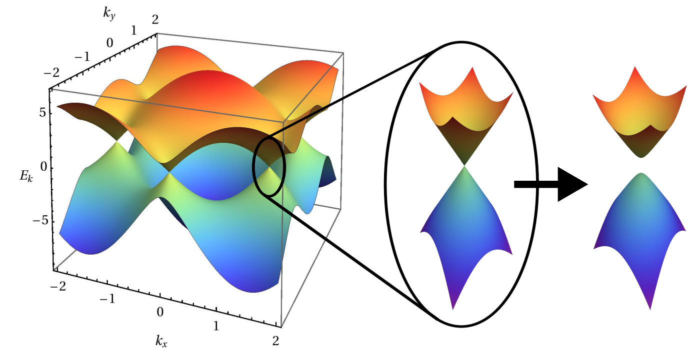
Lifshitz Transition in Graphene

- **semimetal insulator (SDW) transition:**

Ulybyshev, Buividovich, Katsnelson, Polikarpov, PRL 111 (2013) 056801

Smith, LvS, PRB 89 (2014) 195429



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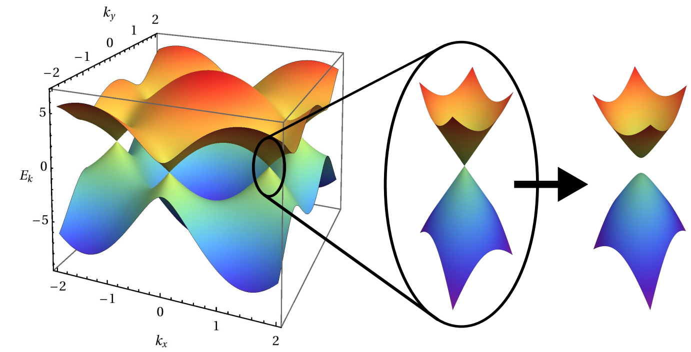

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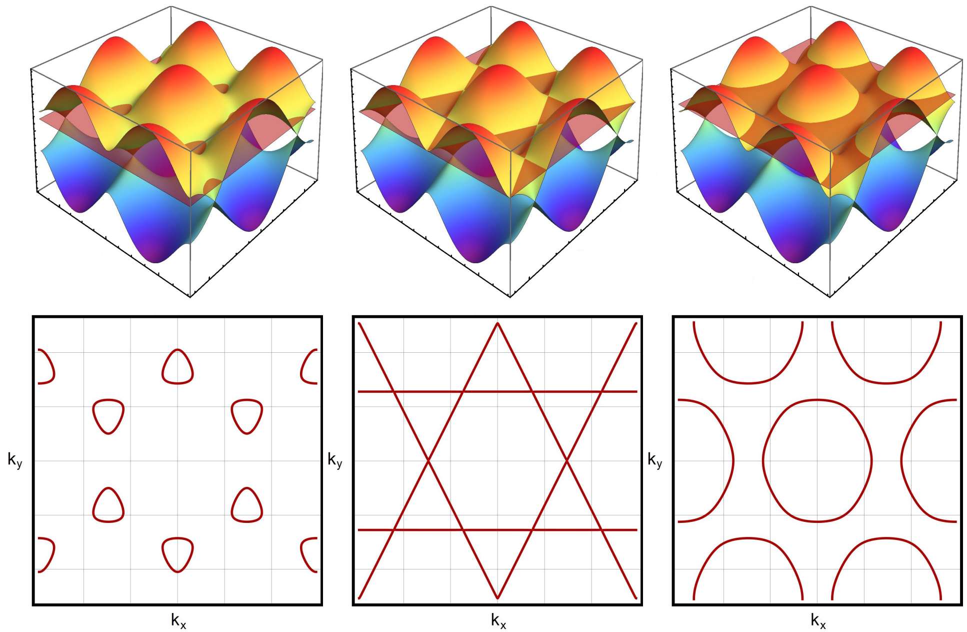

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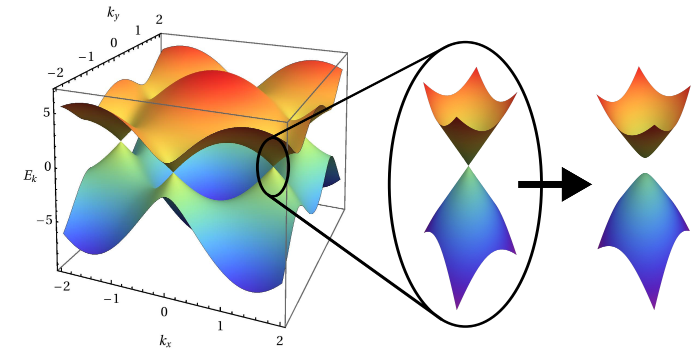

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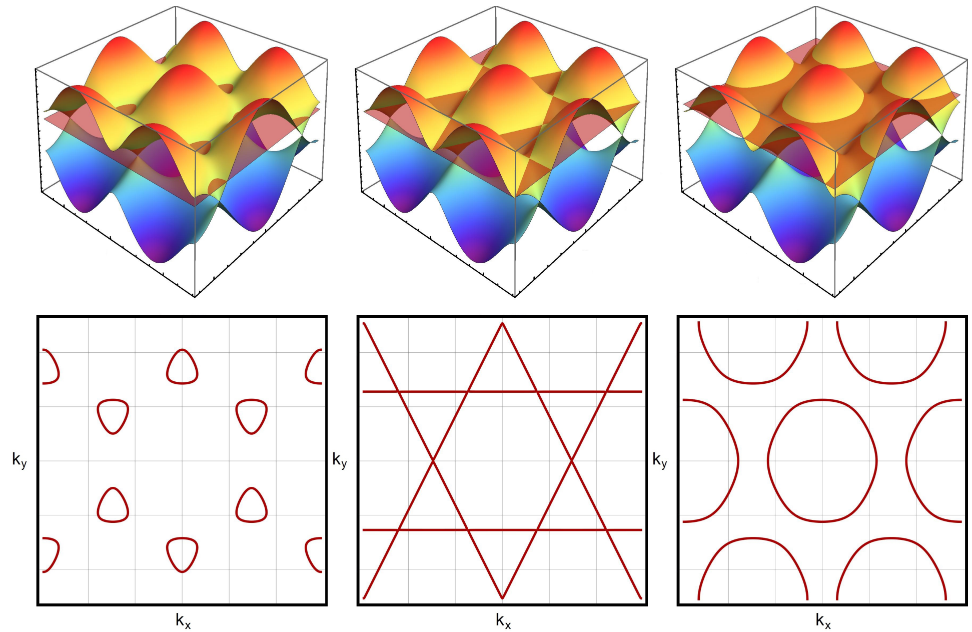

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$$\Omega_{\text{sing}} = \frac{3 g_\sigma \kappa}{2\pi^2} \frac{z^2}{2} \ln|z|$$

$$z = (|\mu| - \kappa)/\kappa$$

Lifshitz Transition in Graphene

- **free tight-binding theory:**

particle-hole susceptibility

$$\chi(z) = \frac{3g_\sigma}{2\pi^2\kappa}\Big(-\ln|z| + 2\ln 2 + \mathcal{O}(z)\Big)$$

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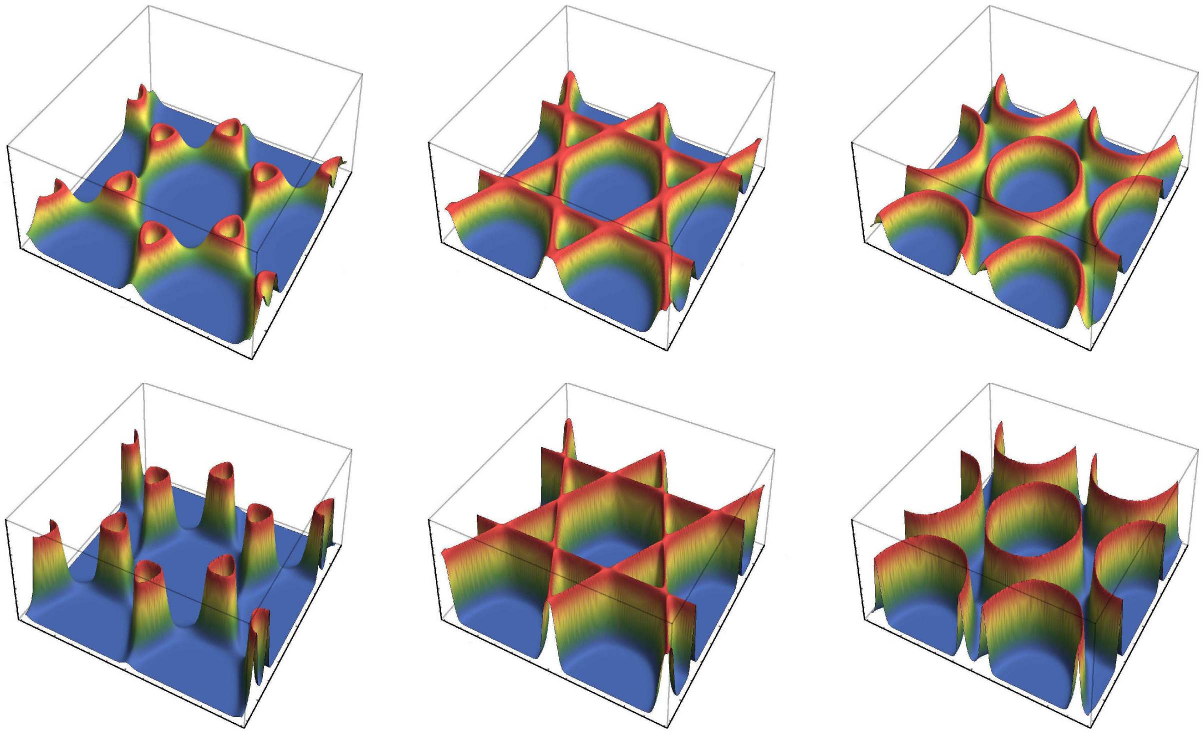

lower T

Lifshitz Transition in Graphene

- **free tight-binding theory:**

  at van Hove sing. ($z = 0$) for $T \to 0$:

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at $T = 0$, finite size, N_c unit cells:

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- **with interactions (HMC):**

μ for finite charge-carrier density $\Rightarrow$ sign problem

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Lifshitz Transition in Graphene

- **free tight-binding theory:**

at van Hove sing. ($z = 0$) for $T \to 0$:

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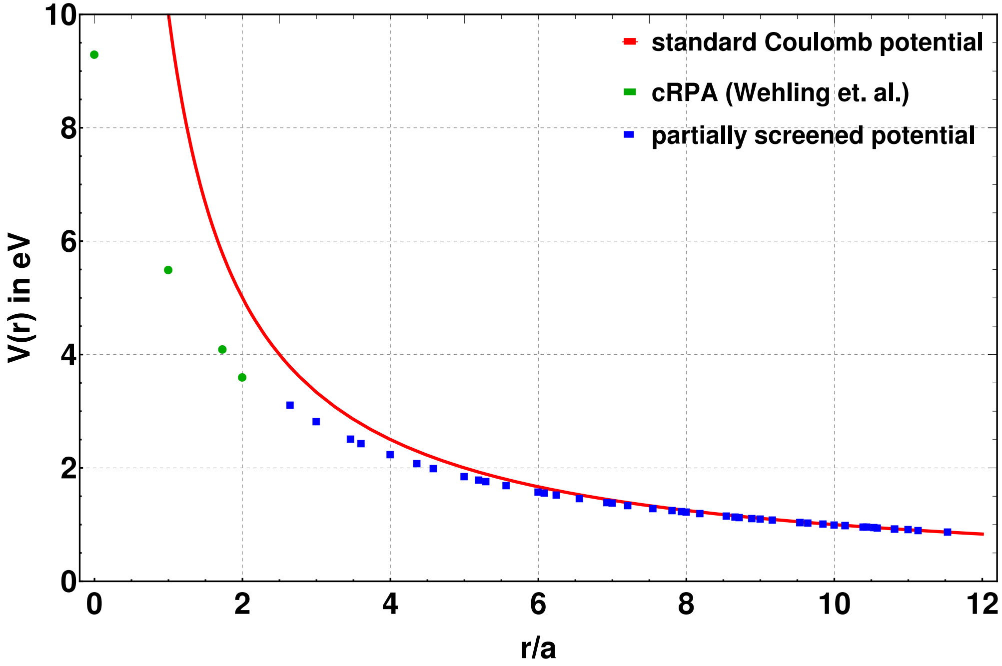
$\pm\mu$ for spin up/down, finite spin density $\Rightarrow$ analogous to isospin chemical potential

Lifshitz Transition in Graphene

- **interactions (screened Coulomb):**
  scaled uniformly by $\lambda \in [0,\ 1]$

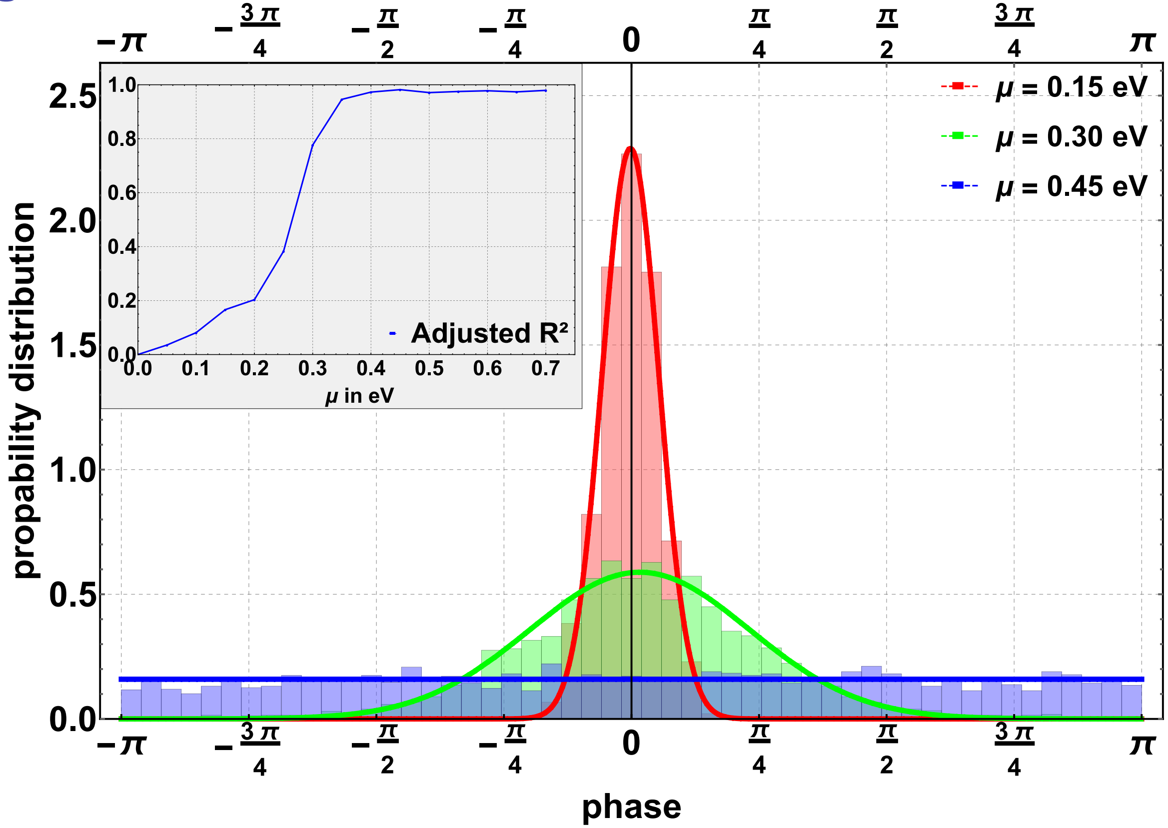


Wehling *et al.*, PRL 106 (2011) 236805

Smith, LvS, PRB 89 (2014) 195429

Lifshitz Transition in Graphene

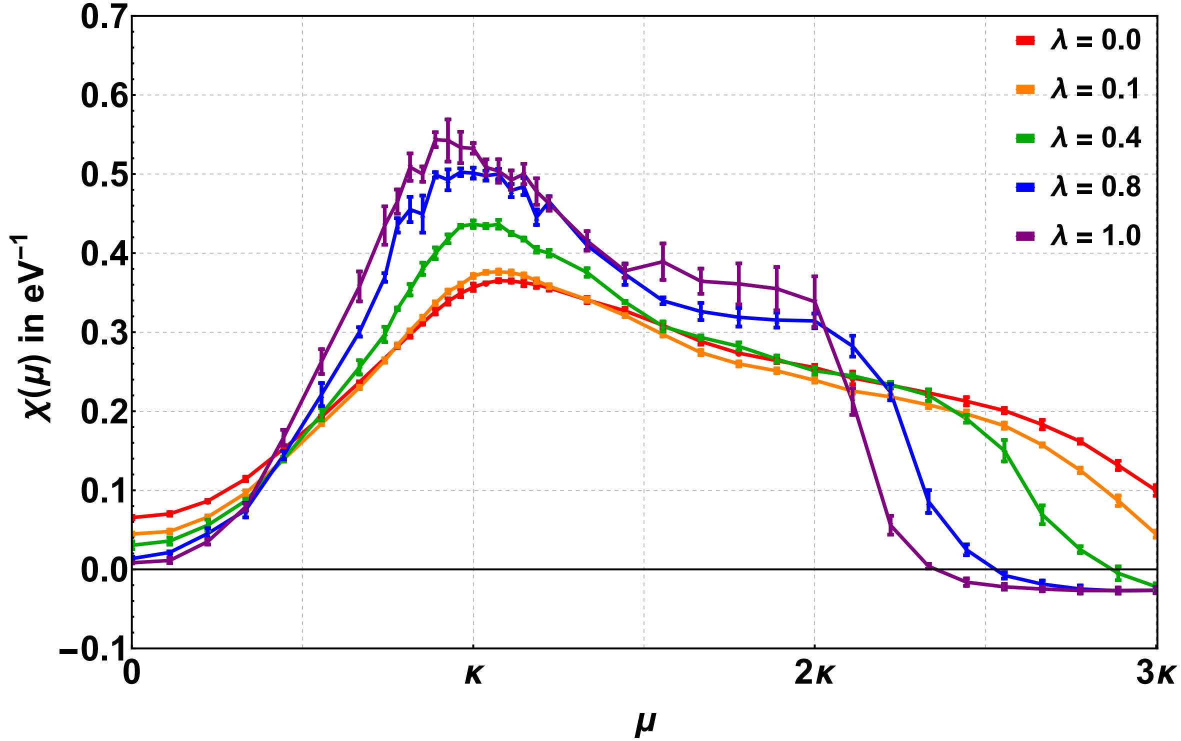
- reweighting?



$\lambda = 0.1$, 6×6 lattice at $\beta = 2$ eV^{-1}

Lifshitz Transition in Graphene

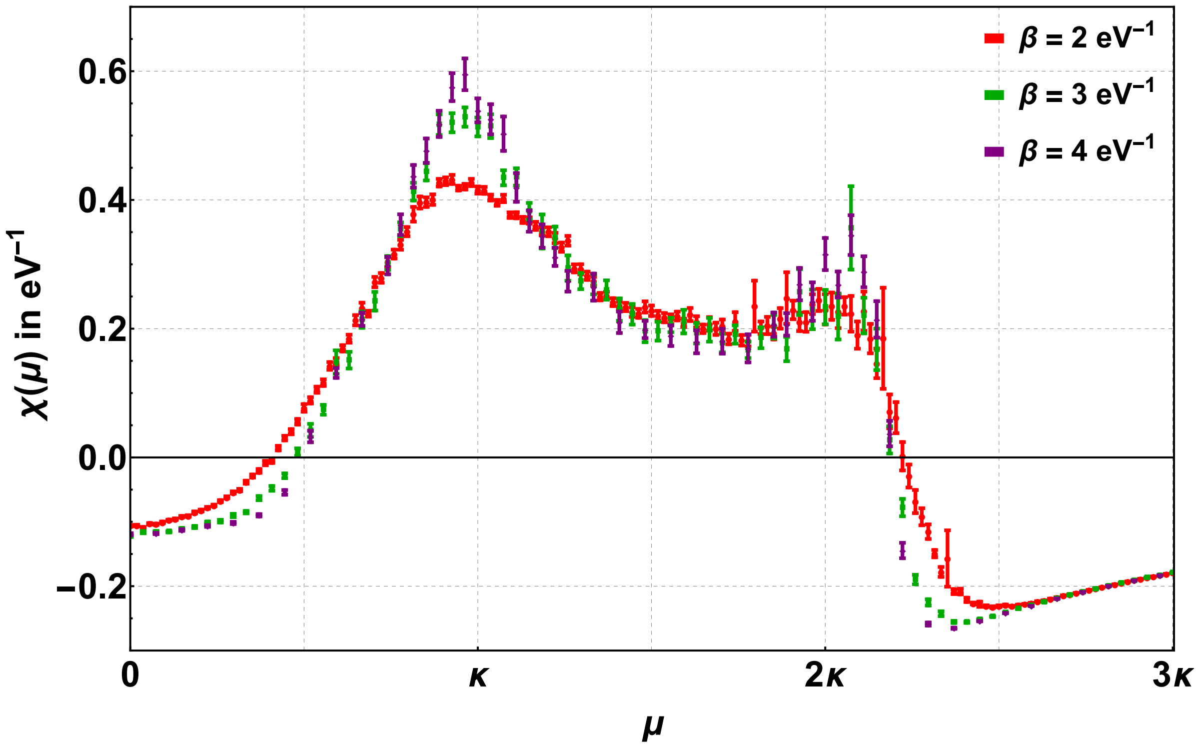
- spin density:



12×12 lattice at $\beta = 2$ eV^{-1}, N_t between 12 and 96, with continuum extrapolated (time discretisation)

Lifshitz Transition in Graphene

- **spin density:**
  lower temperatures

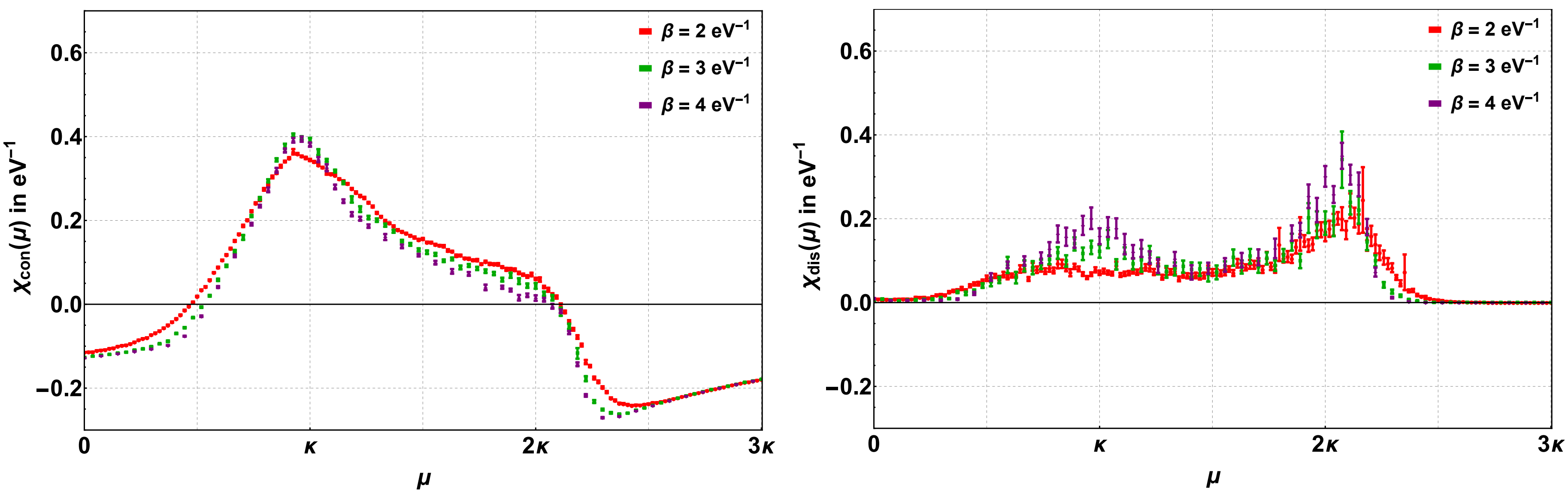


$\lambda = 1,\ N_t = 12, 18$ and 24

Lifshitz Transition in Graphene

- **spin density:**
  lower temperatures

connected susceptibility | disconnected susceptibility



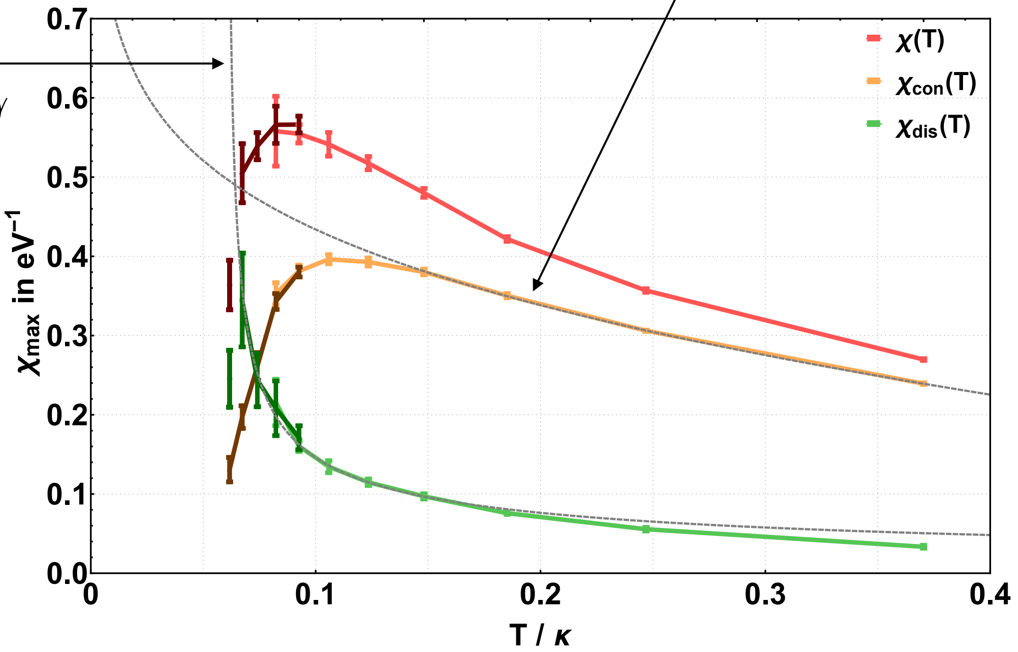
$N_t = 12, 18$ and 24

Lifshitz Transition in Graphene

- **ferromagnetic susceptibility peaks:**
  critical temperature

$$\kappa\,\chi_{\rm con}^{\rm max} = \frac{3}{\pi^2}\ln\left(\frac{\kappa}{T}\right) + b + c\,\frac{T}{\kappa}$$

$$\chi_{\rm dis}^{\rm max} = k\left|\frac{T-T_c}{T_c}\right|^{-\gamma}$$



β_c [eV^{-1}]	T_c [κ]	γ	k [eV^{-1}]
6.1(5)	0.060(5)	0.52(6)	0.12(1)

Conclusions

- **Two-Color QCD with Two Flavors of Staggered Quarks**

  improved action, away from bulk phase ➟ continuum Goldstone spectrum

- **G_2-QCD**

  G_2-nuclear matter, effective theory for heavy quarks, understand generic features in two dimensions (way cheaper)

- **QCD at Finite Isospin Density**

  neutron stars, QCD epoch of Early Universe

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Thank you for your attention!