Flux tubes at finite temperature

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EUROPEAN COOPERATION IN SCIENCE & TECHNOLOGY

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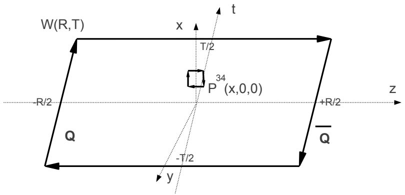
- We study the flux tubes produced by static quark-antiquark and quark-quark charges at finite temperature in pure gauge SU(3) lattice QCD.
- This is relevant both for the study of flux tubes and strings, and for the interaction of heavy quarks and other color sources in heavy ion collision physics.
- To signal the flux tubes, we compute the square densities of the chromomagnetic and chromoelectric fields with plaquettes, in a gauge invariant framework.
- We study the existence and non-existence of flux tubes both below and above the deconfinement phase transition temperature, T_c.
- Using the Lagrangian density as a profile distribution, we also compute the widths of the flux tubes and study their widening as a function of the inter-charge distance.

At zero temperature,

$$\rho_{\mu\nu} = \frac{\langle \mathrm{Tr}\, W \,\Box_{\mu\nu} \rangle}{\langle \mathrm{Tr}\, W \rangle} - \langle \Box_{\mu\nu} \rangle \rightarrow a^4 \left(\left\langle F^2_{\mu\nu} \right\rangle_{q\bar{q}} - \left\langle F^2_{\mu\nu} \right\rangle_{\mathrm{vac}} \right)$$

where W is the Wilson loop and $\Box_{\mu\nu}$ is the plaquette in the (μ, ν) plane,

$$\Box_{\mu\nu} = 1 - \frac{1}{N_c} \mathrm{Tr} \left[U_\mu(s) U_\nu(s+\mu) U^\dagger_\mu(s+\nu) U^\dagger_\nu(s) \right]$$

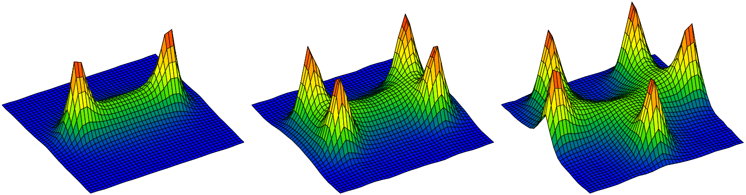


$$\left\langle E_i^2 \right\rangle = -\rho_{i,0} \qquad \text{and} \qquad \left\langle B_i^2 \right\rangle = \rho_{j,k}$$

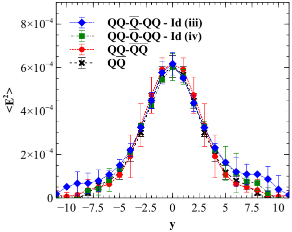
and the Lagrangian ($\mathcal{L}$) density is given by

$$\mathcal{L} = \frac{1}{2} \left(\left\langle E^2 \right\rangle - \left\langle B^2 \right\rangle \right)$$

Flux tubes at Zero Temperature



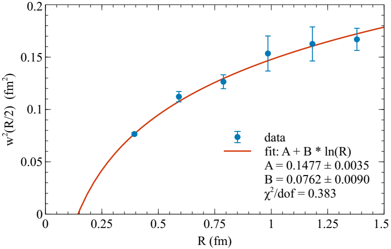
Flux tube profile:



Results in lattice spacing units, $a = 0.07261(85)$ *fm* or $a^{-1} = 2718(32)$ *MeV*
$24^3 \times 48$ lattice volume with $\beta = 6.2$

Widening in the mediator plane

Square of the width of the flux tube in the mediator plane.
Fit of the flux tube width to the leading order one-loop computation in effective string theory[a]



The B parameter can be compared with the theoretical leading order[a] value for the factor of the logarithmic term,

$$B = \frac{D-2}{2\pi\sigma} = 0.0640028\,\mathrm{fm}^2$$

obtained using a string tension of $\sqrt{\sigma} = 0.44\,\mathrm{GeV}$.
The width complies, almost within one standard deviation, with the logarithmic widening obtained at leading order in the Nambu-Gotto effective string theory.

[a] F. Gliozzi et al. JHEP 1011, 053 (2010), arXiv:1006.2252.

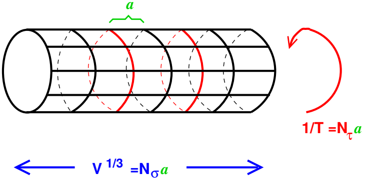
Color fields at finite temperature:

- Not feasible to use the same technique at zero temperature (the Wilson loop)
  - lattice time dimension is related with the temperature.
  - number of points in the time dimension much smaller than in zero temperature.
- At finite temperature the order parameter is the Polyakov loop:

$$L(x) = \frac{1}{N_c} \Pi_{t=1}^{N_t} U_4(x,t)$$

and

$$L = \frac{1}{V_s} \sum_x \mathrm{Tr}\, L(x)$$



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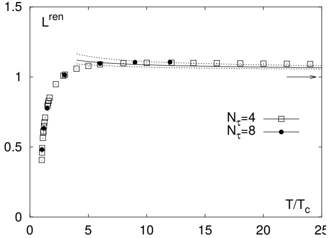
  - $T < T_c : \langle L \rangle = 0$ (center symmetry)
  - $T > T_c : \langle L \rangle \neq 0$ (spontaneous breaking)



High temperature limit, $L^{ren} = 1$, reached from above as expected from PT
Clearly non-perturbative effects below $5T_c$

Polyakov loop correlation function and free energy:

- $F_{Q\bar{Q}} = -T \ln \left(\left\langle \mathrm{Tr}\, L(x) \mathrm{Tr}\, L^\dagger(x+r) \right\rangle \right)$ (gauge invariant)
- $F_1 = -T \ln \left(\left\langle \mathrm{Tr}\, L(x) L^\dagger(x+r) \right\rangle \right)$ (GF)
- $F_8 = -T \ln \left(\frac{9}{8} \left\langle \mathrm{Tr}\, L(x) \mathrm{Tr}\, L^\dagger(x+r) \right\rangle - \frac{1}{8} \left\langle \mathrm{Tr}\, L(x) L^\dagger(x+r) \right\rangle \right)$ (GF)

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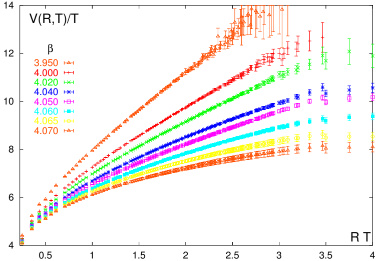
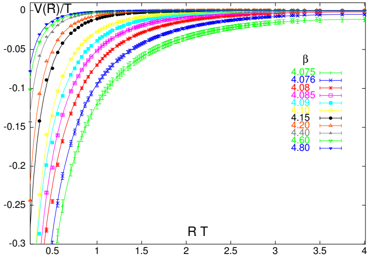
For $T < T_c$

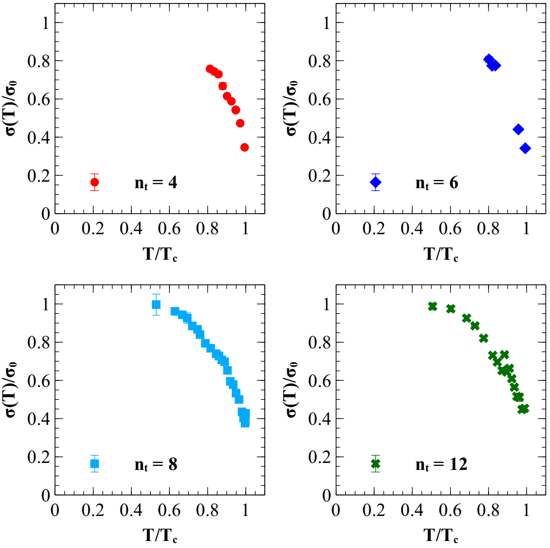


For $T > T_c$



Kaczmarek, Olaf et al. Phys.Rev. D62 (2000)

- String Tension as function of the temperature from $F_{Q\bar{Q}}$



N. Cardoso, P. Bicudo, Phys.Rev. D85 (2012)

Flux tubes at Finite Temperature

The central observables that govern the event in the flux tube can be extracted from the correlation of a plaquette with the Polyakov loops,

$$f_{\mu\nu}(r,x) = \frac{\beta}{a^4}\left[\frac{\langle \mathcal{O}(r)\,\Box_{\mu\nu}(x)\rangle}{\langle \mathcal{O}(r)\rangle} - \langle \Box_{\mu\nu}\rangle\right]$$

x denotes the distance of the plaquette from the line connecting the sources and r is the quark separation.

- $Q\bar{Q}$ case: $\mathcal{O}(r) = \mathrm{Tr}\, L(0)\, \mathrm{Tr}\, L^\dagger(r)$
- QQ case: $\mathcal{O}(r) = \mathrm{Tr}\, L(0)\, \mathrm{Tr}\, L(r)$

where

$$L(s) = \frac{1}{N_c} \Pi_{t=1}^{N_t} U_4(s,t)$$

is the Polyakov loop.

Similar to zero temperature, with $\mathcal{O}(r) = \mathrm{Tr}\, W$ where W is the Wilson loop.

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- QQ case: $\mathcal{O}(r) = \mathrm{Tr}\, L(0)\, \mathrm{Tr}\, L(r)$

or

$$f_{\mu\nu}(r,x) = \frac{\beta}{a^4}\left[\frac{\langle \mathcal{O}(r)\,\Box_{\mu\nu}(x)\rangle - \langle \mathcal{O}(r)\Box_{\mu\nu}(x_R)\rangle}{\langle \mathcal{O}(r)\rangle}\right]$$ [1]

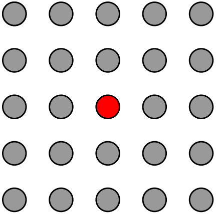
where x_R is the reference point placed far from the quark sources. with

$$\Box_{\mu\nu}(s) = \frac{1}{N_c}\mathrm{Tr}\left[U_\mu(s)U_\nu(s+\mu)U_\mu^\dagger(s+\nu)U_\nu^\dagger(s)\right]$$

the plaquette in the (μ,ν) plane.

[1] Y. c. Peng and R. W. Haymaker, SU(2) flux distributions on finite lattices, Phys. Rev. D **47**, 5104 (1993)

Multihit



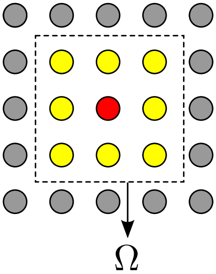
Replace each temporal link by it's thermal average

$$U_4 \rightarrow \bar{U}_4 = \frac{\int dU_4 \, U_4 \, e^{\beta \mathrm{Tr}\left[U_4 F^\dagger\right]}}{\int dU_4 \, e^{\beta \mathrm{Tr}\left[U_4 F^\dagger\right]}}$$

R. Brower et al., Nucl.Phys. B190, 1981.
G. Parisi et al., Phys.Lett. B128, 1983.

Extended Multihit

Replace each temporal link by it's thermal average with the first N neighbors fixed
Instead of taking the thermal average of a temporal link with the first neighbors, we fix the higher order neighbors, and apply the heat-bath algorithm to all the links inside, averaging the central link.



$$U_4 \to \bar{U}_4 = \frac{\int [\mathcal{D}U_4]_\Omega \, U_4 \, e^{\beta \sum_{\mu,s} \mathrm{Tr}\left[U_\mu(s) F^\dagger(s)\right]}}{\int [\mathcal{D}U_4]_\Omega \, e^{\beta \sum_{\mu,s} \mathrm{Tr}\left[U_\mu(s) F^\dagger(s)\right]}}$$

N. Cardoso et al., Phys. Rev. D 88, 2013.

By using $N = 2$ we are able to greatly improve the signal, when compared with the error reduction achieved with the simple multihit.
Of course, this technique is more computer intensive than simple multihit, while being simpler to implement than multilevel.
The only restriction is $R > 2N$ for this technique to be valid.

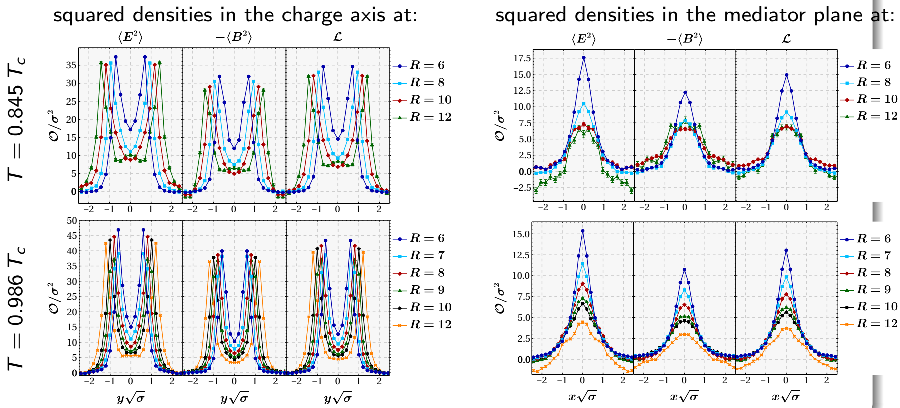
Lattice ensembles:

Volume	β	T/T_c	$a\sqrt{\sigma}$	# config.
32^4	6.0	0	0.219718	1100
$48^3 \times 8$	5.96	0.845	0.235023	5990
$48^3 \times 8$	6.0534	0.986	0.201444	5990/5110*
$48^3 \times 8$	6.13931	1.127	0.176266	5990
$48^3 \times 8$	6.29225	1.408	0.141013	5990
$48^3 \times 8$	6.4249	1.690	0.117513	5990

where σ is the string tension at zero temperature.

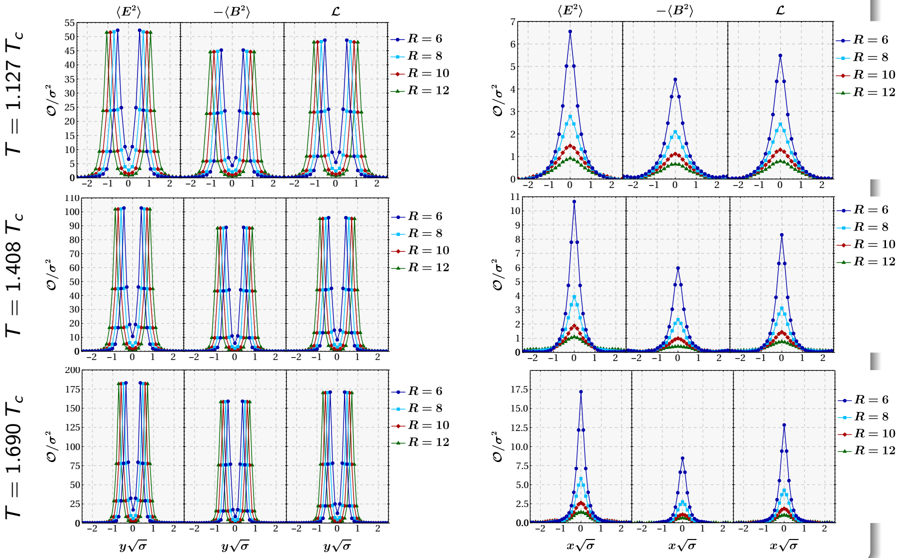
All the computations were done in NVIDIA GPUs using CUDA.

Results for the $Q\bar{Q}$ system



Results for the $Q\bar{Q}$ system

squared densities in the charge axis at: squared densities in the mediator plane at:



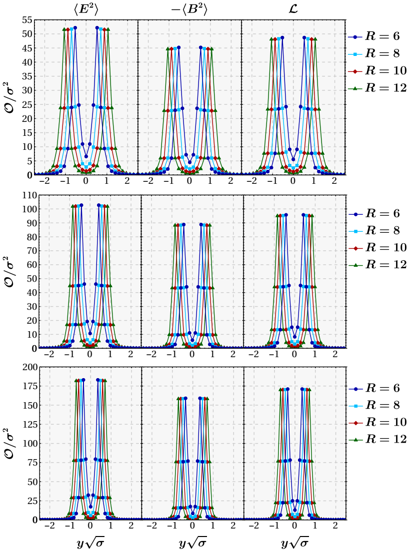
Results for the QQ system

$T = 1.127\ T_c$

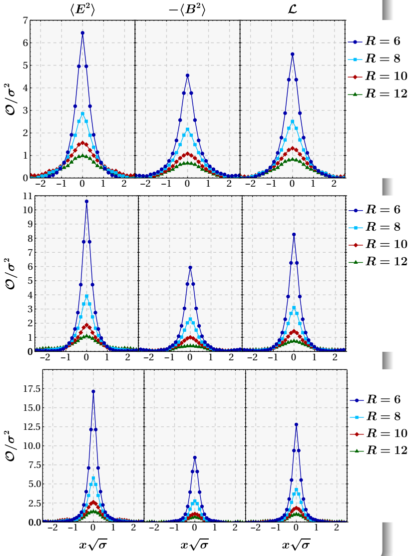
$T = 1.408\ T_c$

$T = 1.690\ T_c$

squared densities in the charge axis at:

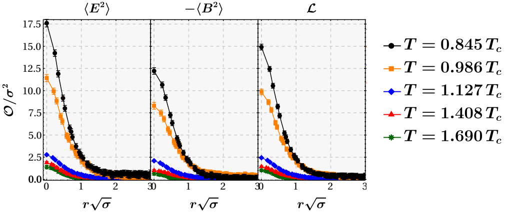


squared densities in the mediator plane at:

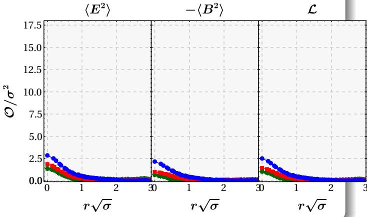


Flux profiles in the mediator plane for $R = 1.41\sqrt{\sigma}$

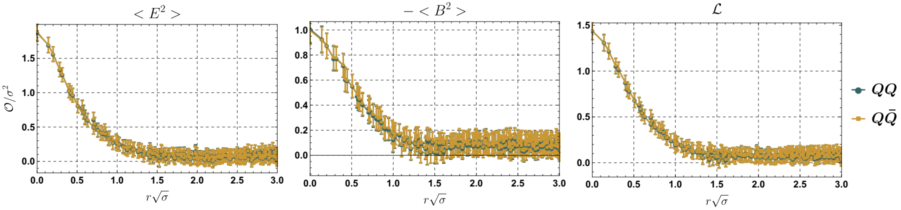
- $Q\bar{Q}$



- QQ

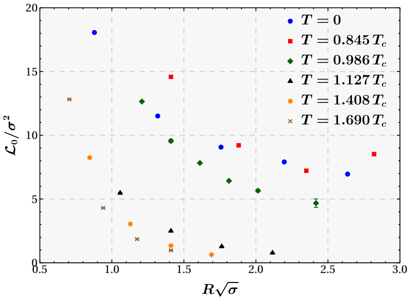


- $T = 1.408\,Tc$

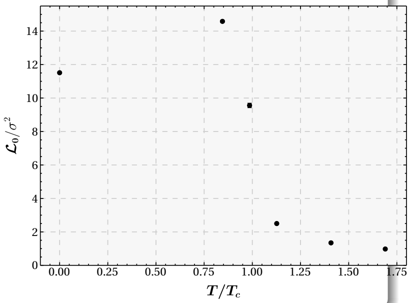


Results for the central axial $\mathcal{L}_0$ for the system $Q\bar{Q}$:

- as a function of inter-charge distance

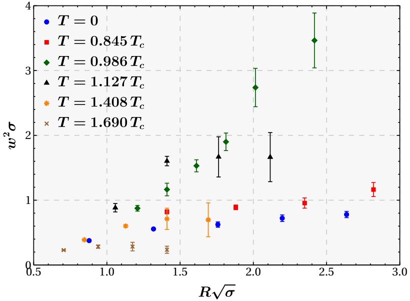


- Temperature dependence for $R = 1.41\sqrt{\sigma}$

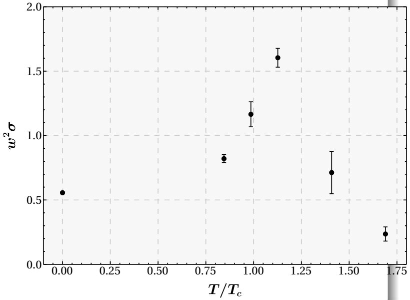


Results for the width, w^2, for the system $Q\bar{Q}$:

- as a function of inter-charge distance.

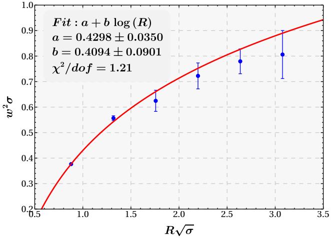


- Temperature dependence for $R = 1.41\sqrt{\sigma}$.



Widening of the flux tube, $Q\bar{Q}$

- $T = 0$



Results

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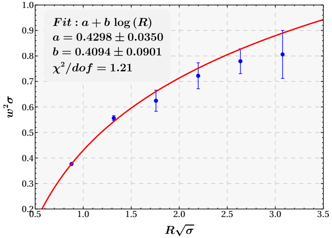
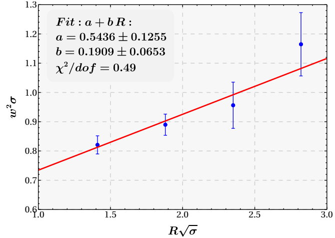
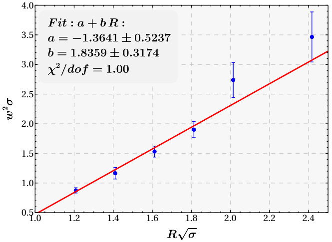


- $T = 0.845\ T_c$



- $T = 0.986\ T_c$



Summary

- We compute the square densities of the chromomagnetic and chromoelectric fields produced by different Polyakov loop sources, above and below the phase transition

- As the distance increase between the sources, the fields square densities decrease. Below the deconfinement critical temperature, this decrease is moderate and is consistent with the widening of the flux tube as already seen in studies at zero temperature

- Moreover the field intensity clearly decreases when the temperature increases, as expected from the critical curve for the string tension

- Above the deconfinement critical temperature, at $T > T_c$, the fields rapidly decrease to zero as the quarks are pulled apart, qualitatively consistent with screened Coulomb-like fields

- While the width of the flux tube below the phase transition temperature increases with the separation between the quark-antiquark, above the phase transition we find no evidence for widening

- In the same perspective, the QQ and the $Q\bar{Q}$ square fields are essentially similar

Thanks