NEW EXACT AND PERTURBTIVE SOLUTIONS OF RELATIVISTIC HYDRO

A COLLECTION OF RECENT RESULTS

MÁTÉ CSANÁD (EÖTVÖS U) @ THOR LISBON MEETING, JUNE 13, 2018

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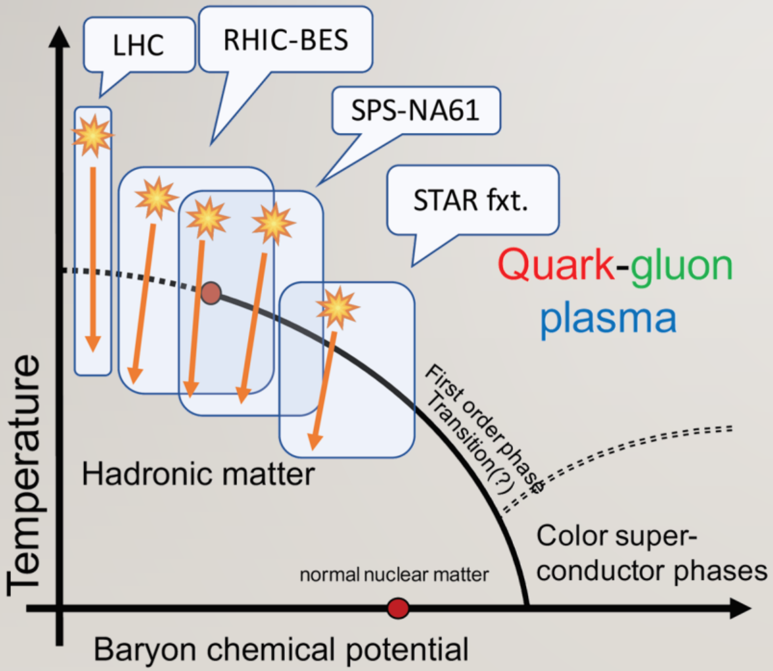
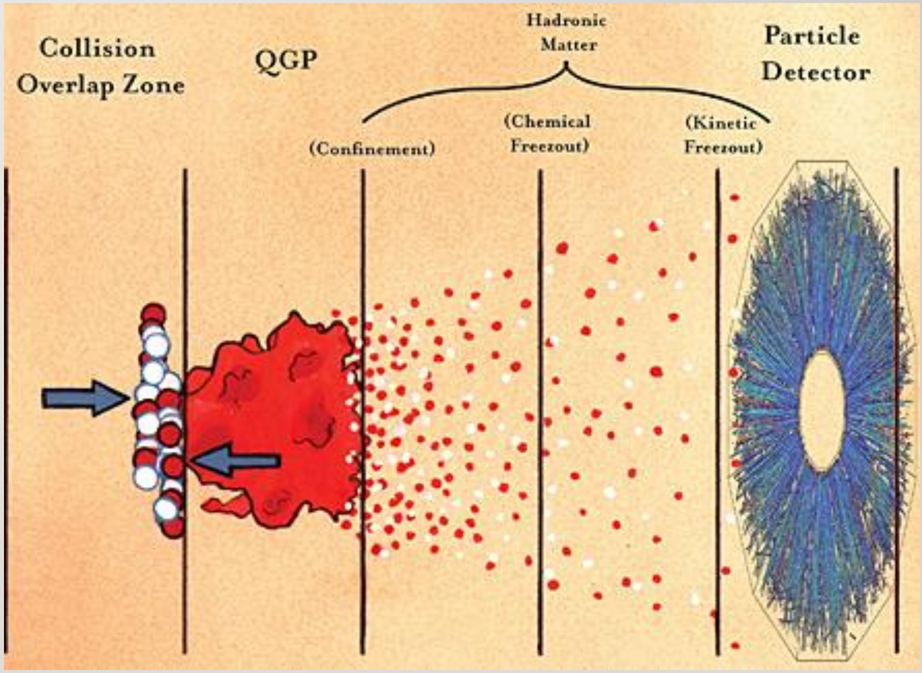
2/27 PHASES OF QUARK MATTER

- An evolution throughout many phases
- Modeling possible with hydrodynamics (?)





EXACT HYDRO HISTORY & BASICS

- Relativistic hydrodynamics: established by Landau (for p+p!)
- Exact, analytic solutions important: connect initial and final state
- Famous solutions by Landau&Khalatnikov and Hwa&Bjorken

  L. D. Landau, Izv. Akad. Nauk Ser. Fiz. 17, 51 (1953)
  I.M. Khalatnikov, Zhur. Eksp. Teor. Fiz. 27, 529 (1954)
  R. C. Hwa, Phys. Rev. D 10, 2260 (1974)
  J. D. Bjorken, Phys. Rev. D 27, 140 (1983)

- Discovery of sQGP: many new solutions

  See e.g. this review: de Souza, Koide, Kodama, Prog. Part. Nucl. Phys. 86, 35 (2016)

- Analytic solutions capture many features of data

  MCs, Vargyas, Eur. Phys. J. A 44, 473 (2010)
  MCs, Szabo, Phys. Rev. C 90, 054911 (2014)

- Still lacking: non-spherical 3D, accelerating, realistic solutions
- Linearized hydro: perturbations

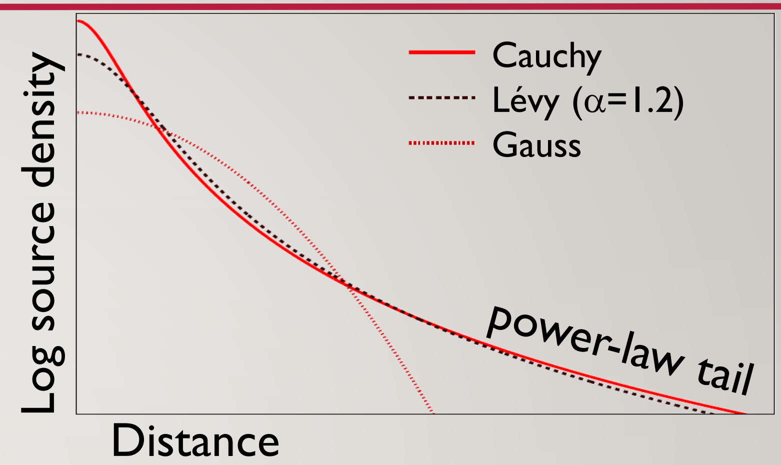
  Kurgyis, MCs, Universe 3 (2017) no.4, 84
  Shi, Liao and Zhuang, Phys.Rev. C90 (2014) no.6, 064912

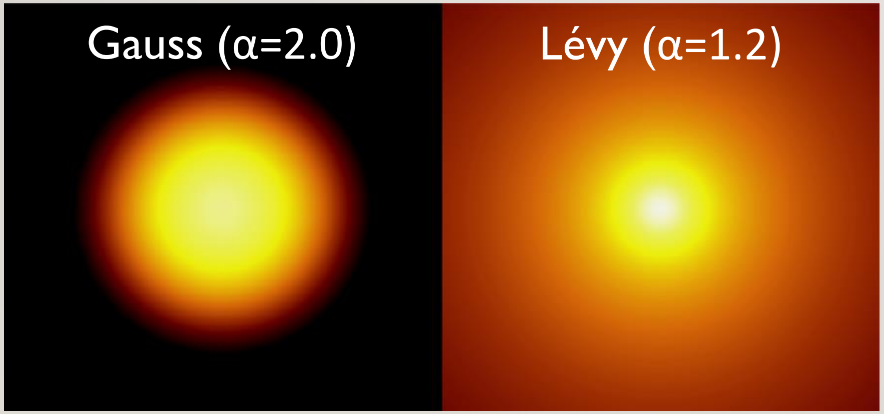
4/27 A NEW CHALLENGE

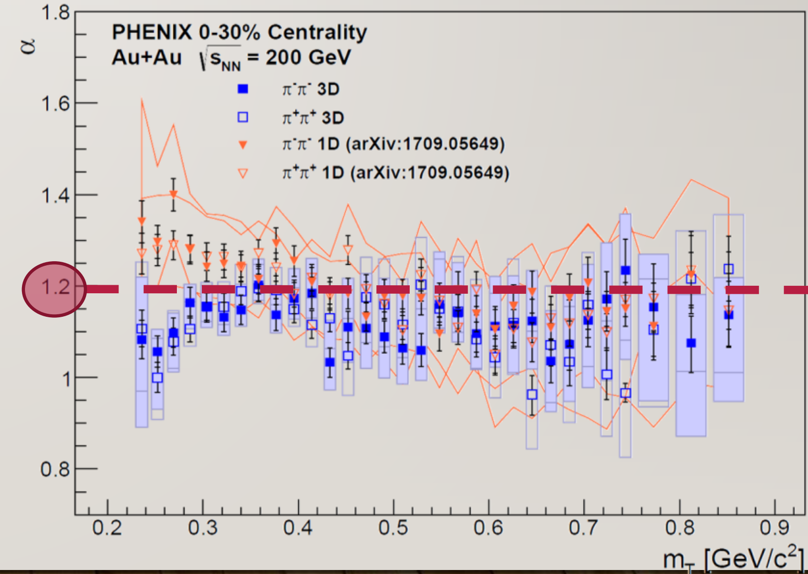
- PHENIX observing Lévy sources:

$$\mathcal{L}(\alpha, R; r) = \frac{1}{(2\pi)^3} \int d^3q e^{iqr} e^{-\frac{1}{2}|qR|^\alpha}$$

- Shape parameter: Gauss if $\alpha = 2$, power-law tail if $\alpha < 2$
- How to reconcile with hydro?
  - Exponential cutoff $\exp(-p^\mu u_\mu / T)$ in Boltzmann-Jüttner?
  - Rescattering?







THE PERTURBATIVE METHOD

- Method: perturbed equations for a known solution
- Linearized hydro equations:

$$(\kappa+1)\delta p u^\mu \partial_\mu u^\nu + (\kappa+1) p \delta u^\mu \partial_\mu u^\nu + (\kappa+1) p u^\mu \partial_\mu \delta u^\nu = (g^{\mu\nu} - u^\mu u^\nu)\partial_\mu \delta p - \delta u^\mu u^\nu \partial_\mu p - u^\mu \delta u^\nu \partial_\mu p$$

$$\kappa \delta u^\mu \partial_\mu p + \kappa u^\mu \partial_\mu \delta p + (\kappa+1)\delta p \partial_\mu u^\mu + (\kappa+1) p \partial_\mu \delta u^\mu = 0$$

$$u^\mu \partial_\mu \delta n + \delta n \partial_\mu u^\mu + \delta u^\mu \partial_\mu n + n \partial_\mu \delta u^\mu = 0$$

- Need a specific „base" solution
- Example: standing fluid

$$\kappa\partial_0\delta p + (\kappa+1)p\partial_\mu\delta u^\mu = 0$$

$$(\kappa+1)p\partial_0\delta u^\mu - Q^{\mu\nu}\partial_\mu\delta p = 0$$

with $Q^{\mu\nu} = \delta^{\mu 1}\delta^{\nu 1} - g^{\mu\nu}$

- Result: waves

$$\partial_0^2\delta p = c_s^2\Delta p$$

Method similar to Shi, Liao and Zhuang

Phys.Rev. C90 (2014) no.6, 064912 [arXiv:1405.4546]

6/27 A NEW CLASS OF PERTURBATIVE SOLUTIONS

- Hubble-flow: $u^\mu = \frac{x^\mu}{\tau}, p = p_0 \left(\frac{\tau_0}{\tau}\right)^{3+\frac{3}{\kappa}}, n = n_0 \left(\frac{\tau_0}{\tau}\right)^3 \mathcal{N}(s), u^\mu \partial_\mu s = 0$
- Describes observables, including HBT and higher order flow

  MCs, Szabo, Phys. Rev. C 90, 054911 (2014),

  MCs, Vargyas, Eur. Phys. J. A 44, 473 (2010)

- Perturbative solution on top of Hubble-flow possible:

$$u^\mu = \frac{x^\mu}{\tau} \quad \rightarrow \quad \delta u^\mu = \delta \cdot F(\tau) g(x_\nu) \chi(S) \partial^\mu S$$

$$p = p_0 \left(\frac{\tau_0}{\tau}\right)^{3+\frac{3}{\kappa}} \quad \rightarrow \quad \delta p = \delta \cdot p_0 \pi(S) \left(\frac{\tau_0}{\tau}\right)^{3+\frac{3}{\kappa}}$$

$$n = n_0 \left(\frac{\tau_0}{\tau}\right)^3 \mathcal{N}(S) \quad \rightarrow \quad \delta n = \delta \cdot n_0 \left(\frac{\tau_0}{\tau}\right)^3 h(x_\nu) \nu(S)$$

Kurgyis, MCs, Universe 3 (2017) no.4, 84, arXiv:1711.05446

RESCRITIONS FOR PERTURBATIVE SOLUTIONS

- Flow profile $\chi(S)$, pressure profile $\pi(S)$, density profile $\nu(S)$
- Auxiliary functions $F(\tau), g(x^\nu), h(x^\nu)$
- These are related to each other as:

$$\frac{\chi'(S)}{\chi(S)} = -\frac{\partial_\mu \partial^\mu S}{\partial_\mu S \partial^\mu S} - \frac{\partial_\mu S \partial^\mu \ln g(x_\nu)}{\partial_\mu S \partial^\mu S}$$

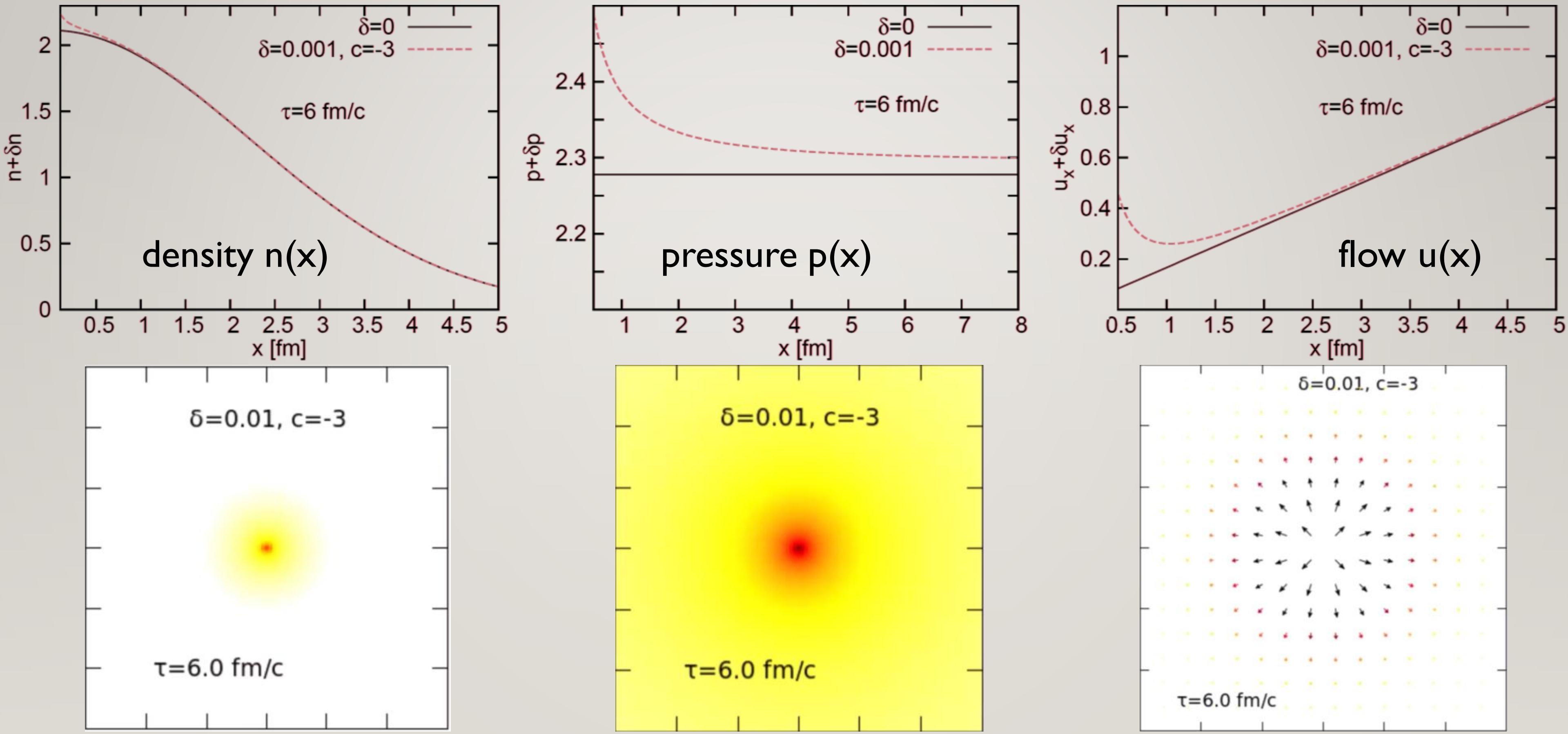
$$\frac{\pi'(S)}{\chi(S)} = (\kappa + 1)\left[F(\tau)\left(u^\mu \partial_\mu g(x_\nu) - \frac{3g(x_\nu)}{\kappa\tau}\right) + F'(\tau) g(x_\nu)\right]$$

$$\frac{\nu(S)}{\chi(S)\mathcal{N}'(S)} = -\frac{F(\tau) g(x_\nu) \partial_\mu S \partial^\mu S}{u^\mu \partial_\mu h(x_\nu)}$$

- Left side: only depends on scale variable S!
- Many solutions possible, various scaling variables and profiles

TIME EVOLUTION OF PERTURBATIONS

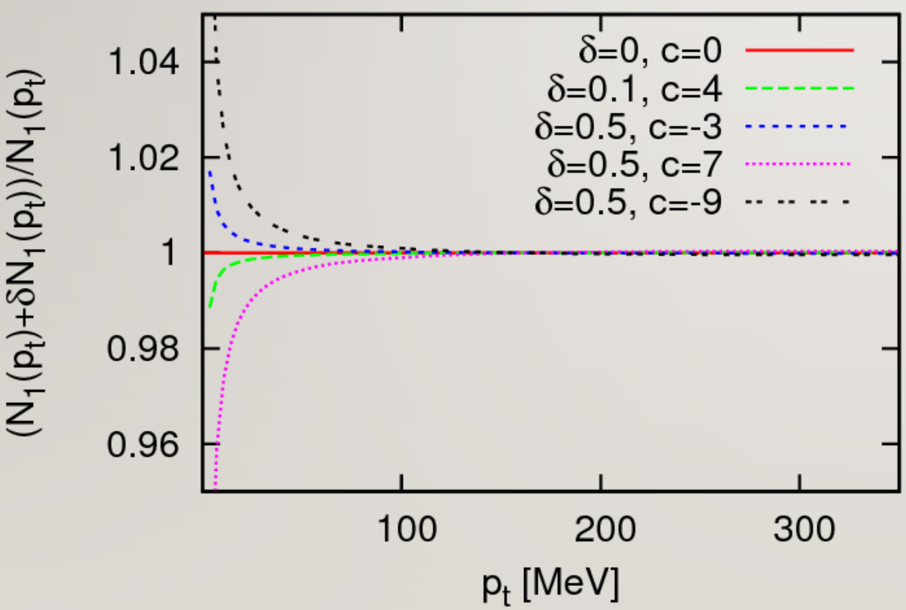
- An example perturbation from the class of solutions:

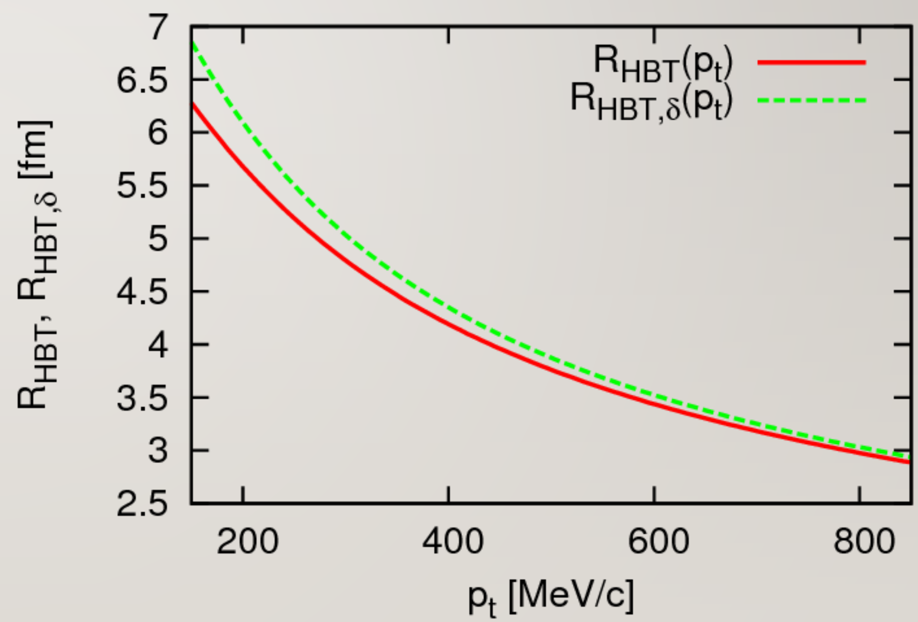


Kurgyis, MCs, Universe 3 (2017) no.4, 84, arXiv:1711.05446

9/27 PERTURBATIONS OF THE OBSERVABLES

- Observables calculable via usual Jüttner-Boltzmann source w/ Cooper-Fry
- Spectra and correlations obtain a perturbative component
- Hubble-flow observables stable against small perturbations





Kurgyis, MCs, Universe 3 (2017) no.4, 84, arXiv:1711.05446

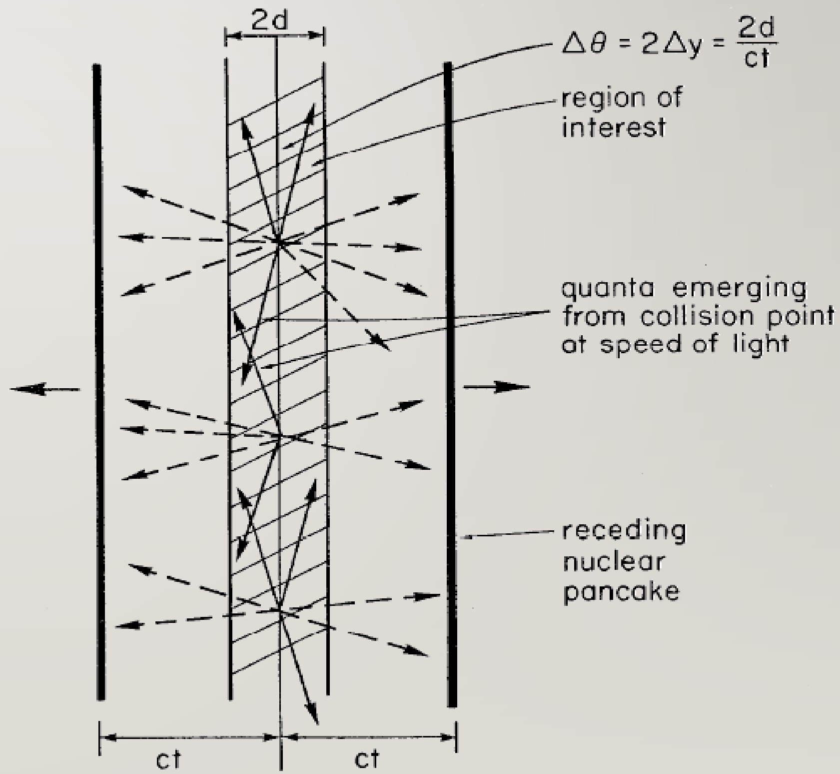
THE BJORKEN-ESTIMATE

- The original idea: energy density based on dE/dy
- QGP critical $\epsilon_c \sim 1$ GeV/fm³ (from $\epsilon_c = (6-8)\times T_c^4$)
- Result (~2000x cited)

$$E = N\frac{dE}{dy}\Delta y = N\frac{dE}{dy}\frac{1}{2}\frac{2d}{t} = \epsilon A d$$

$$\epsilon_{\mathrm{Bj}} = \frac{1}{R^2\pi\tau_0}\frac{dE}{d\eta} = \frac{\langle E\rangle}{R^2\pi\tau_0}\frac{dN}{d\eta}$$

- Boost invariant flow
  Phys.Rev. D27 (1983)
- Needs correction!



AN ANALYTIC SOLUTION WITH ACCELERATION

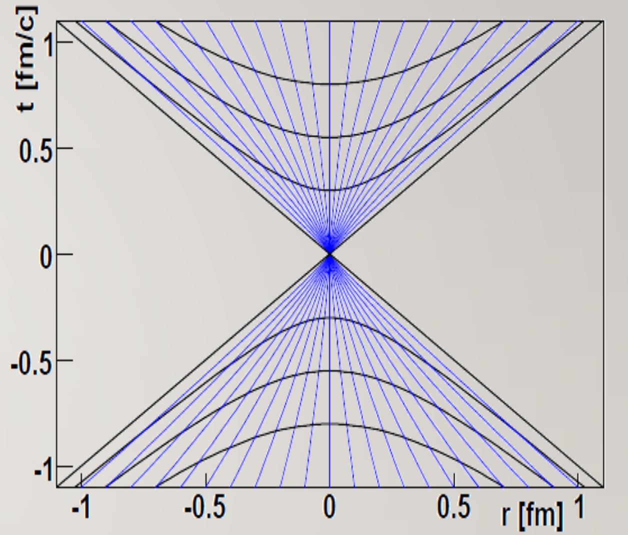
- The CNC solution in $1+d$ dimensions

Csörgő, Nagy, Csanád, Phys.Lett. B663 (2008) 306-311

Nagy, Csörgő, Csanád, Phys.Rev. C77 (2008) 024908

$$v = \tanh \lambda\eta,\ p = p_0\left(\frac{\tau_0}{\tau}\right)^{\lambda d\frac{\kappa+1}{\kappa}}\left(\cosh\frac{\eta}{2}\right)^{-(d-1)\phi_\lambda}$$

$$\sigma = \sigma_0 \nu(s)\left(\frac{p}{p_0}\right)^{\frac{\kappa}{\kappa+1}}, T = \frac{T_0}{\nu(s)}\left(\frac{p}{p_0}\right)^{\frac{\kappa}{\kappa+1}}$$

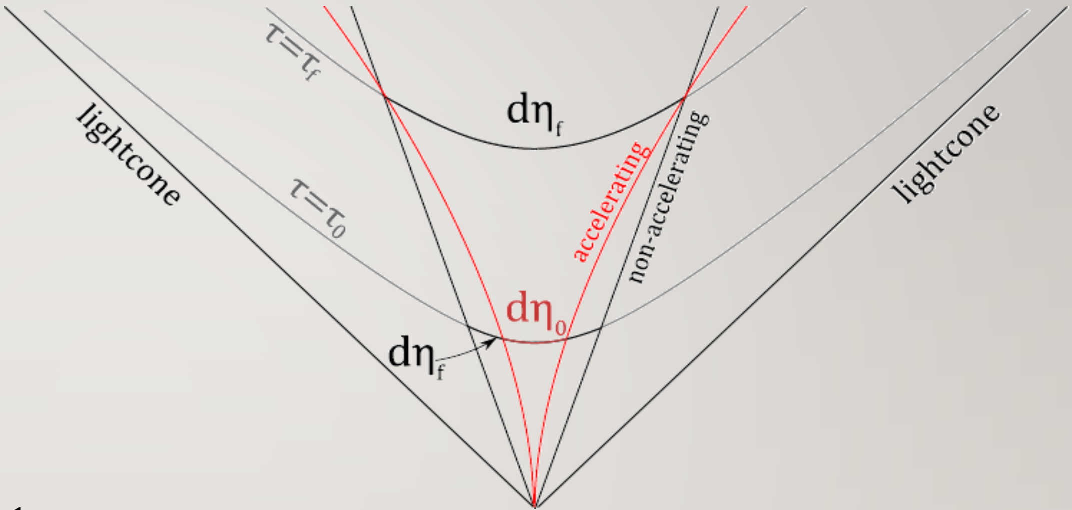


- Classes of solutions:

	λ	d	κ	ϕ_λ
Hwa-Bjorken	1	$\in \mathbb{R}$	$\in \mathbb{R}$	0
Fixed acceleration, any dim.	2	$\in \mathbb{R}$	d	0
→ **$d=1, \kappa=1$, any acceleration**	$\in \mathbb{R}$	**1**	**1**	**0**
Fixed deceleration	$1/2$	$\in \mathbb{R}$	1	$(\kappa+1)/\kappa$
Fixed acceleration	$3/2$	$\in \mathbb{R}$	$(4d-1)/3$	$(\kappa+1)/\kappa$

AN ADVANCED ENERGY DENSITY ESTIMATE

- Fact: dN/dy not flat
- Finiteness & acceleration
  - Acceleration parameter λ
- Corrections needed:
  - $y \neq \eta$ & $\eta_{\text{final}} \neq \eta_{\text{initial}}$
  - Work done by pressure



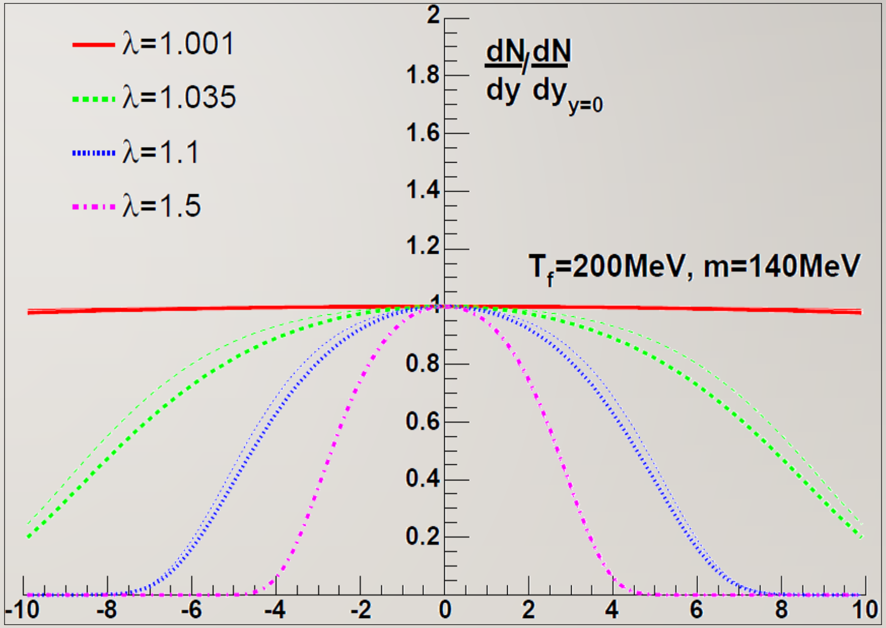
- Corrected estimate for $\kappa = 1$

$$\epsilon = \epsilon_{\text{Bj}}(2\lambda - 1)\left(\frac{\tau_f}{\tau_i}\right)^{\lambda-1}, \qquad \tau = \lambda\tau_{\text{Bj}} = \lambda\sqrt{\frac{m_T}{T_f}}R_{\text{long}}$$

- Björken estimate: only for $\kappa = \infty$ (dust EoS)
- Will come back to this soon

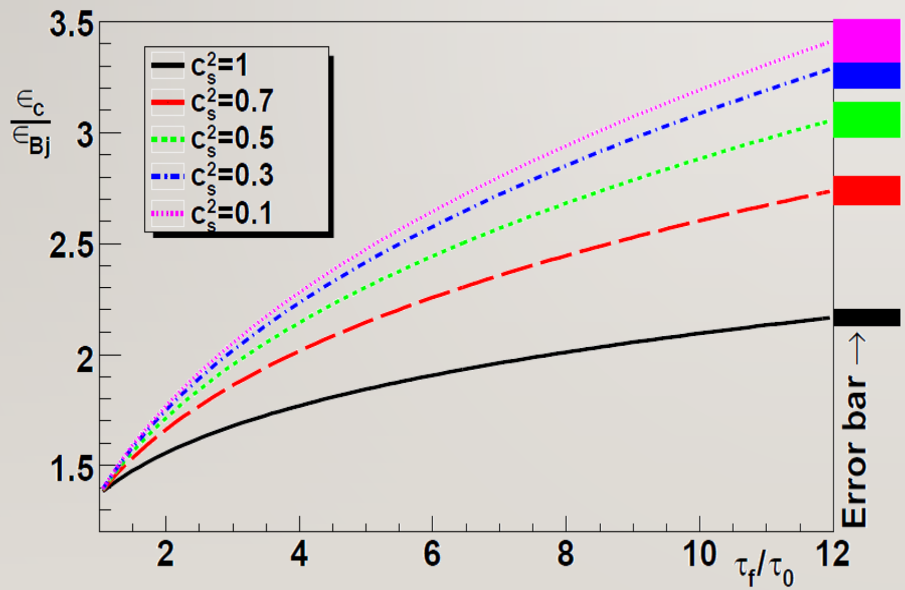
THE PSEUDORAPIDITY DENSITY FROM CNC

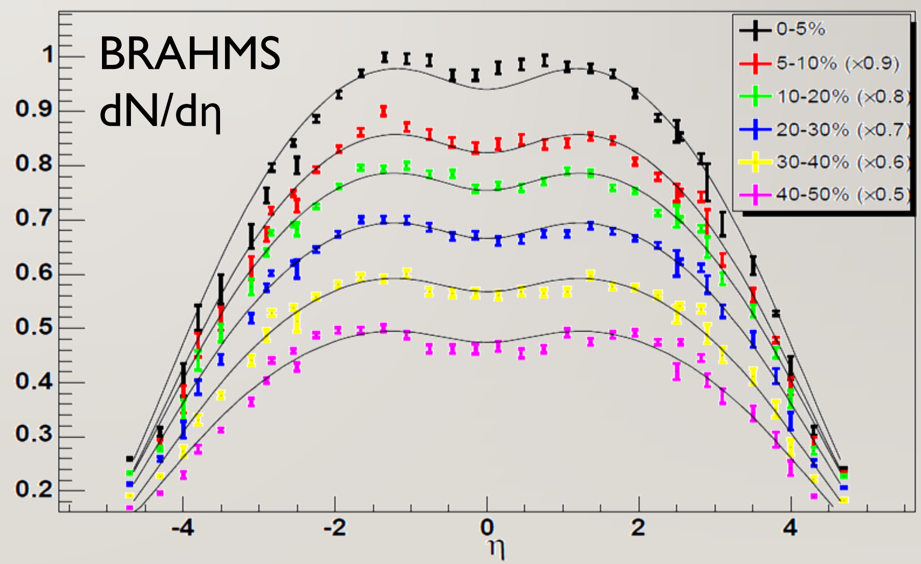
- $\frac{dN}{dy} \cong N_0 \cosh^{-\frac{\alpha}{2}-1}\left(\frac{y}{\alpha}\right) \exp\left[-\frac{m}{T_f}\cosh^{\alpha}\frac{y}{\alpha}\right]$
- Main parameter: $\alpha = \frac{2\lambda-1}{\lambda-1}$
- Particle mass m,
- Freeze-out temp. T_f
- Measure acceleration from rapidity distributions
- Extension to more complex flows?



INITIAL ENERGY DENSITY AT RHIC

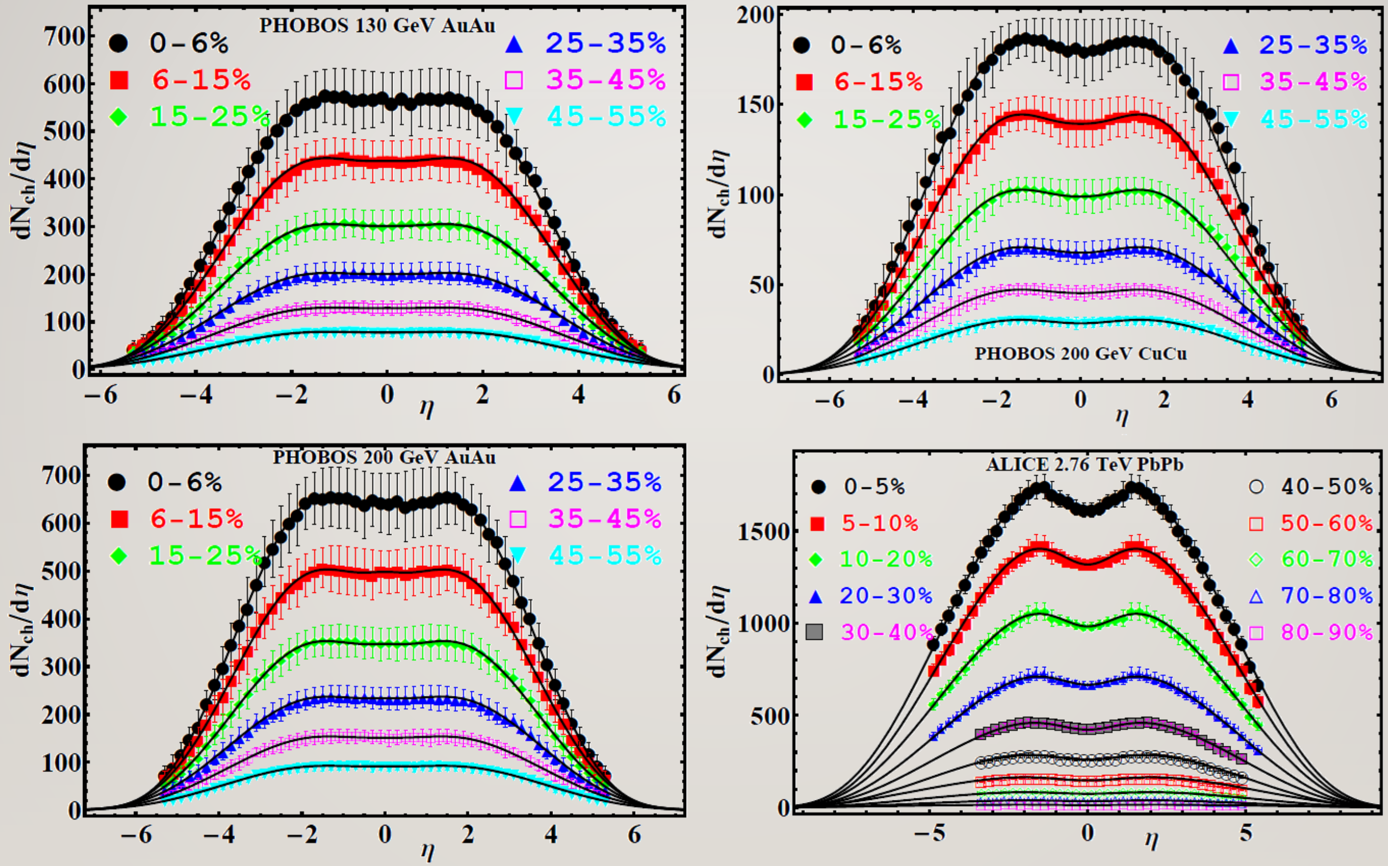
- Bjorken estimate from BRAHMS: $\epsilon_{\rm Bj} = \frac{\langle E \rangle}{R^2 \pi \tau_0} \frac{dN}{d\eta} \cong 5$ GeV/fm3
- Advanced estimate: $\epsilon = \epsilon_{\rm Bj}(2\lambda - 1)\left(\tau_f/\tau_i\right)^{(\lambda-1)}$
- Correction: 2-3x, result ~15 GeV/fm3, QCD agreement!
- Corresponds to Tini ≅ 2Tc ≅ 340 MeV, confirmed by g spectra





PSEUDORAPITY DENSITIES IN A+A

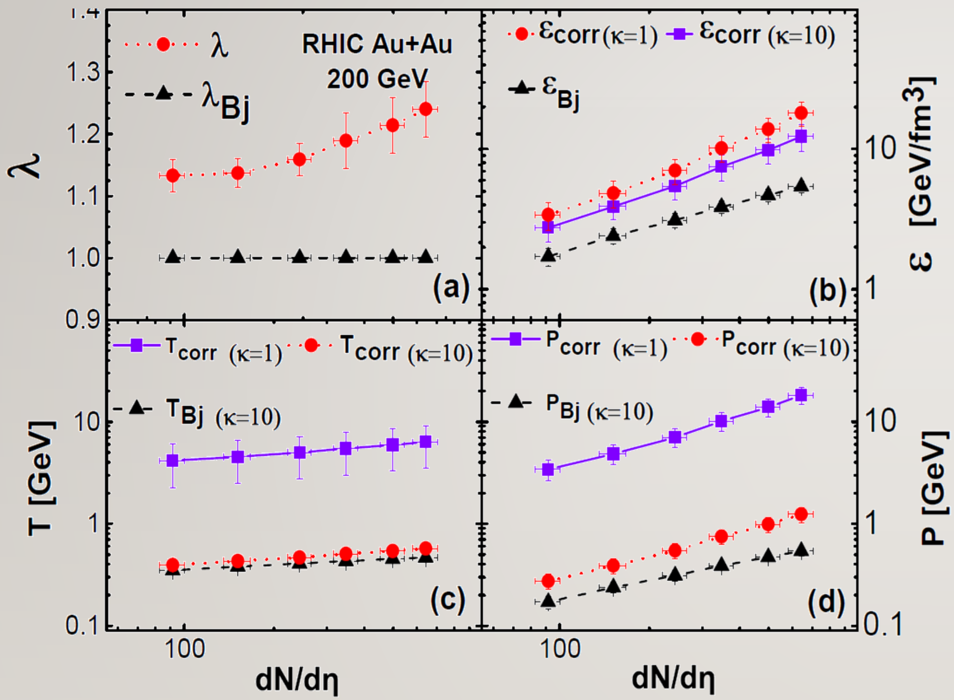
- Described well from RHIC to LHC

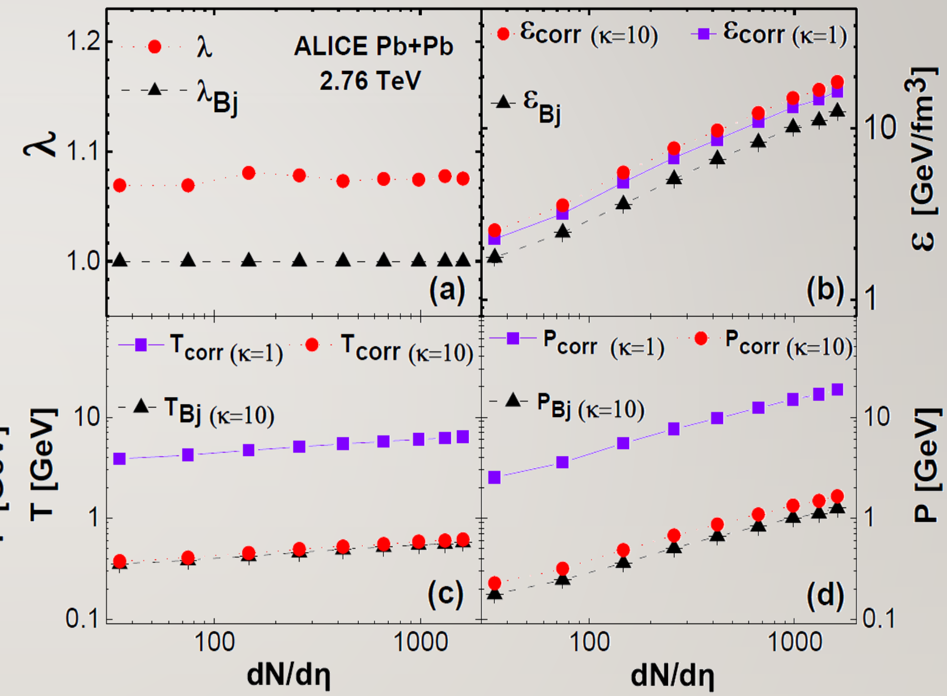


Jiang, Yang, MCs, Csörgő, Phys. Rev. C 97, 064906, 2018

16/27 ENERGY DENSITIES IN AA, RHIC TO LHC

- Effect of acceleration and conjectured effect of Equation of State





Jiang, Yang, MCs, Csörgő, Phys. Rev. C 97, 064906, 2018

- Effect of EoS: important to understand analytically!
- What about p+p?

BJORKEN ENERGY DENSITY ESTIMATE IN PP

- Rough estimate via the Bjorken formula: $\epsilon_{\mathrm{Bj}} = \frac{\langle E\rangle}{R^2\pi\tau_0}\frac{dN}{d\eta}$
  - Number of particles at midrapidity: 1.5×5.89
  - Average energy: $\langle m_t\rangle = \langle E\rangle = 0.562\ \mathrm{GeV}$
  - Transverse size of the system $\mathrm{R}^2\pi = \sigma_{\mathrm{tot}}^2/4\sigma_{\mathrm{el}} = 9.8\ \mathrm{fm}^2$
  - Formation time $\tau_0 = 1\ \mathrm{fm}/c$ (conservative estimate)
- Energy density from this:

$$\epsilon_{\mathrm{Bj}}(7\ \mathrm{TeV}) = \frac{1}{R^2\pi\tau_0}\frac{dE}{d\eta} = \frac{\langle E\rangle}{R^2\pi\tau_0}\frac{dn}{d\eta} = \frac{0.562 \times 1.5 \times 5.89}{1.76^2\pi}\frac{\mathrm{GeV}}{\mathrm{fm}^3} = 0.507\frac{\mathrm{GeV}}{\mathrm{fm}^3}$$

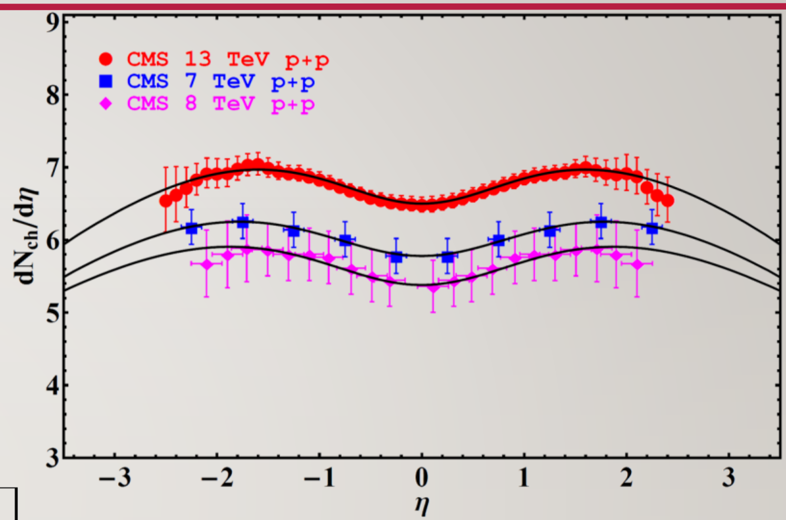
$$\epsilon_{\mathrm{Bj}}(8\ \mathrm{TeV}) = \frac{1}{R^2\pi\tau_0}\frac{dE}{d\eta} = \frac{\langle E\rangle}{R^2\pi\tau_0}\frac{dn}{d\eta} = \frac{0.571 \times 1.5 \times 6.17}{1.80^2\pi}\frac{\mathrm{GeV}}{\mathrm{fm}^3} = 0.519\frac{\mathrm{GeV}}{\mathrm{fm}^3}$$

MCs, Csörgő, Jiang, Yang, Universe 3 (2017) no.1, 9, arXiv:1609.07176

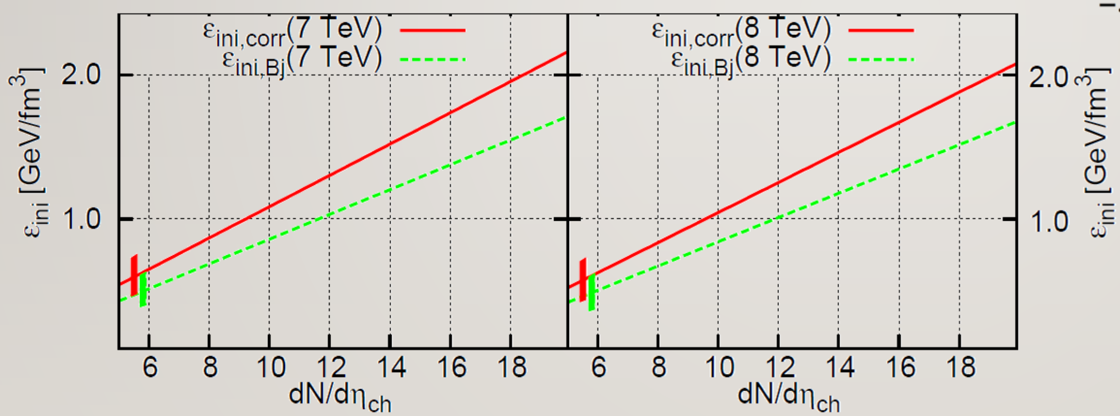
- This is at average multiplicity; compare to $\epsilon_{\mathrm{crit}} \approx 1\frac{\mathrm{GeV}}{\mathrm{fm}^3}$

18/27 ENERGY DENSITY IN P+P

- Data from p+p well described
  - 7 TeV: $\lambda = 1.073, \ \epsilon_{corr} = 0.645 \frac{GeV}{fm^3}$
  - 8 TeV: $\lambda = 1.067, \ \epsilon_{corr} = 0.641 \frac{GeV}{fm^3}$
  - 13 TeV: $\lambda = 1.065, \epsilon_{corr} = 0.692 \frac{GeV}{fm^3}$



- Multiplicity dependence



- Energy density above 1 GeV/fm^3 for multiplicites above ~10! EoS dependence?

MCs, Csörgő, Jiang, Yang, Universe 3 (2017) no.1, 9, arXiv:1609.07176 + manuscript in preparation

A NEW CLASS OF EXACT SOLUTIONS

- How to reconcile the Bjorken estimate ($\kappa = \infty$) with hydro?
- Search for new solutions with flow $u^\mu = (\cosh\Omega(\eta), \sinh\Omega(\eta))$
- Energy and Euler equations become:

$$\partial_\eta \Omega + \kappa\left[\tau\partial_\tau + \tanh(\Omega - \eta)\,\partial_\eta\right]\ln T = 0$$
$$\partial_\eta \ln T + \tanh(\Omega - \eta)\left(\tau\partial_\tau \ln T + \partial_\eta \Omega\right) = 0$$

- A new class of solutions emerges, if one relaxes self-similarity
- These will be implicit, introducing

$$\eta(H) = \Omega(H) - H$$

$$\Omega(H) = \frac{\lambda}{\sqrt{\lambda - 1}\sqrt{\kappa - \lambda}}\,\mathrm{atan}\left(\frac{\sqrt{\kappa - \lambda}}{\sqrt{\lambda - 1}}\tanh H\right)$$

Csörgő, Kasza, MCs, Jiang, Universe 2018, 4(6), 69 arXiv:1805.01427

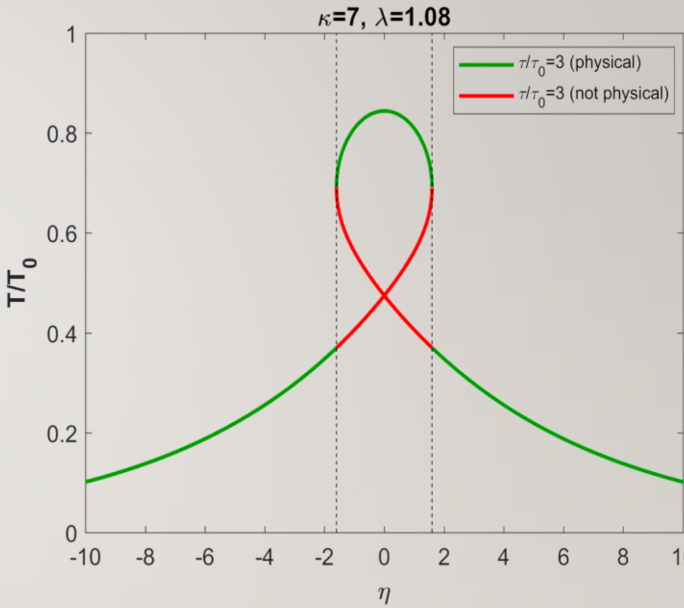
THE NEW CLASS OF SOLUTIONS

$$u^\mu = (\cosh\Omega(H), \sinh\Omega(H))$$

$$\sigma(\tau,H) = \sigma_0 \left(\frac{\tau_0}{\tau}\right)^\lambda \nu(s) \left[1+\frac{\kappa-1}{\lambda-1}\sinh^2 H\right]^{-\lambda/2}$$

$$T(\tau,H) = T_0 \left(\frac{\tau_0}{\tau}\right)^{\frac{\lambda}{\kappa}} \frac{1}{\nu(s)} \left[1+\frac{\kappa-1}{\lambda-1}\sinh^2 H\right]^{-\lambda/2\kappa}$$

$$s(\tau,H) = \left(\frac{\tau_0}{\tau}\right)^{\lambda-1} \sinh H \left[1+\frac{\kappa-1}{\lambda-1}\sinh^2 H\right]^{-\lambda/2}$$

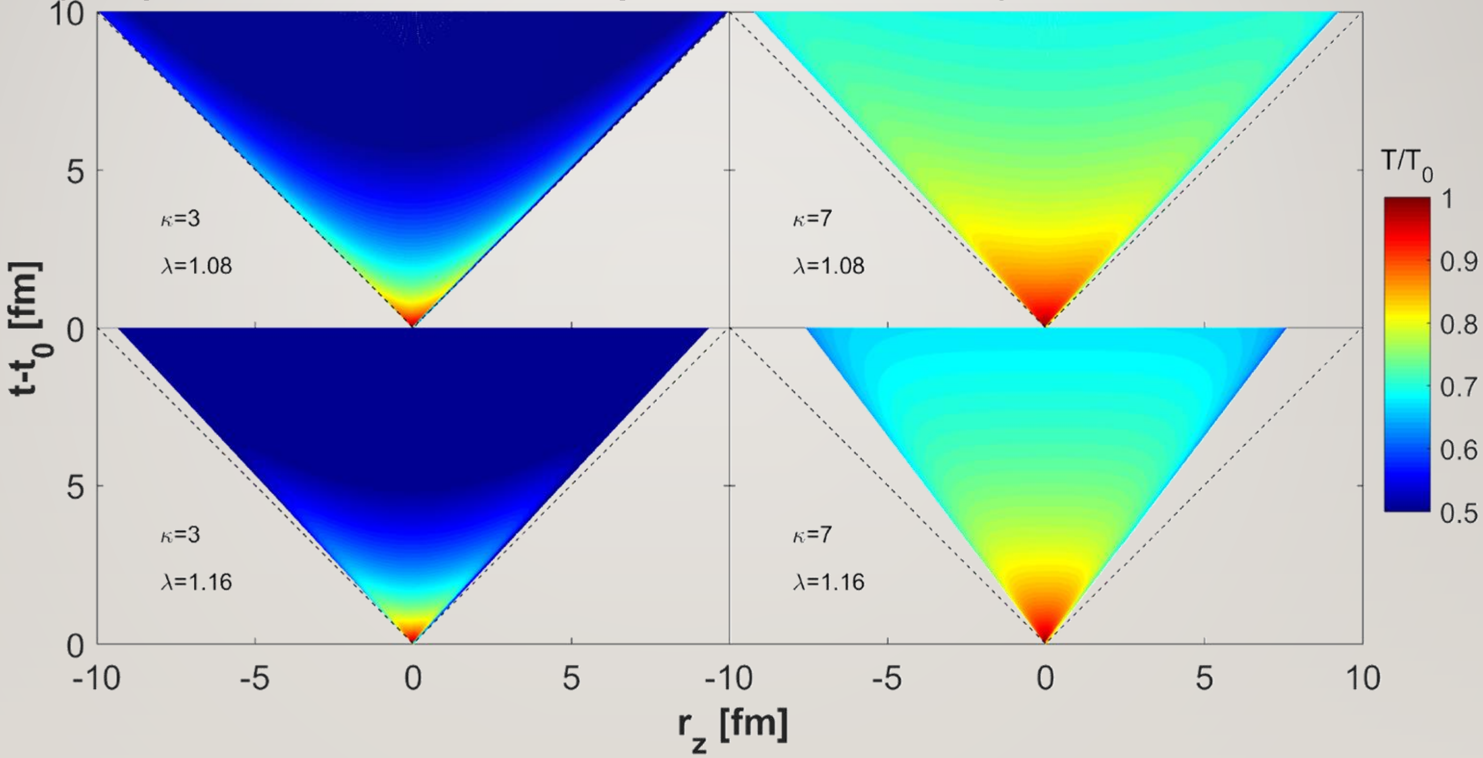


- Quantities given parametrically as $(\eta(H), \Omega(\mathrm{H}))$
- Simplification: limit the solution in η where $\eta \to \Omega$ univalent (functional)
- Not self-similar: Coordinate dependence not only via scaling variable s
- Explicit and exact solution

Csörgő, Kasza, MCs, Jiang, Universe 2018, 4(6), 69 arXiv:1805.01427

TEMPERATURE EVOLUTION

- Recall: limited domain in η, where $(\eta(H), \Omega(\mathrm{H}))$ relation functional
- Strong dependence on EoS, analytic understanding



Csörgő, Kasza, MCs, Jiang, Universe 2018, 4(6), 69 arXiv:1805.01427

OBSERVABLES

- Rapidity density calculable in saddle-point approximation

$$\frac{dN}{dy} \cong N_0 \cosh^{-\frac{\alpha(\kappa)}{2}-1}\left(\frac{y}{\alpha(1)}\right) \exp\left[-\frac{m}{T_f}\left(\cosh^{\alpha(\kappa)}\frac{y}{\alpha(1)} - 1\right)\right]$$

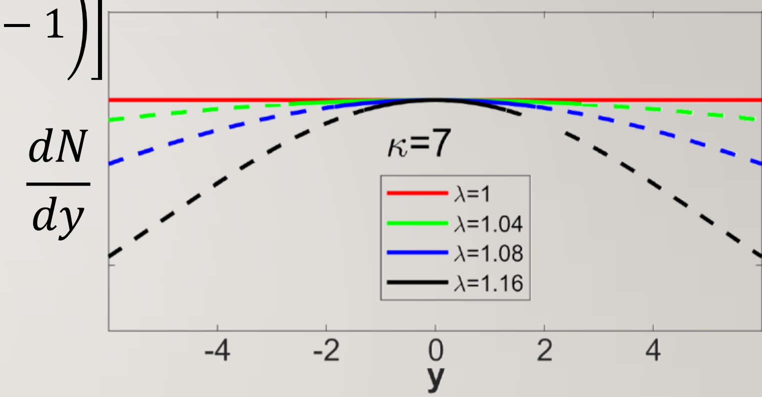
where $\alpha(\kappa) = \frac{2\lambda-\kappa}{\lambda-\kappa}$ was introduced

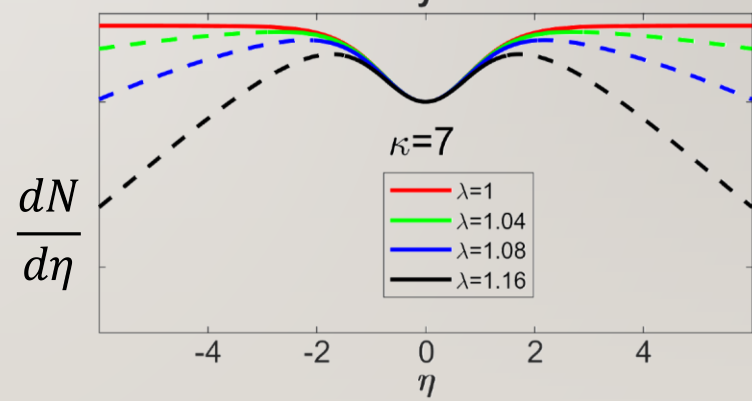
- Normalization:

$$N_0 = \frac{R^2\pi\tau_f}{(2\pi\hbar)^3}\sqrt{\frac{(2\pi T_f m)^3}{\lambda(2\lambda-1)}}\exp\left(-\frac{m}{T_f}\right)$$

- Pseudorapidity density as parametric $\eta(y) \to \frac{dN}{d\eta}(y)$ curve
- Using Jacobian: $\frac{dy}{d\eta} = \frac{\langle p_t(y)\rangle \cosh\eta(y)}{\sqrt{m^2+\langle p_t(y)\rangle^2\cosh^2\eta(y)}}$

and $\langle p_t(y)\rangle = \frac{\sqrt{T_f^2+2mT_f}}{1+\frac{\alpha(\kappa)}{2\alpha(1)}\frac{T_f+m}{T_f+2m}y^2}$

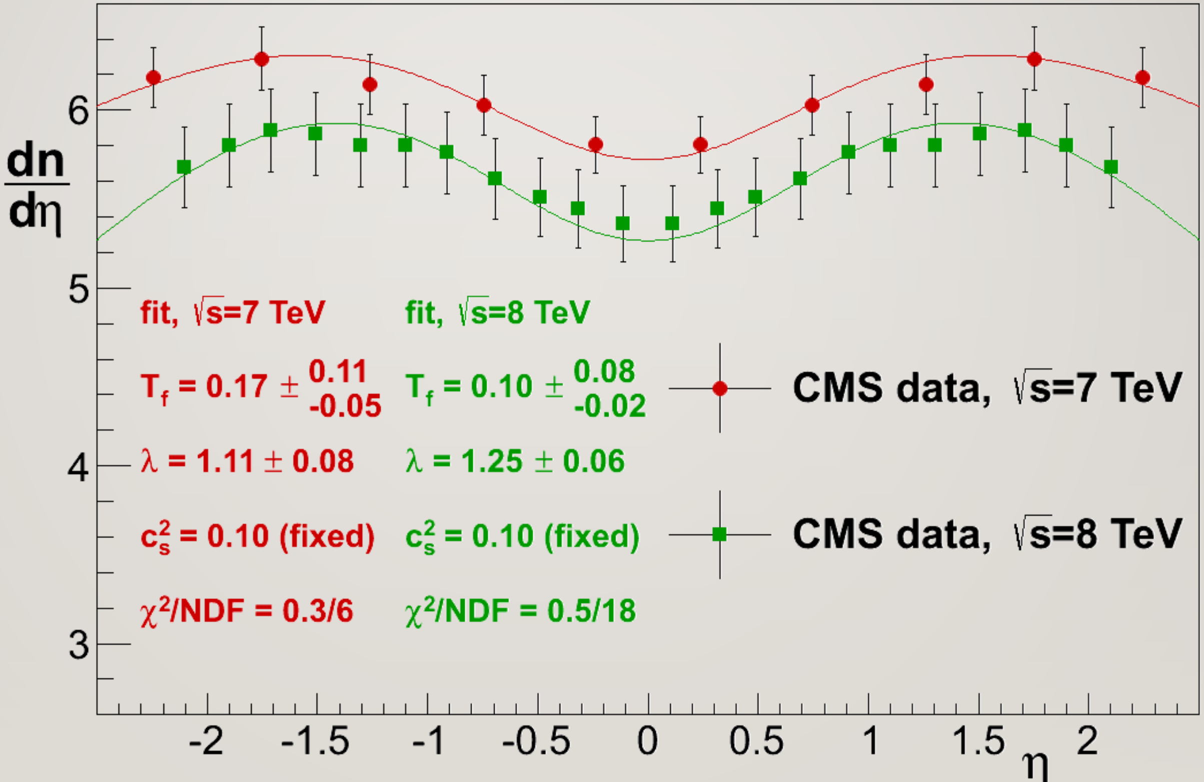




Csörgő, Kasza, MCs, Jiang, Universe 2018, 4(6), 69 arXiv:1805.01427

COMPARISON TO DATA

- Description valid in limited η interval only
- Result very close to CNC solution



Csörgő, Kasza, MCs, Jiang, Universe 2018, 4(6), 69 arXiv:1805.01427

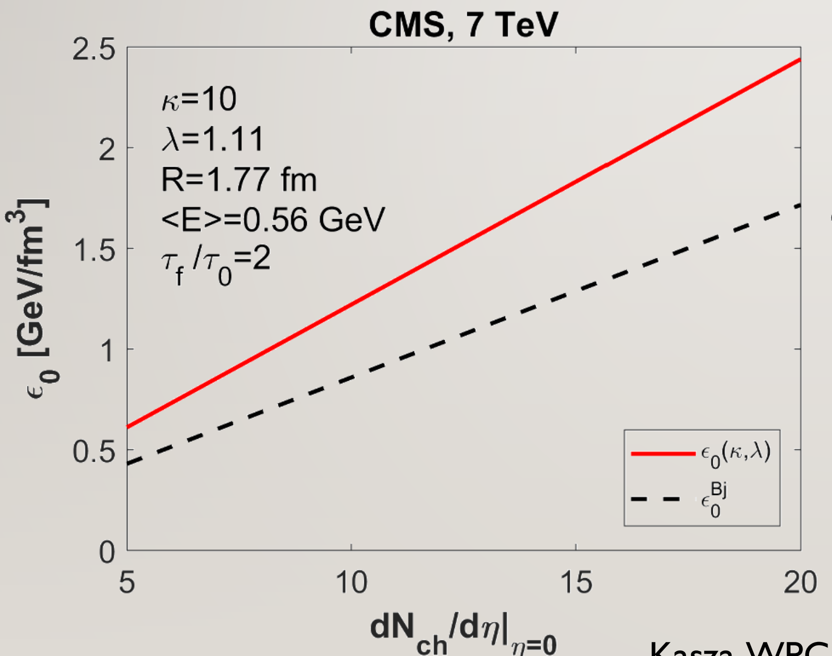
WHAT ABOUT THE ENERGY DENSITY?

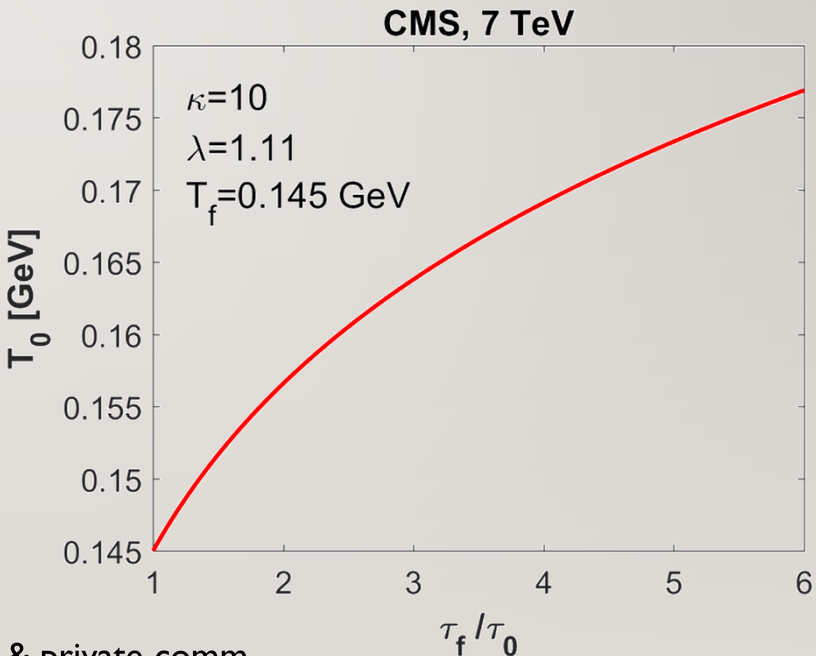
- Bjorken estimate: $\epsilon_{\mathrm{Bj}} = \frac{\langle E \rangle}{R^2 \pi \tau_0} \frac{dN}{d\eta}$
  - Valid only for dust EoS, $\kappa = \infty$
- CNC solution, finiteness and acceleration (only these effects), valid for $\kappa = 1$
  - Correction factor: $(2\lambda - 1)\left(\frac{\tau_f}{\tau_0}\right)^{\lambda - 1}$
- Work done by pressure (without acceleration, just the expansion)
  - Correction factor: $\left(\frac{\tau_f}{\tau_0}\right)^{\lambda}$
- CKCJ solution, exact EoS dependent result
  - Correction factor: $(2\lambda - 1)\left(\frac{\tau_f}{\tau_0}\right)^{\left(1+\frac{1}{\kappa}\right)\lambda - 1}$
- Energy density: $\epsilon = \frac{dN}{d\eta} \frac{\langle E \rangle}{R^2 \pi \tau_0} (2\lambda - 1)\left(\frac{\tau_f}{\tau_0}\right)^{\left(1+\frac{1}{\kappa}\right)\lambda - 1}$

Csörgő, Kasza, WPCF2018&private comm.

DETERMINING THE INITIAL STATE

- Dependence on EoS: from direct photons and/or lattice QCD
- Dependence on multiplicity: plug in measured value
- Dependence on final/initial time: largest source of uncertainty
- What about the effect of viscosity?

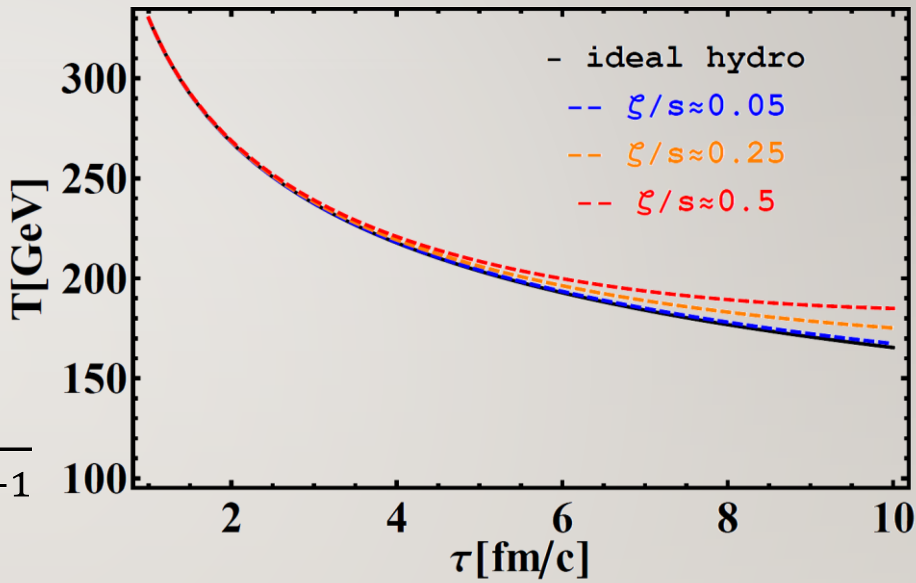




Kasza, WPCF2018 & private comm.

A NEW VISCOUS SOLUTION

- A new analytic solution with bulk viscosity
- $u^\mu = \frac{x^\mu}{\tau}$
- $n = n_0 \left(\frac{\tau_0}{\tau}\right)^3 \nu(s)$
- $p = p_0 \left(\frac{\tau_0}{\tau}\right)^{3\frac{\kappa+1}{\kappa}} + \frac{\zeta}{\kappa}\frac{3}{\tau}\frac{3}{3\frac{\kappa+1}{\kappa}-1}$
- $T = T_0 \left(\frac{\tau_0}{\tau}\right)^{\frac{3}{\kappa}} + \frac{\zeta}{\kappa n_0}\frac{3}{\tau}\left(\frac{\tau}{\tau}\right)^3 \mathcal{T}(s)\frac{3}{3\frac{\kappa+1}{\kappa}-1}$



- Viscous heating at late stages
- Note: shear viscosity cancels for Hubble-flow!
- New shear viscous analytic solutions in preparation

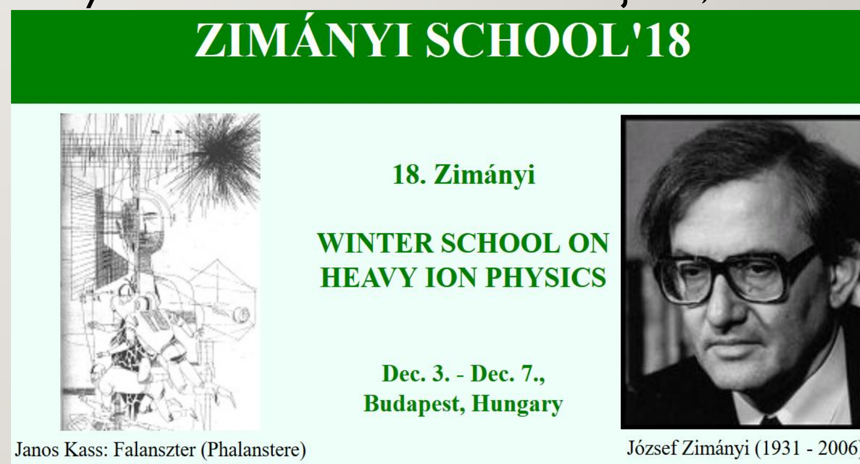
Jiang, Yang, Csörgő, Kasza, Nagy, MCs, in preparation

SUMMARY

- Many new results in exact/analytic hydro
- Perturbations on top of Hubble-flow
  - Allows to introduce complicated anisotropies
  - To be expanded to other solutions
- Advanced energy density estimates
  - Björken estimate: no acceleration, no pressure
  - Advanced estimates based on exact solutions
  - High energy densities reached in LHC p+p
- New accelerating families of solutions
  - Arbitrary acceleration, arbitrary EoS
  - EoS dependent energy density estimate

THANK YOU FOR YOUR ATTENTION!

Ceterum censeo
~~Carthaginem esse delendam~~
… if you are interested in these subjects, come to:



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BACKUP