

# Probing the gluon Wigner distribution in nuclei

Lund U.

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**COST WG1&2 meeting,  
Lisboa, Portugal**

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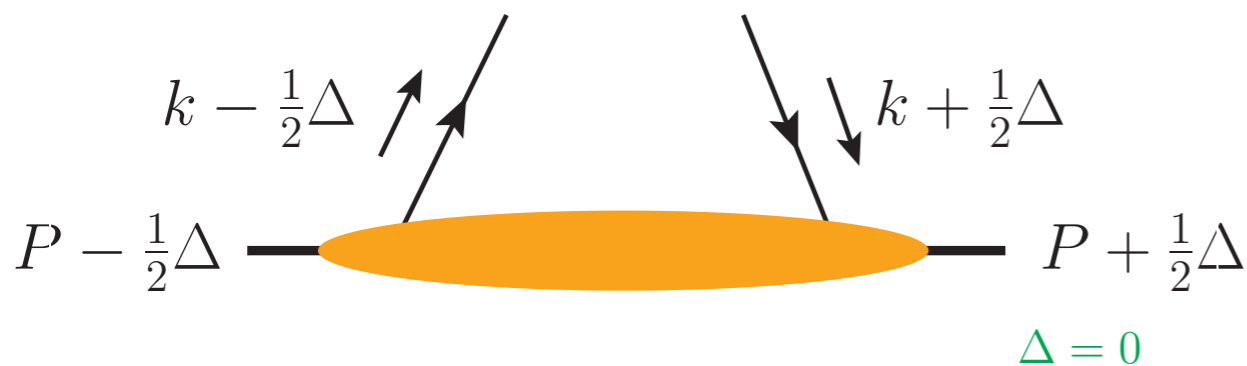
- ✓ **Nucleon tomography: phase space distributions**
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*Y. Hagiwara, Y. Hatta, R. Pasechnik, M. Tasevsky, O. Teryaev, Phys. Rev. D96 (2017) 034009  
arXiv:1706.01765*

# Nucleon tomography: phase space distributions

What do we know about the nucleon?

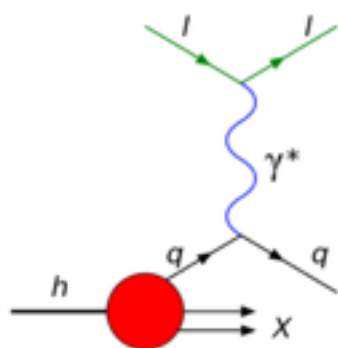
It is a complicated object!



$$H(k, P, \Delta) = (2\pi)^{-4} \int d^4 z e^{izk} \times \langle p(P + \frac{1}{2}\Delta) | \bar{q}(-\frac{1}{2}z) \Gamma q(\frac{1}{2}z) | p(P - \frac{1}{2}\Delta) \rangle$$

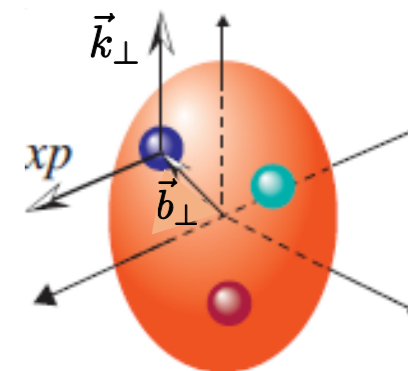
parton correlation function

Partons also experience a transverse motion at a given impact parameter!



$f(k, P)$  parton correlation function

$H(k, P, \Delta)$



$\int dk^-$

$\xi = 0$

$H(x, k, \xi, b) \xleftrightarrow{\text{FT}} H(x, k, \xi, \Delta)$

GTMD

$\vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$

$\int dk^-$   $W(x, k, b)$  Wigner distribution

$\int d^2 k$

$\xi = 0$

$H(x, \xi, b) \xleftrightarrow{\text{FT}} H(x, \xi, \Delta^2)$  GPD

TMD

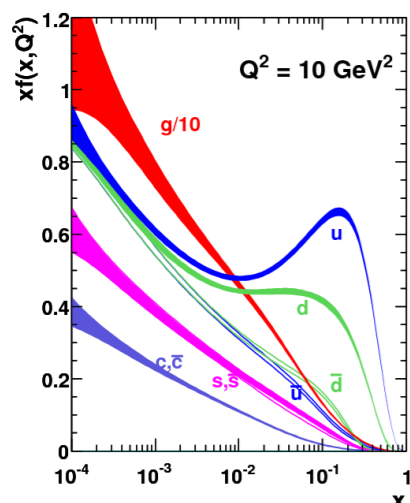
$\int dx x^{n-1}$

$f(x, z) \xleftrightarrow{\text{FT}} f(x, k)$

$f(x, b)$  impact parameter distribution

$\sum_{k=0}^n A_{nk}(\Delta^2) (2\xi)^k$

GFFs



$f(x)$  PDF

$F_n(b) \xleftrightarrow{\text{FT}} F_n(\Delta^2)$  form factor

$\xi = 0$

Figure from Ref. M. Diehl, arXiv: 1512.01328

# Nucleon 5D tomography: the “mother distribution”

✓ 5D tomography: Generalised TMD (GTMD)

Meissner, Metz, Schlegel (2009)...

Husimi distribution

Y. Hagiwara, Y. Hatta (2015)...

Wigner'1932

**Wigner distribution**

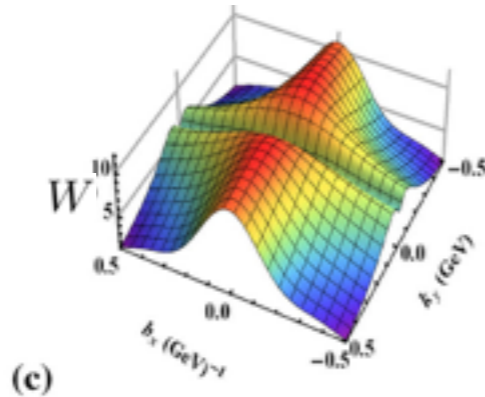
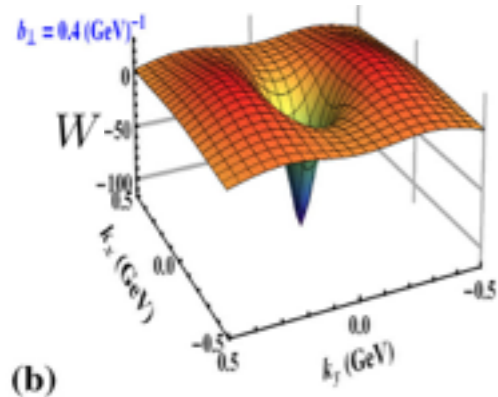
Belitsky, Ji, Yuan (2004); Ji (2003);  
Lorce, Pasquini (2011); Y. Hatta (2011)...

+ many more studies...

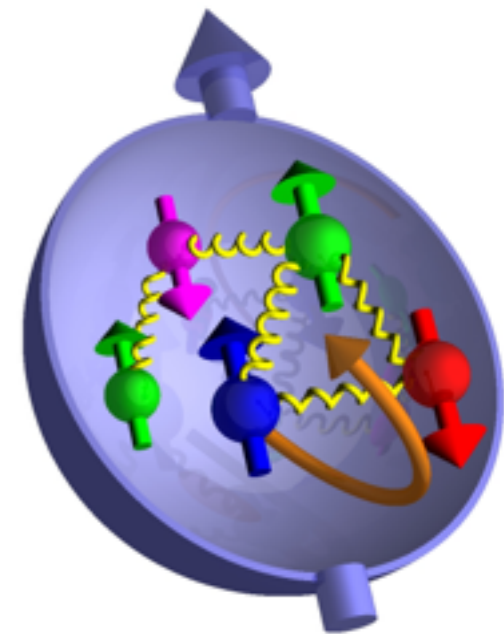
Example: leading-twist quark Wigner distribution

$$W(x, \vec{k}_\perp, \vec{b}_\perp) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{i\vec{b}_\perp \cdot \vec{\Delta}_\perp} \int \frac{dz^- d^2 z_\perp}{16\pi^3} e^{ixP^+ z^- - i\vec{k}_\perp \cdot \vec{z}_\perp} \langle P - \frac{\Delta}{2} | \bar{q}(-z/2) \gamma^+ q(z/2) | P + \frac{\Delta}{2} \rangle$$

J. More et al, PRD'17



**Non-trivial correlation between the transverse momentum and the impact parameter due to orbital angular momentum!**



Spin decomposition of the nucleon:

$$\frac{1}{2} \Delta \Sigma + \Delta G + L^q + L^g \equiv \frac{1}{2}$$

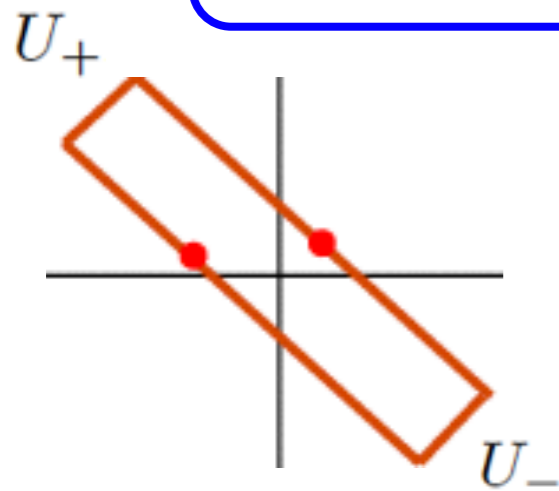
$$L = \int dx d^2 b_\perp d^2 k_\perp (\vec{b}_\perp \times \vec{k}_\perp)_z \cdot W(x, \vec{k}_\perp, \vec{b}_\perp) \quad \text{canonical orbital angular momentum}$$

Wigner/GTMD distributions provide the most complete information on partonic “image” of the nucleon!

# The gluon Wigner distribution at small x: dipole picture

**From quark to gluon:**  $\bar{\Psi}(\vec{r} - \xi/2)\Gamma\Psi(\vec{r} + \xi/2) \rightarrow F^{+\nu}(\vec{r} - \xi/2)F_{\nu}^+(\vec{r} + \xi/2)$

$$xW(x, \vec{q}_{\perp}, \vec{b}_{\perp}) = \frac{2}{P^+(2\pi)^3} \int dz^+ d^2\vec{z}_{\perp} \int \frac{d^2\vec{\Delta}_{\perp}}{(2\pi)^2} e^{i\vec{q}_{\perp}\cdot\vec{z}_{\perp} - ixP^-z^+} \\ \times \left\langle P + \frac{\vec{\Delta}_{\perp}}{2} \left| \text{Tr} \left[ U_+ F_a^{+i} \left( \vec{b}_{\perp} + \frac{z}{2} \right) U_- F_a^{+i} \left( \vec{b}_{\perp} - \frac{z}{2} \right) \right] \right| P - \frac{\vec{\Delta}_{\perp}}{2} \right\rangle$$



**Staple-shaped Wilson lines:**  $U_{\pm} \equiv U[0, \pm\infty; 0]U[\pm\infty, z^+; \vec{z}_{\perp}]$

$$U[z_1^+, z_2^+; \vec{z}_{\perp}] \equiv \mathcal{P} \exp \left( ig \int_{z_1^+}^{z_2^+} dz^+ \hat{A}^-(z^+, \vec{z}_{\perp}) \right)$$

$x \ll 1 \quad e^{-ixP^-z^+} \approx 1$

**Y. Hatta, B. W. Xiao, F. Yuan, PRL 116, 202301 (2016)**

$$xW_g(x, \mathbf{k}, \mathbf{b}_{\perp}) = \frac{2N_c}{\alpha_S} \int \frac{d^2\mathbf{r}}{(2\pi)^2} e^{i\mathbf{k}\cdot\mathbf{r}} \left( \frac{1}{4} \nabla_{\mathbf{b}_{\perp}}^2 - \nabla_{\mathbf{r}}^2 \right) S_Y(\mathbf{r}, \mathbf{b}_{\perp}) \quad Y = \ln \frac{1}{x}$$

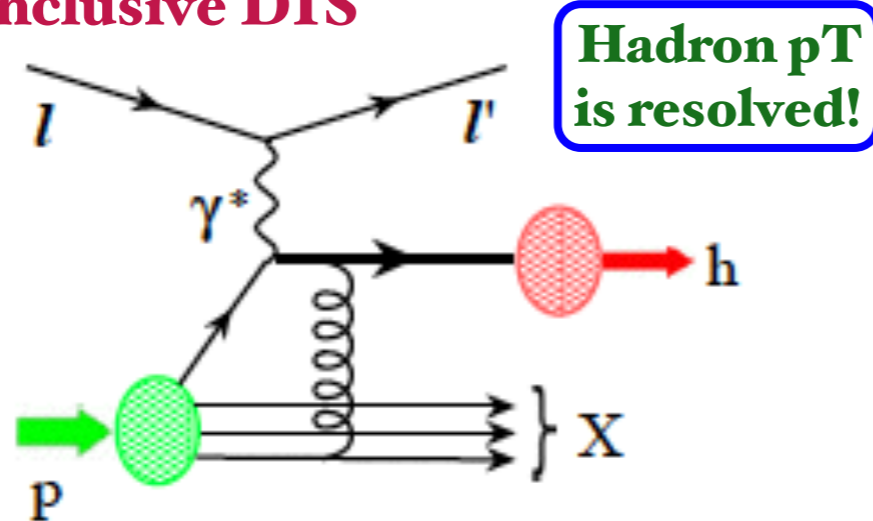
**Dipole S-matrix:**  $S_Y(\vec{q}_{\perp}, \vec{\Delta}_{\perp}) = \int \frac{d^2\vec{r}_{\perp} d^2\vec{b}_{\perp}}{(2\pi)^4} e^{i\vec{\Delta}_{\perp}\cdot\vec{b}_{\perp} + i\vec{q}_{\perp}\cdot\vec{r}_{\perp}} \left\langle \frac{1}{N_c} \text{Tr} U \left( \vec{b}_{\perp} + \frac{\vec{r}_{\perp}}{2} \right) U^{\dagger} \left( \vec{b}_{\perp} - \frac{\vec{r}_{\perp}}{2} \right) \right\rangle_Y$

# Nucleon tomography: relevant processes

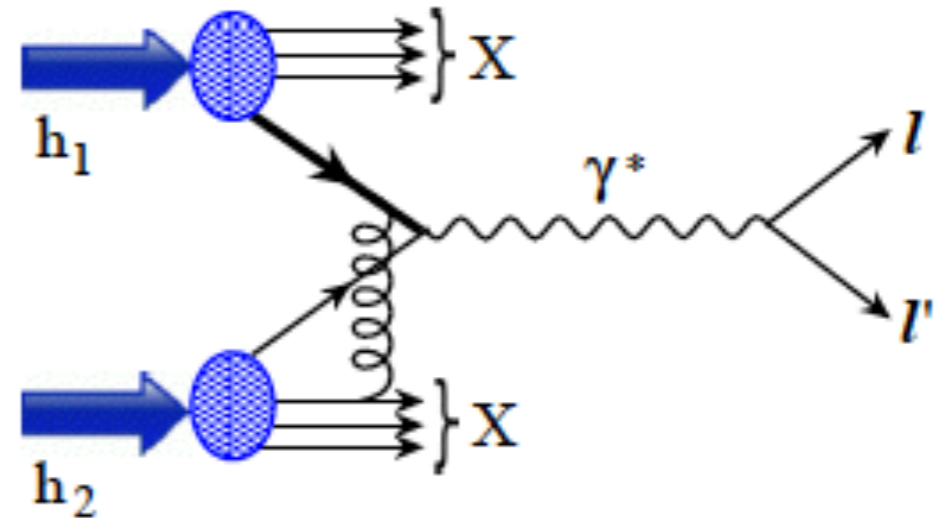
Combination of TMD and GPD provide a deep 3D picture of the quark and gluon content of the nucleon

## Semi-Inclusive DIS

**TMD**



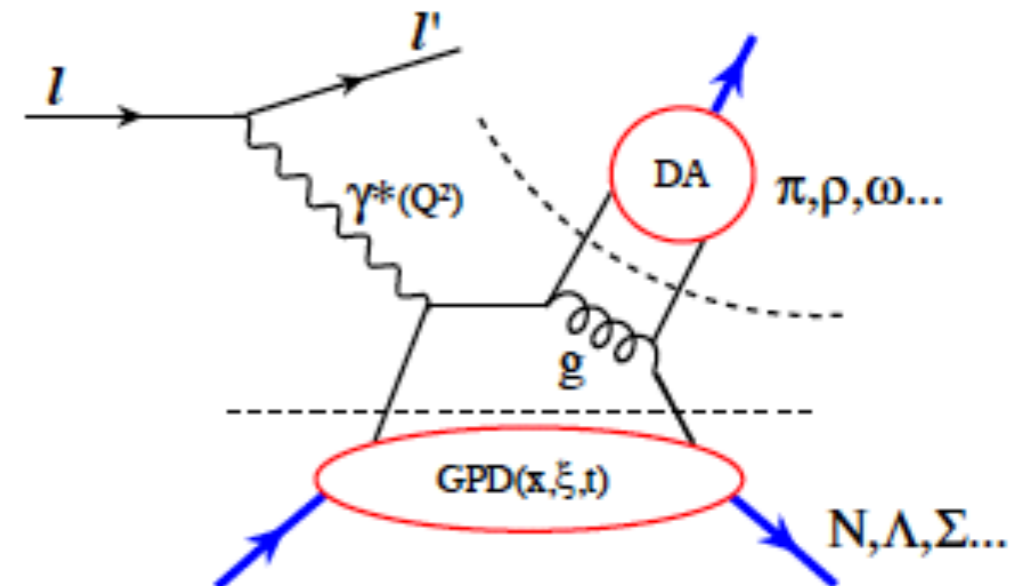
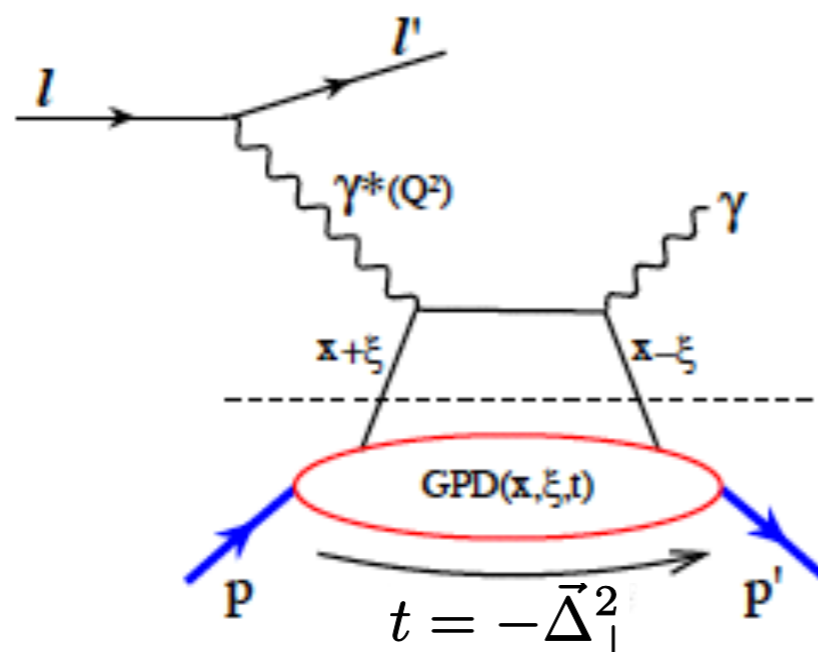
## Drell-Yan



Review: e.g. N. Stefanis et al.  
arXiv: 1612.03077

## Deeply-Virtual Compton Scattering

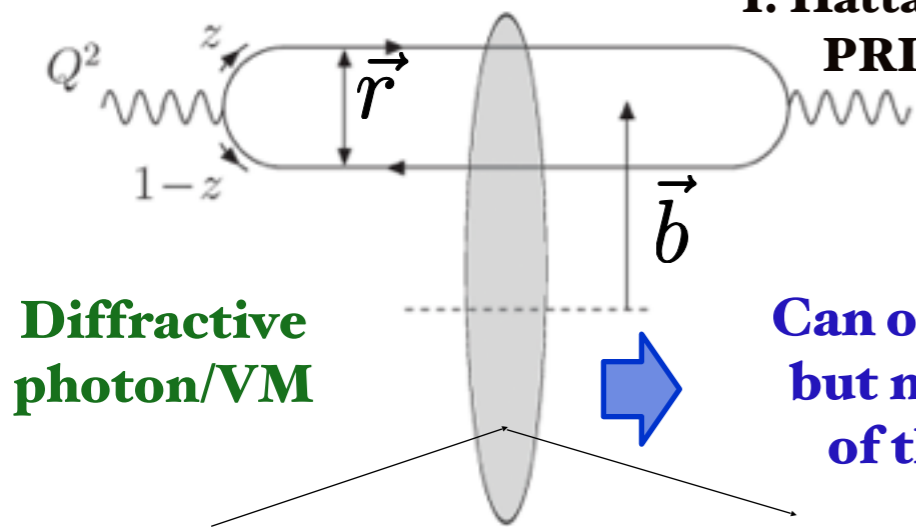
**GPD**



What about accessing the 5D Wigner/GTMD distributions?

# Gluon Wigner from diffractive DIS processes

Y. Hatta, B. W. Xiao, F. Yuan,  
PRD95, 114026 (2017)

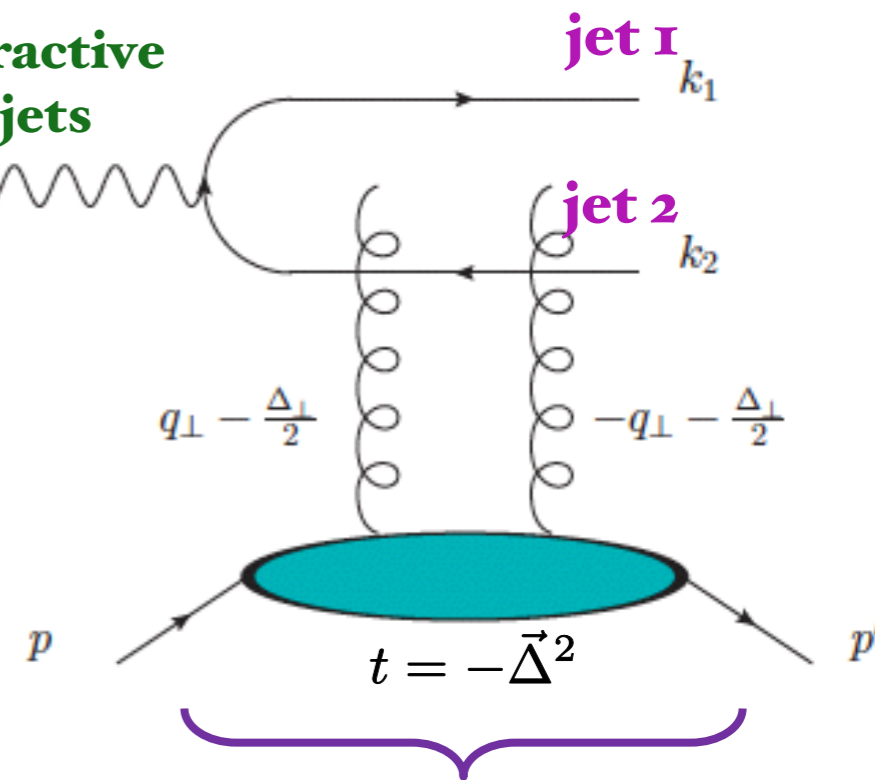


**Diffractive photon/VM**

Can only access b-profile but not kT-dependence of the gluon Wigner!

T. Altinoluk et al, PLB758, 373 (2016)

**Diffractive dijets**



**Dijet observables:**

**Proton recoil momentum:**

$$\vec{k}_{1\perp} + \vec{k}_{2\perp} = -\vec{\Delta}_{\perp}$$

**Dijet relative momentum:**

$$\vec{P}_{\perp} = \frac{1}{2}(\vec{k}_{2\perp} - \vec{k}_{1\perp})$$

Y. Hatta, B. W. Xiao, F. Yuan,  
PRL 116, 202301 (2016)

$$\frac{d\sigma}{d\vec{P}_{\perp} d\vec{\Delta}_{\perp}} \propto |\vec{M}|^2, \quad \vec{M}(\vec{P}_{\perp}, \vec{\Delta}_{\perp}) = \int \frac{d^2\vec{q}_{\perp}}{2\pi} \frac{\vec{P}_{\perp} - \vec{q}_{\perp}}{(\vec{P}_{\perp} - \vec{q}_{\perp})^2 + \epsilon_f^2} S_Y(\vec{q}_{\perp}, \vec{\Delta}_{\perp})$$

for small- $Q^2$      $\vec{q}_{\perp} \sim \vec{P}_{\perp}$

**Advantage!**

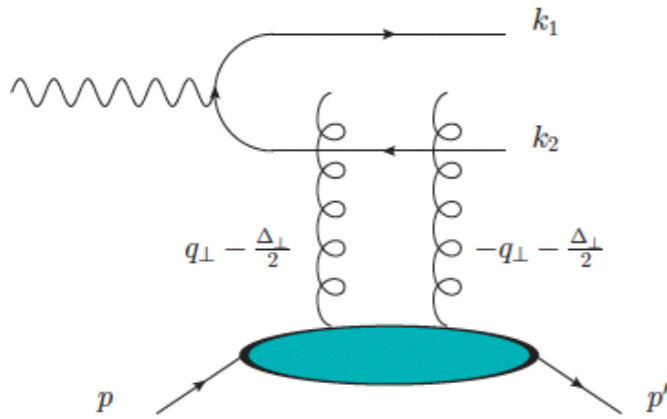
$$\frac{d\sigma}{d\vec{P}_{\perp} d\vec{\Delta}_{\perp}} \propto \left( S_Y(\vec{P}_{\perp}, \vec{\Delta}_{\perp}) \right)^2$$

**Fourier transform of the dipole S-matrix!**

# Elliptic Wigner distribution and dipole orientation

Y. Hatta, B. W. Xiao, F. Yuan, PRL 116, 202301 (2016)

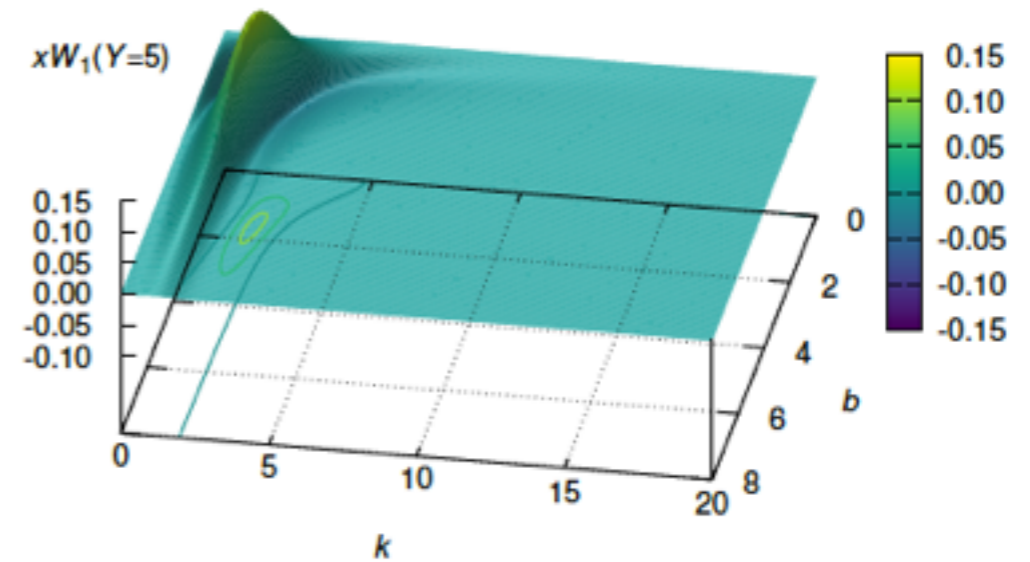
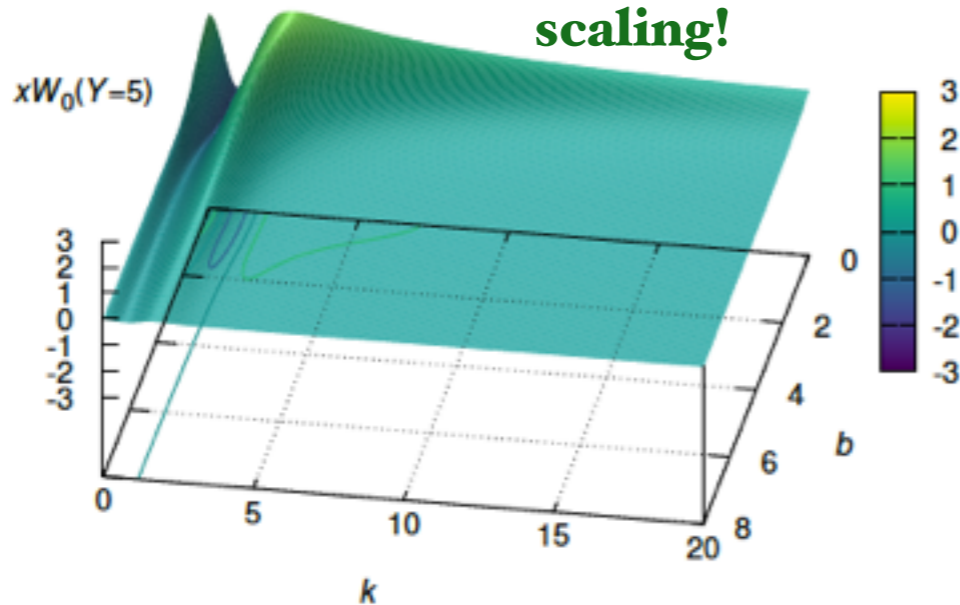
Y. Hagiwara, Y. Hatta, T. Ueda, PRD 94, 094036 (2016)



$$W(x, b, k) = W_0(x, b, k) + 2 \cos 2(\phi_k - \phi_b) W_1(x, b, k) + \dots$$

“Elliptic” gluon Wigner

Geometric scaling!



**BK equation with SO(3) symmetry (CGC)**

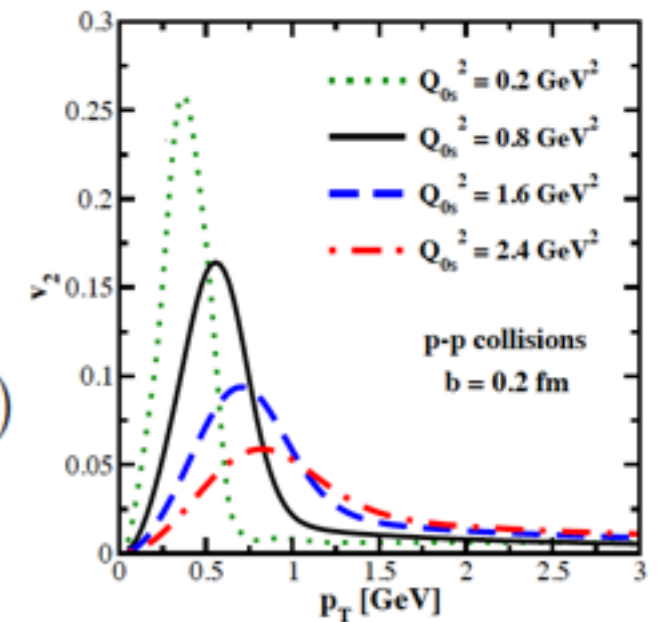
Gubser (2011)

E. Iancu and A. Rezaeian, PRD95, 094003 (2017)

**McLerran-Venugopalan model for the dipole S-matrix**

$$S(b, r) = \exp\{-N_{2g}(b, r)\}$$

$$N_{2g}(b, r, \theta) = \mathcal{N}_0(b, r) + \mathcal{N}_\theta(b, r) \cos(2\theta)$$



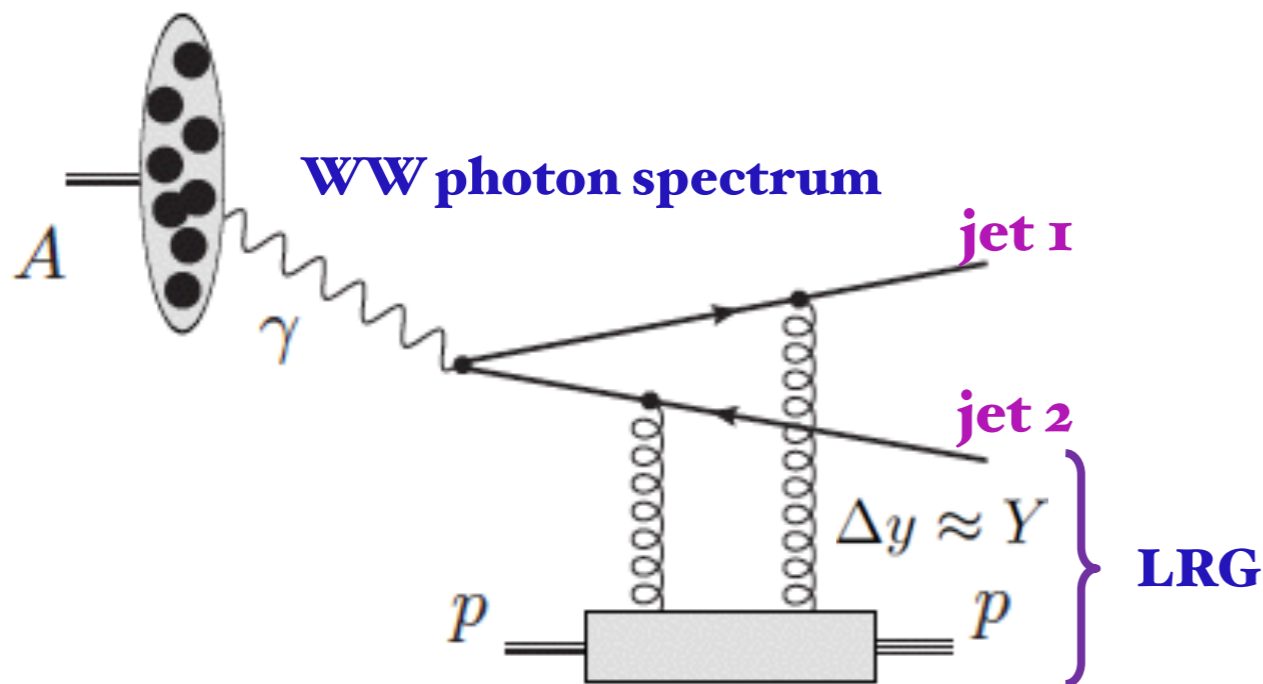
Dipole orientation effects



Elliptic flow, gluon transversity, angular correlation in DVCS etc

# Accessing the gluon Wigner from exclusive dijets in UPC

Y. Hagiwara, Y. Hatta, RP, M. Tasevsky, O. Teryaev,  
PRD 96, 034009 (2017)



photon-target diffractive amplitude

$$\vec{M}(\vec{P}_\perp, \vec{\Delta}_\perp) = \int \frac{d^2 \vec{q}_\perp}{2\pi} \frac{\vec{P}_\perp - \vec{q}_\perp}{(\vec{P}_\perp - \vec{q}_\perp)^2} S(\vec{q}_\perp, \vec{\Delta}_\perp)$$

“Isotropic”

“Elliptic”

$$S(\vec{q}_\perp, \vec{\Delta}_\perp) = S_0(q_\perp, \Delta_\perp) + 2 \cos 2(\phi_q - \phi_\Delta) \tilde{S}(q_\perp, \Delta_\perp)$$

Photon-target cross section:

$$\frac{d\sigma^{p\gamma}}{dy_1 dy_2 d^2 \vec{k}_{1\perp} d^2 \vec{k}_{2\perp}} = N_c \alpha_{em} (2\pi)^2 q^+ \delta(k_1^+ + k_2^+ - q^+) \sum_f e_f^2 2z(1-z)(z^2 + (1-z)^2) |\vec{M}|^2 \quad z = \frac{k_{1\perp} e^{y_1}}{k_{1\perp} e^{y_1} + k_{2\perp} e^{y_2}}$$

Nucleus-target cross section:



$$\frac{d\sigma^{pA}}{dy_1 dy_2 d^2 \vec{k}_{1\perp} d^2 \vec{k}_{2\perp}} \approx \underbrace{\omega \frac{dN}{d\omega}}_{\text{photon flux}} \frac{2(2\pi)^4 N_c \alpha_{em}}{P_\perp^2} \sum_f e_f^2 z(1-z)(z^2 + (1-z)^2) (A^2 + 2 \cos 2(\phi_P - \phi_\Delta) AB)$$

$$S_0(P_\perp, \Delta_\perp) = -\frac{1}{P_\perp} \frac{\partial}{\partial P_\perp} A(P_\perp, \Delta_\perp)$$

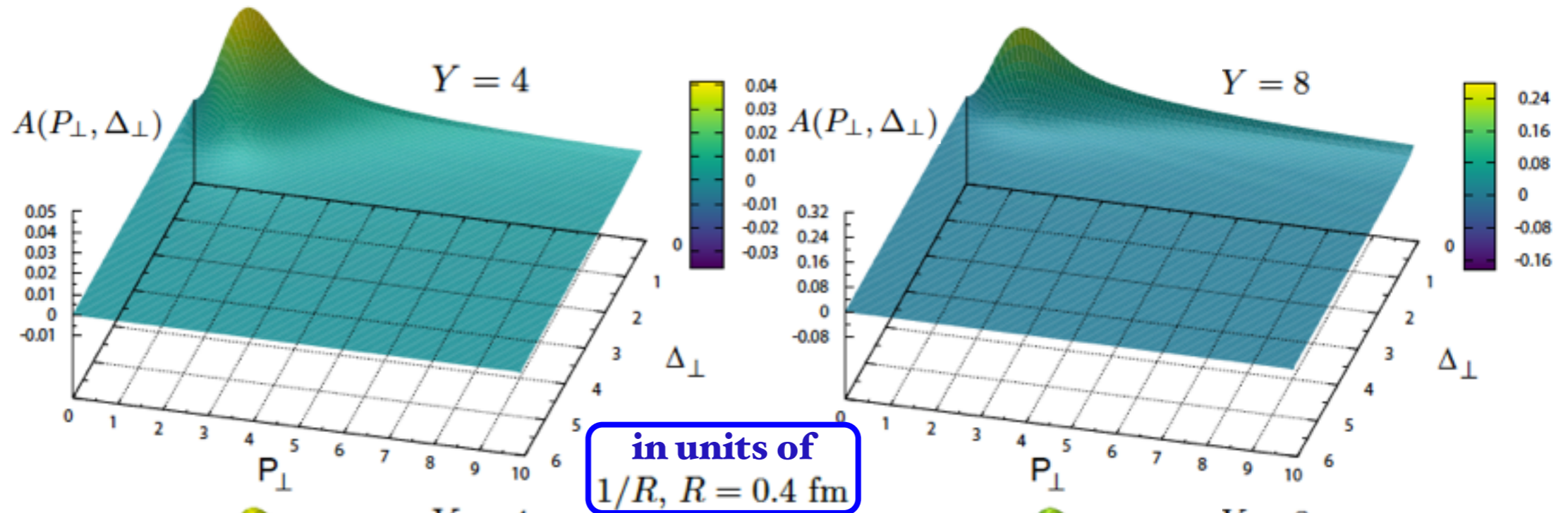
$$\tilde{S}(P_\perp, \Delta_\perp) = -\frac{\partial B(P_\perp, \Delta_\perp)}{\partial P_\perp^2} + \frac{2}{P_\perp^2} \int_0^{P_\perp^2} \frac{dP'_\perp{}^2}{P'_\perp{}^2} B(P'_\perp, \Delta_\perp)$$

Separate measurements  
of A and B

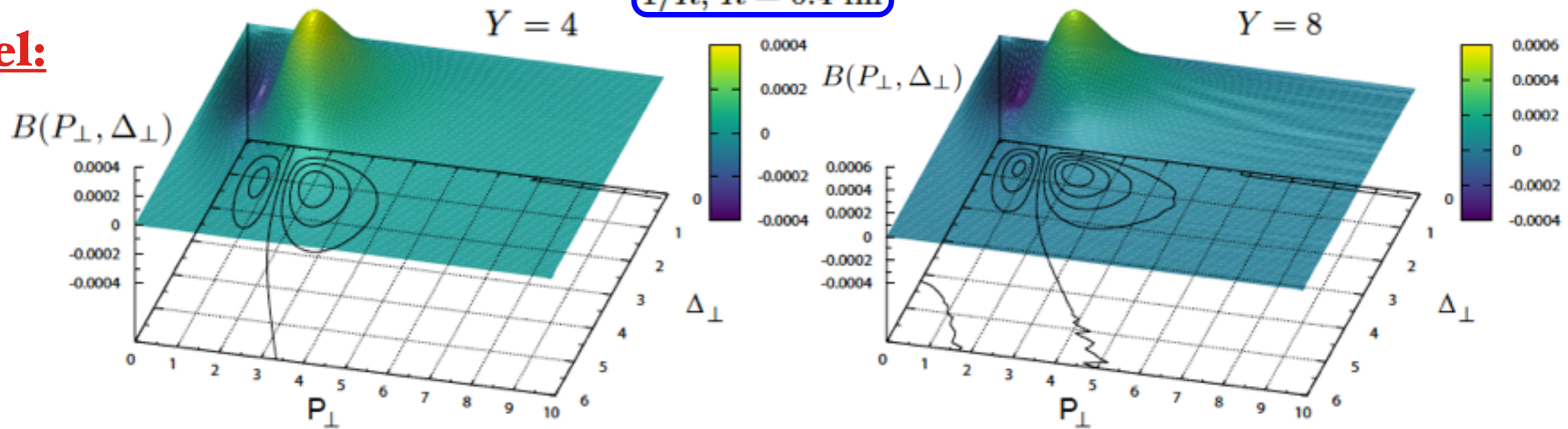


full information  
about the gluon Wigner!

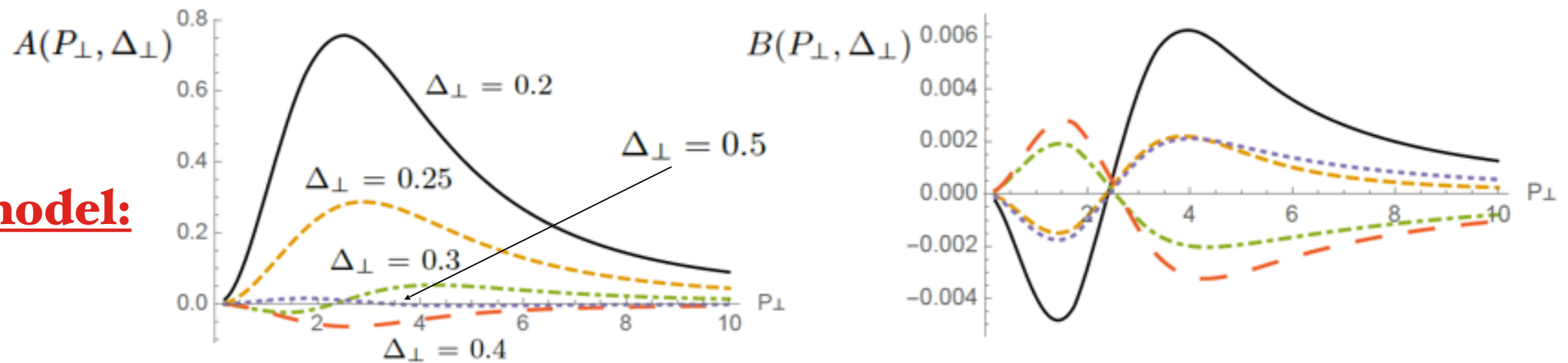
# CGC results: BK vs MV



**BK model:**



**MV model:**



# Conclusions

- ✓ Quasi-probability (quark and gluon) Wigner distributions represent 5D snapshot of the hadronic structure and contain full information about it equivalent to knowing the exact wave functions of partons in the nucleon.
- ✓ The elliptic gluon Wigner distribution contains an important info on azimuthal angle correlation due to dipole orientation effects and is responsible e.g. for the elliptic flow and angular correction in exclusive dijet production.
- ✓ One of the most promising ways to access the gluon Wigner distribution is by measuring the differential cross section of exclusive dijet production in ultraperipheral pA/AA collisions. A dedicated analysis for a given experiment is necessary.