

and beauty
Charmed \backslash states from lattice QCD

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in collaboration with:

Christian B. Lang, Luka Leskovec, Daniel Mohler, Richard Woloshyn

Outline

- I will discuss hadronic states that have to be address by simulating the scattering of two mesons on the lattice
- States slightly below threshold : $B_{s0}, B_{s1}, D_{s0}, D_{s1}, X(3872)$
Flavor according to quark model: $\underline{s} b$ $\underline{s} c$ $\underline{c} c$
Quark models expected $D_{s0}, D_{s1}, X(3872)$ above threshold - in contrast to exp
 B_{s0}, B_{s1} are still missing from experiment, so we present predictions
First simulation of such “deuterium-like” bound states, but in mesonic system
- Charmonium resonances above open-charm threshold: $\Psi(3770)$
- Search for flavor exotic states: Z_c^+ , $\underline{c} c \underline{d} u$
- Conclusions

Lattice setup

	PACS-CS	
	Ensemble (1)	Ensemble (2)
$N_L^3 \times N_T$	$16^3 \times 32$	$32^3 \times 64$
N_f	2	2+1
a [fm]	0.1239(13)	0.0907(13)
L [fm]	1.98(2)	2.90(4)
m_π [MeV]	266(3)(3)	156(7)(2)

Fermilab method for heavy quarks :

[El Khadra, Kronfeld et al, 1997]

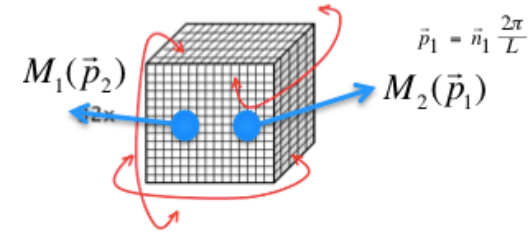
$$E_M(p) = M_1 + \frac{\mathbf{p}^2}{2M_2} - \frac{a^3 W_4}{6} \sum_i p_i^4 - \frac{(\mathbf{p}^2)^2}{8M_4^3} + \dots$$

Rest hadron energies have sizable discretization errors but these largely cancel in splittings

Discrete energy spectrum from correlators

Example: meson channel with given J^{PC}

$$\mathcal{O} = \bar{q}\Gamma q, \quad \bar{q}\Gamma' q, \quad (\bar{q}\Gamma_1 q)(\bar{q}\Gamma_2 q), \quad [\bar{q}\bar{q}][qq]$$



$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) | 0 \rangle = \sum_n Z_i^n Z_j^{n*} e^{-E_n t} \quad Z_i^n = \langle 0 | \mathcal{O}_i | n \rangle$$

Energies and overlaps extracted using GEVP $C(t)u^{(n)}(t) = \lambda^{(n)}(t)C(t_0)u^{(n)}(t)$

All physical states with given J^{PC} appear as energy levels E_n in principle : single particle, two-particle,...

channel : "eigenstates"

$J^{PC} = 0^+, \bar{s}b$: B_{s0}, BK

$J^{PC} = 1^{++}, \bar{c}c$: $\chi_{c1}, X(3872), DD^*,$

$J^{PC} = 1^{+-}, \bar{c}c\bar{d}u$: $Z_c^+, J/\psi \pi^+, \dots$

Two-meson states:

- In experiment: two-meson decay products with continuous E.
- On lattice: discrete E due to finite box and periodic BC.

Bound state and narrow resonance:

lead to extra energy level (in addition to two-meson levels)

Search for exotics:

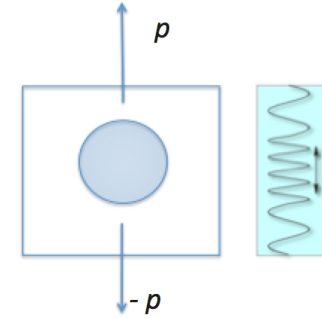
search for "extra" energy level (in addition to two-meson levels)

Scattering of two mesons

at total momentum $P=0$

$$E_n(L) \xrightarrow{E_n = E_{M1}(p) + E_{M2}(-p)} p \xrightarrow{\text{Luscher's eq.}} \delta(p)$$

$$\cot \delta(p) = \frac{2\mathcal{Z}_{00}(1; (\frac{pL}{2\pi})^2)}{Lp\sqrt{\pi}}$$



Scattering matrix for partial wave l :

$$S(p) = e^{2i\delta_l(p)}, \quad S(p) = 1 + 2iT(p), \quad T(p) = \frac{1}{\cot(\delta_l(p)) - i}$$

Bound state:

$$\cot[\delta(p_B)] = i, \quad p_B^2 < 0$$

$$m_B = E_{M1}(p_B) + E_{M2}(-p_B)$$

Resonance:

$$T(p) = \frac{-\sqrt{s} \Gamma(p)}{s - m_R^2 + i\sqrt{s} \Gamma(p)}$$

$$\Gamma(p) = g^2 \frac{p^{2l+1}}{s}$$

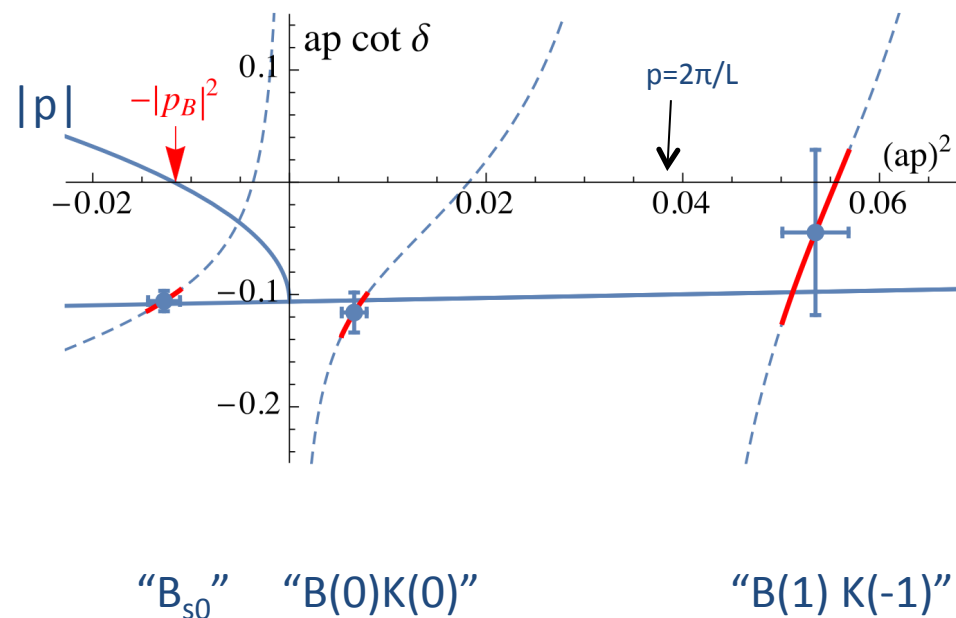
$$\frac{p^{2l+1}}{\sqrt{s}} \cot \delta(p) = \frac{1}{g^2} (m_R^2 - s)$$

Bound states slightly below threshold

Mass prediction for missing B_{s0} below BK threshold

- Scalar B_s state has not been experimentally found yet.
- It is expected slightly below BK threshold.
- We simulated BK scattering on Ensemble (2).

$$J^P = 0^+ : \mathcal{O} = \bar{s}b, B(0)K(0), B(1)K(-1)$$



$$T(p) = \frac{1}{\cot(\delta_l(p)) - i}$$

The scattering matrix has a pole at the position of the bound state (on the first Riemann sheet)

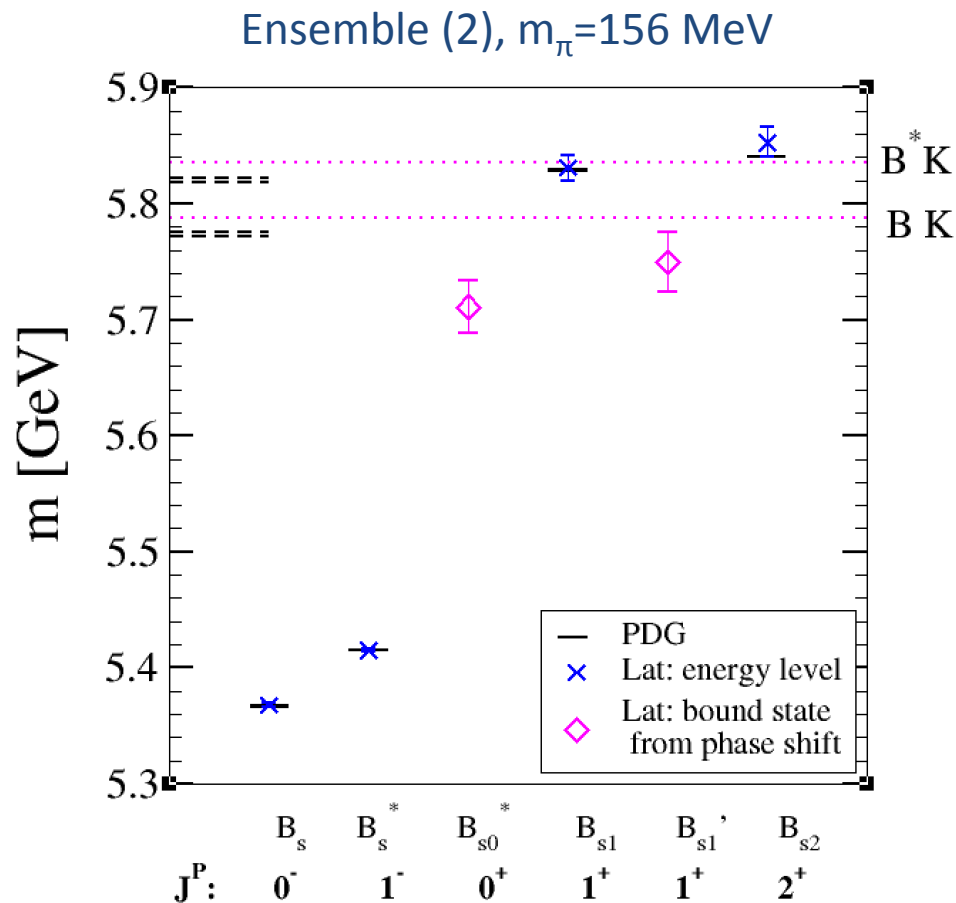
$$p_B = i|p_B| \quad \cot \delta(p_B) = i$$

$$p_B \cot \delta(p_B) = -|p_B|$$

$$m_B^{lat} = E_B(p_B) + E_K(-p_B)$$

[C. Lang, D. Mohler, S.P.,
R. Woloshyn: 1501.0164]

Mass prediction for missing B_{s0} and B_{s1}



Quantities shown:

for two bound states :

$$m_B = (m_B - E_{th})^{lat} + E_{th}^{exp}$$

for other states :

$$m = (m - \bar{m})^{lat} + \bar{m}^{lat}$$

for dotted lattice thresholds :

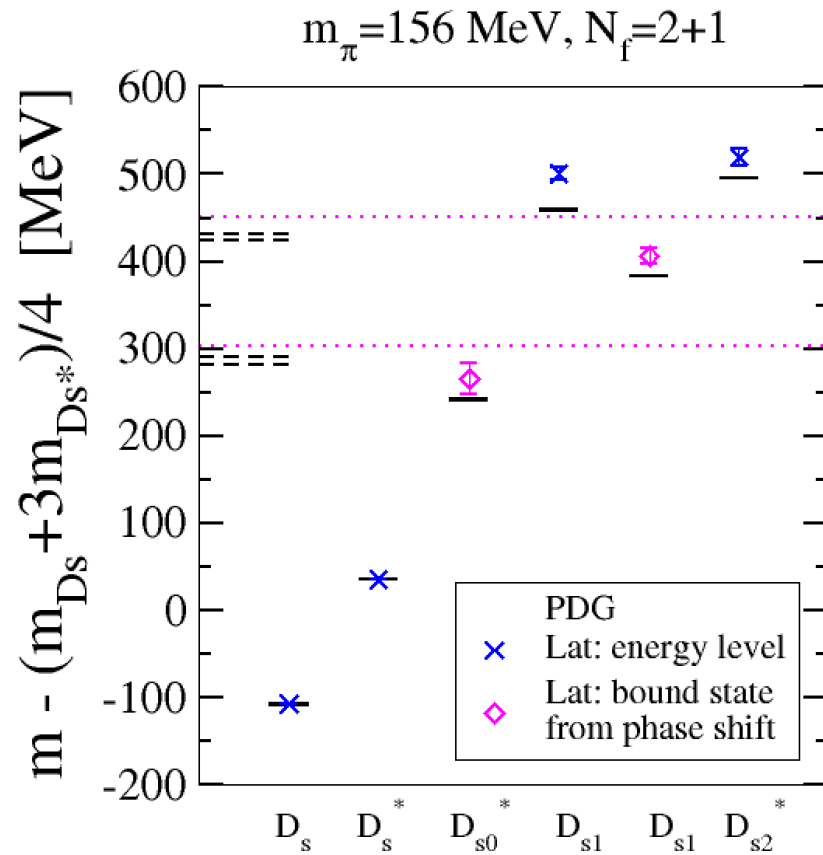
$$E_{th} = (E_{th} - \bar{m})^{lat} + \bar{m}^{lat}$$

$$\bar{m} \equiv \frac{1}{4}(m_{B_s} + 3m_{B_s^*})$$

- B_{s1}' and B_{s2} agree well with exp
- B_{s0} and B_{s1} are predictions for yet unobserved states (errors contain statistical and several sources of systematical uncertainties)

[C. Lang, D. Mohler, S.P.,
R. Woloshyn: 1501.0164]

D_{s0} and D_{s1} below DK and D^*K thresholds

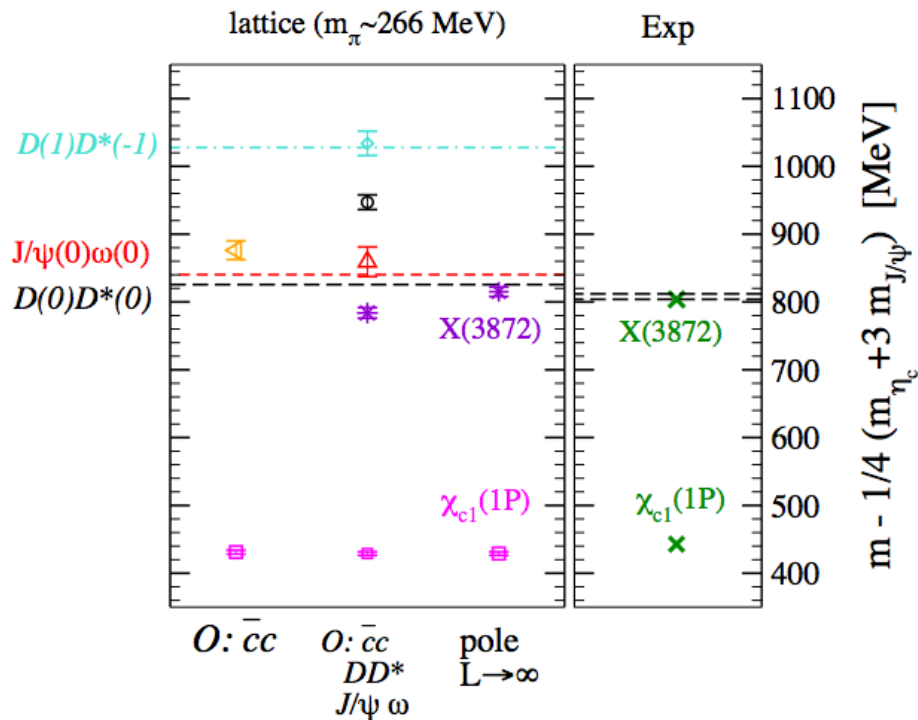


- Analogy, just b replaced with c
- In this case D_{s0} and D_{s1} have been observed experimentally
- Quark models expected them above thresholds but they were found below them
- Our post-dictions agree with measured masses

[D. Mohler, C. Lang, L. Leskovec, S.P., R. Woloshyn:
1308.3175, Phys. Rev. Lett 2013
1403.8103, PRD 2014]

X(3872) below DD* threshold

\mathcal{O} : $\bar{c}c$, DD^* , $J/\psi\omega$



[S.P. and L. Leskovec : 1307.5172, Phys. Rev. Lett. 2013]

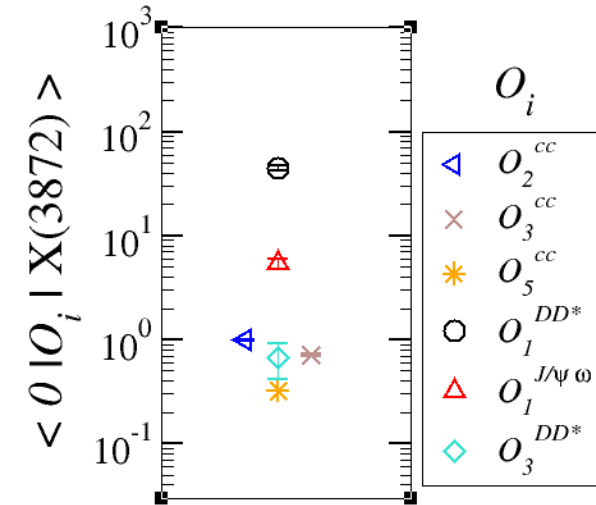
Ensemble (1), $m_{\pi} \approx 266$ MeV, $N_f=2$

Assumptions:

- charm Wick annihilation omitted
- DD* scattering analyzed assuming $J/\psi\omega$ is decoupled (good evidence for that from the lattice data)

cc-like, $J^{PC}=1^{++}$, $I=0$

$$\langle 0 | O_j | X(3872) \rangle$$



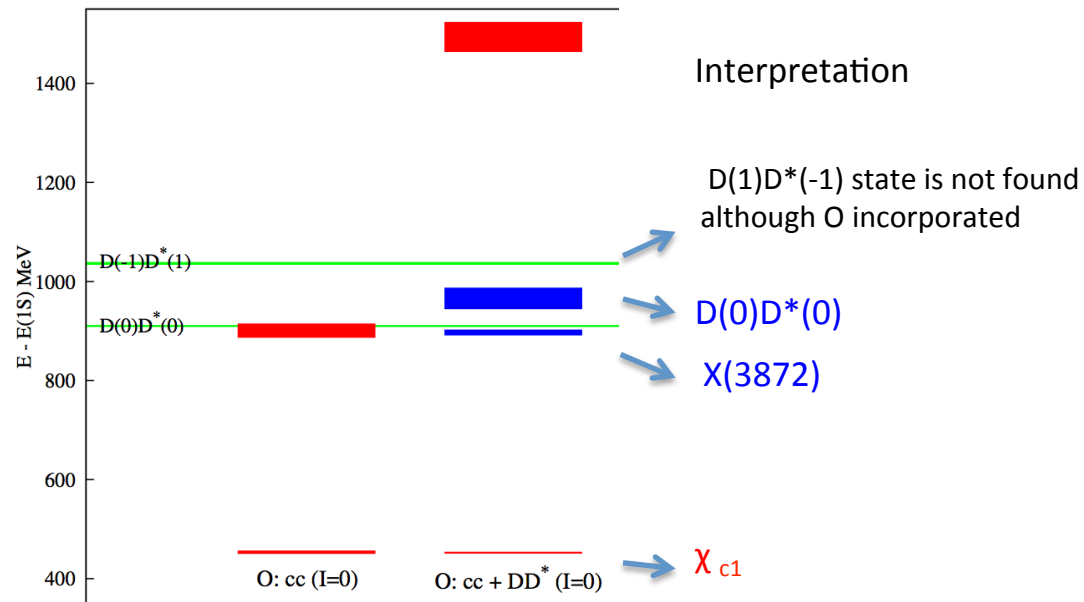
Overlaps normalized to $\langle 0 | O_1^{cc} | X(3872) \rangle$

X(3872)	$m - (m_{D_0} + m_{D_0^*})$
lat	-11 ± 7 MeV
exp	-0.14 ± 0.22 MeV

X(3872) appears only if both cc and DD* interp. used.

New evidence for X(3872) : $J^{PC}=1^{++}, I=0$

$\mathcal{O}: \bar{c} c, DD^*$



X(3872)	$m - (m_{D0} + m_{D0^*})$
lat	-13 ± 6 MeV
exp	-0.14 ± 0.22 MeV

HISQ quarks, $m_u = m_d = m_s/5$, $16^3 \times 48$, $a=0.15$ fm
 [C. DeTar, Song-haeng Lee, et al., 1411.1389]

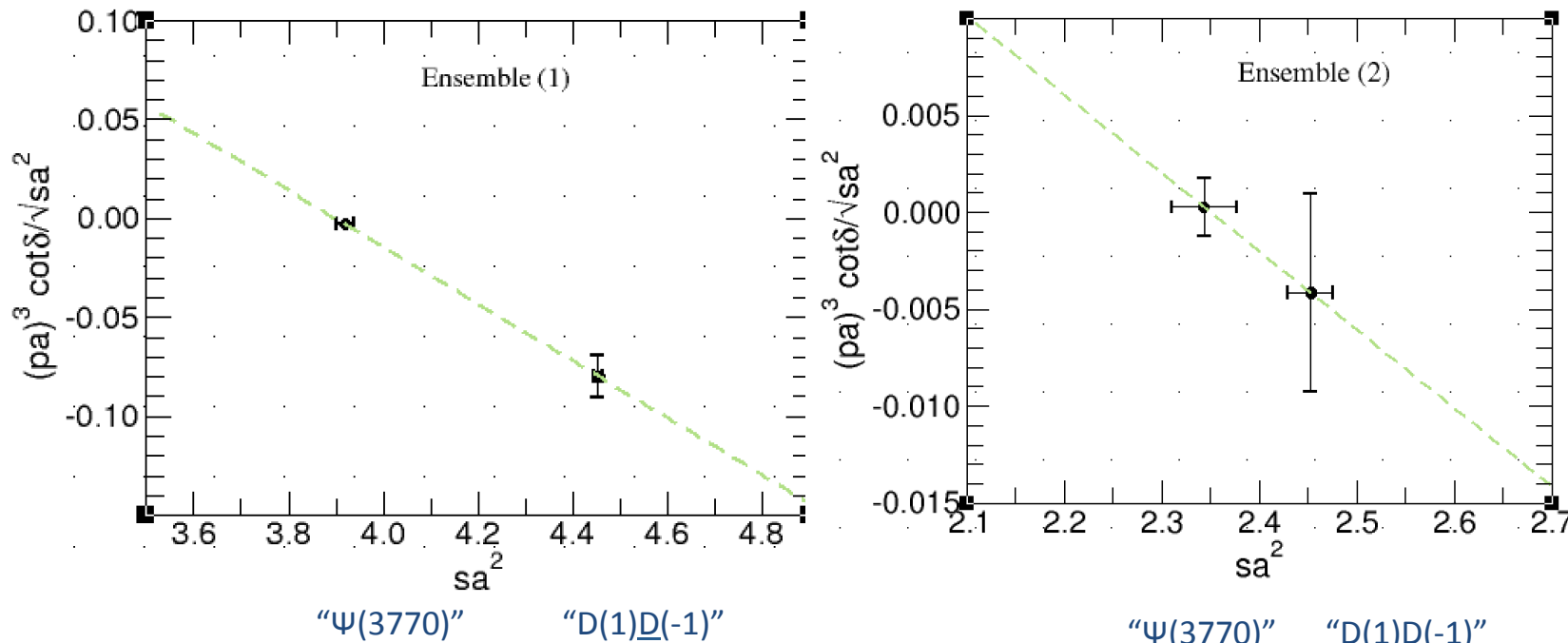
Charmonium resonances

Resonance $\psi(3770)$ in p-wave $D\bar{D}$ scattering

PRELIMINARY

$\bar{c}c$, $J^{PC} = 1^{--}$: J/ψ , $\psi(2S)$ below $D\bar{D}$ threshold

$\psi(3770)$ lowest state above threshold $\Gamma^{\text{exp}} \sim 27 \text{ MeV}$



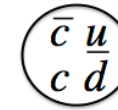
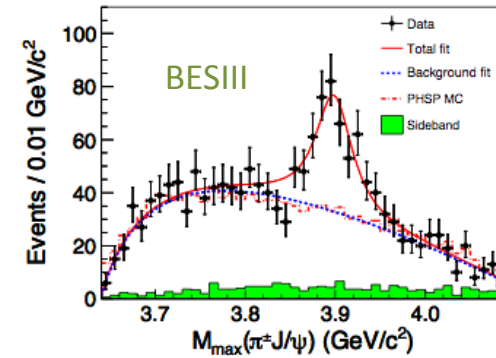
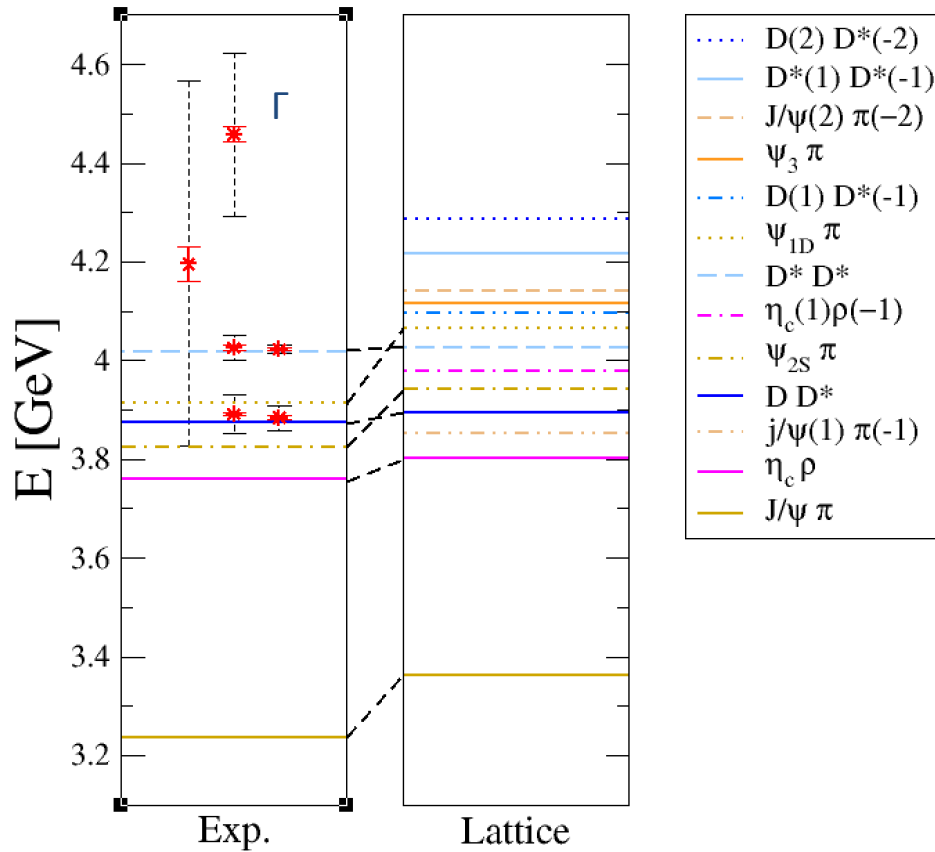
	Mass [MeV]	$g_{\psi(3770)D\bar{D}}$
Ensemble(1)	3785(7)(8)	11.4 (0.8)
Ensemble(2)	3783(49)(10)	21.6(14.9)
Experiment	3773.15(33)	≈ 18.7

$$\frac{p^3}{\sqrt{s}} \cot \delta = \frac{6\pi}{g^2} (m_R^2 - s) \quad \Gamma(s) = \frac{g^2 p^3}{6\pi s}$$

Search for hadrons
with manifestly **exotic** flavor:

ccud (l=1)

Z_c^+ channel : $I^G=1^+, J^{PC}=1^{+-}$



Lattice:

lines represent energies of
13 two-meson states

$$E = E[M_1(p_1)] + E[M_2(p_2)]$$

in non-interacting case

Extracting 13 two-meson states
is a huge challenge!

[S.P., Lang, Leskovec, Mohler, 1405.7612v2, PRD 2015]

Ensemble (2), $m_\pi \approx 266$ MeV, $L \approx 2$ fm, $N_f=2$

Z_c^+ channel : $|G=1^+, J^{PC}=1^{+-}$

Interpolating fields

18 two-meson (MM)

Aiming at 9 two-meson states listed in previous slide

$$\begin{aligned} \mathcal{O}_1^{\psi(0)\pi(0)} &= \bar{c}\gamma_i c(0) \bar{d}\gamma_5 u(0), \\ \mathcal{O}^{\psi(1)\pi(-1)} &= \sum_{e_k=\pm e_{x,y,z}} \bar{c}\gamma_i c(e_k) \bar{d}\gamma_5 u(-e_k), \\ \mathcal{O}^{\eta_c(0)\rho(0)} &= \bar{c}\gamma_5 c(0) \bar{d}\gamma_i u(0), \\ \mathcal{O}_1^{D(0)D^*(0)} &= \bar{c}\gamma_5 u(0) \bar{d}\gamma_i c(0) + \{\gamma_5 \leftrightarrow \gamma_i\}, \\ \mathcal{O}^{D^*(0)D^*(0)} &= \epsilon_{ijk} \bar{c}\gamma_j u(0) \bar{d}\gamma_k c(0), \end{aligned}$$

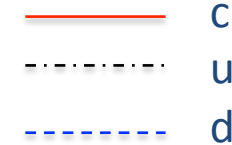
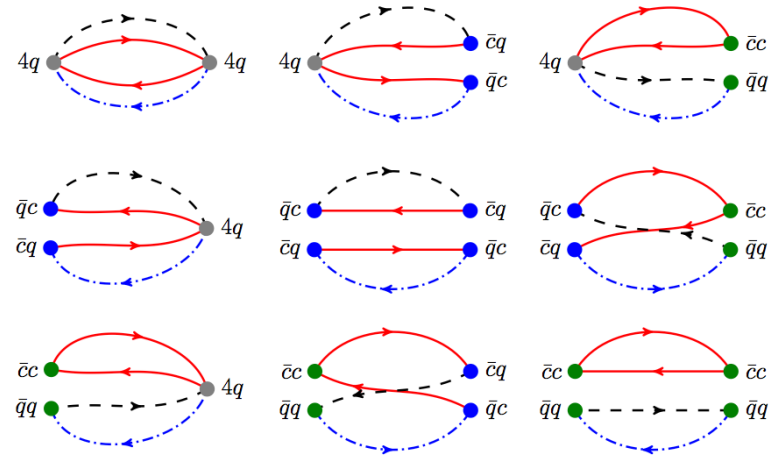
and 13 others ..

4 diquark-antidiquark (4Q)

Aiming to find additional state related to exotic Z_c^+

$$\begin{aligned} \mathcal{O}_1^{4q} &\approx [\bar{c} C \gamma_5 \bar{d}]_{3_c} [c \gamma_i C u]_{\bar{3}_c} \\ \mathcal{O}_2^{4q} &\approx [\bar{c} C \bar{d}]_{3_c} [c \gamma_i \gamma_5 C u]_{\bar{3}_c} \end{aligned}$$

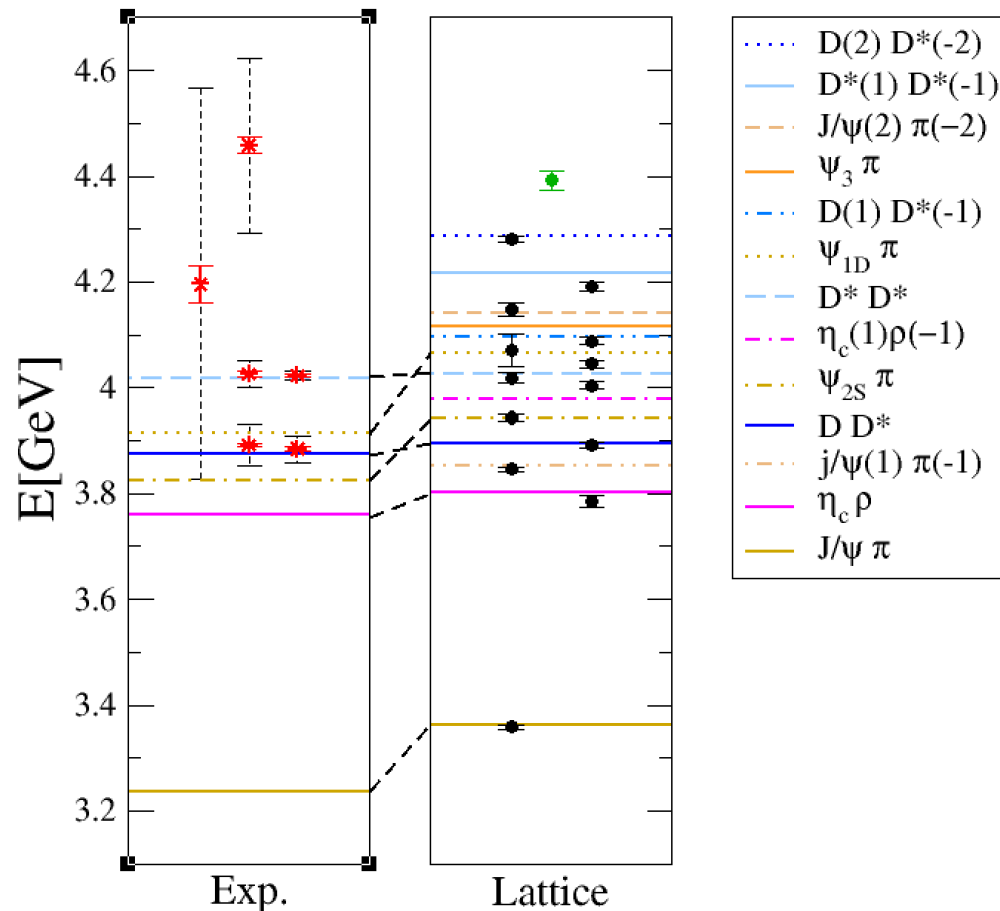
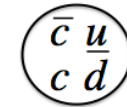
and 2 others ..



Wick contractions

$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t) \mathcal{O}_j^+(0) | 0 \rangle$$

Z_c^+ channel : $|^G=1^+, J^{PC}=1^{+-}$



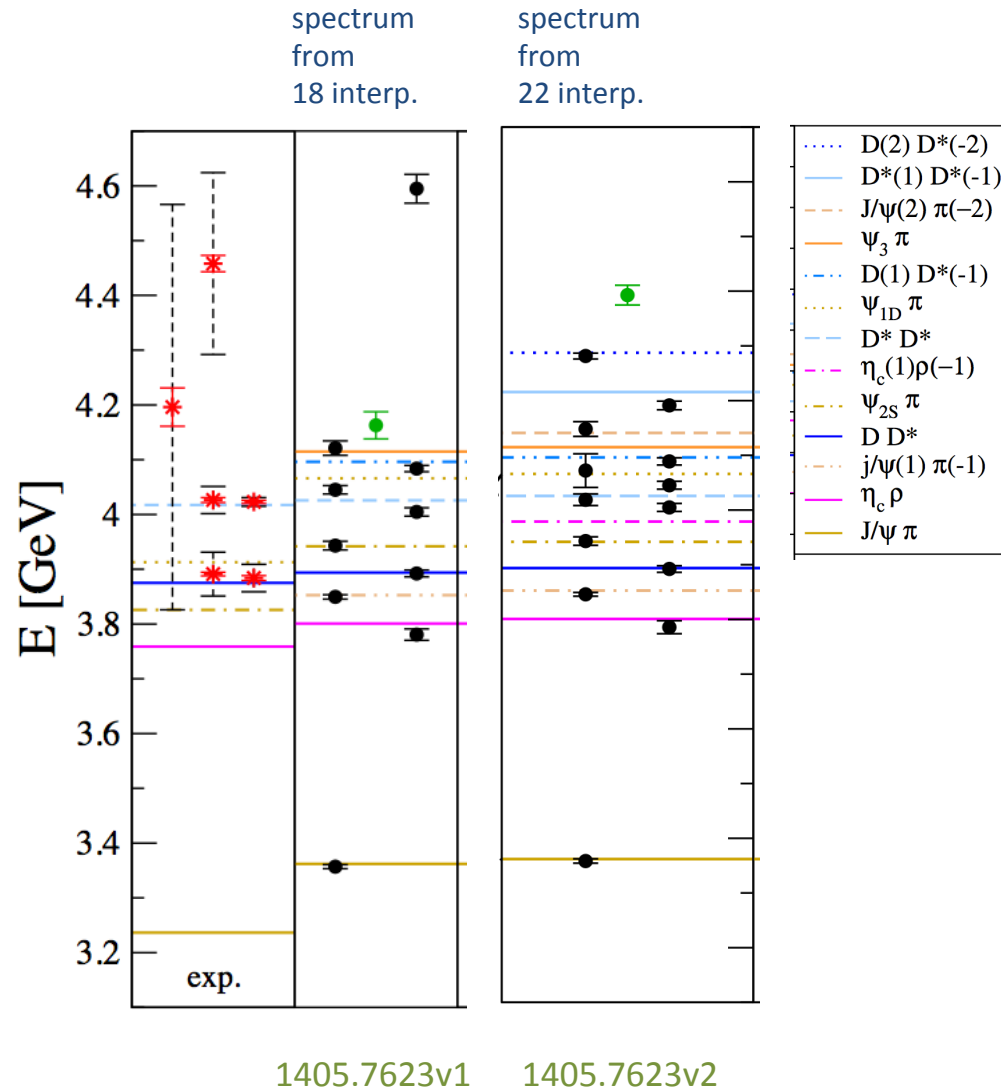
Conclusion:

- we find 13 two-meson states (black circles) as expected
- we find no additional state below 4.2 GeV
- we find no candidate for Z_c below 4.2 GeV

[S.P., Lang, Leskovec, Mohler, 1405.7612v2, PRD 2015]

Ensemble (1), $m_\pi \approx 266$ MeV, $L \approx 2$ fm, $N_f=2$

Z_c^+ channel



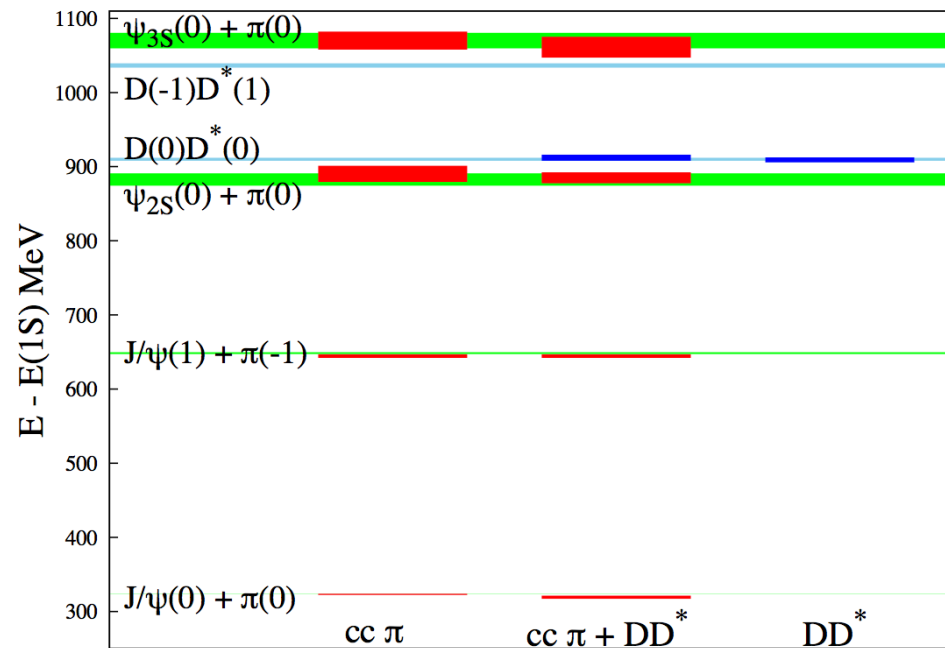
Results from the extended basis:
based on E_n and $Z_i^n = \langle 0 | \mathcal{Q} | n \rangle$

- lowest 13 states (black):
two-meson states
- no extra state below 4.2 GeV
- no extra state at 4.16 GeV
(extended basis gives an extra state at 4.4 GeV)
- attributing a state at 4.16 GeV to Z_c^+ (green) was a premature conclusion
- we can not exclude that state at 4.16 GeV was a linear combination of omitted two-meson states, induced via O^{4q}

Conclusion: we do not find Z_c^+ candidate below 4.2 GeV

Similar conclusion concerning Z_c^+ channel

Z_c channel, $I=1, J^{PC}=1^{+-}$



Only expected two-meson states,
no candidate for Z_c

S.-H. Lee, C. DeTar, H. Na,
Lattice 2014 proc.: 1411.1389
 $m_u=m_s/5; N_f=2+1+1; \text{HISQ}$

Puzzle: why there is no additional eigenstate related to $Z_c^+(3900)$ on the lattice?

Why does such large basis of creation operators not excite observed Z_c^+ (in addition to two-meson states) ?

- ✧ Even more interpolators needed ?
- ✧ Is $m_\pi=266$ MeV to high?
- ✧ Could the neglect of charm annihilation contribution significantly modify the conclusion?
- ✧ Is something wrong with working assumption that Z_c^+ should lead to an additional eigenstate?

Note: we have observed additional levels for resonances: $\rho, K^*, D_0^*, D_1, a_1, b_1, \Psi(3770)$
bound states: $D_{s0}^*, D_{s1}, B_{s0}^*, B_{s1}, X(3872)$

Is Z_c is of different origin and does not lead to additional level?

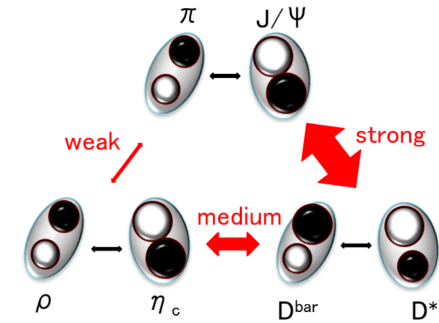
- ✧ Perhaps an additional eigenstate does not have to arise if it is a coupled channel effect: more analytical work is needed

$$\begin{aligned}
 \mathcal{O}_1^{\psi(0)\pi(0)} &= \bar{c}\gamma_i c(0) \bar{d}\gamma_5 u(0), \\
 \mathcal{O}_2^{\psi(0)\pi(0)} &= \bar{c}\gamma_i \gamma_t c(0) \bar{d}\gamma_5 u(0), \\
 \mathcal{O}_3^{\psi(0)\pi(0)} &= \bar{c} \overleftrightarrow{\nabla}_j \gamma_i \overleftrightarrow{\nabla}_j c(0) \bar{d}\gamma_5 u(0), \\
 \mathcal{O}_4^{\psi(0)\pi(0)} &= \bar{c} \overleftrightarrow{\nabla}_j \gamma_i \gamma_t \overleftrightarrow{\nabla}_j c(0) \bar{d}\gamma_5 u(0), \\
 \mathcal{O}_5^{\psi(0)\pi(0)} &= |\epsilon_{ijk}| |\epsilon_{klm}| \bar{c}\gamma_j \overleftrightarrow{\nabla}_l \overleftrightarrow{\nabla}_m c(0) \bar{d}\gamma_5 u(0), \\
 \mathcal{O}_6^{\psi(0)\pi(0)} &= |\epsilon_{ijk}| |\epsilon_{klm}| \bar{c}\gamma_t \gamma_j \overleftrightarrow{\nabla}_l \overleftrightarrow{\nabla}_m c(0) \bar{d}\gamma_5 u(0), \\
 \mathcal{O}_7^{\psi(0)\pi(0)} &= R_{ijk} Q_{klm} \bar{c}\gamma_j \overleftrightarrow{\nabla}_l \overleftrightarrow{\nabla}_m c \bar{d}\gamma_5 u(0), \\
 \mathcal{O}_8^{\psi(0)\pi(0)} &= R_{ijk} Q_{klm} \bar{c}\gamma_t \gamma_j \overleftrightarrow{\nabla}_l \overleftrightarrow{\nabla}_m c \bar{d}\gamma_5 u(0), \\
 \mathcal{O}^{\psi(1)\pi(-1)} &= \sum_{e_k=\pm e_{x,y,z}} \bar{c}\gamma_i c(e_k) \bar{d}\gamma_5 u(-e_k), \\
 \mathcal{O}^{\psi(2)\pi(-2)} &= \sum_{|u_k|^2=2} \bar{c}\gamma_i c(u_k) \bar{d}\gamma_5 u(-u_k), \\
 \mathcal{O}^{\eta_c(0)\rho(0)} &= \bar{c}\gamma_5 c(0) \bar{d}\gamma_i u(0), \\
 \mathcal{O}^{\eta_c(1)\rho(-1)} &= \sum_{e_k=\pm e_{x,y,z}} \bar{c}\gamma_5 c(e_k) \bar{d}\gamma_i u(-e_k), \\
 \mathcal{O}_1^{D(0)D^*(0)} &= \bar{c}\gamma_5 u(0) \bar{d}\gamma_i c(0) + \{\gamma_5 \leftrightarrow \gamma_i\}, \\
 \mathcal{O}_2^{D(0)D^*(0)} &= \bar{c}\gamma_5 \gamma_t u(0) \bar{d}\gamma_i \gamma_t c(0) + \{\gamma_5 \leftrightarrow \gamma_i\}, \\
 \mathcal{O}^{D(1)D^*(-1)} &= \sum_{e_k=\pm e_{x,y,z}} \bar{c}\gamma_5 u(e_k) \bar{d}\gamma_i c(-e_k) + \{\gamma_5 \leftrightarrow \gamma_i\}, \\
 \mathcal{O}^{D(2)D^*(-2)} &= \sum_{|u_k|^2=2} \bar{c}\gamma_5 u(u_k) \bar{d}\gamma_i c(-u_k) + \{\gamma_5 \leftrightarrow \gamma_i\}, \\
 \mathcal{O}^{D^*(0)D^*(0)} &= \epsilon_{ijl} \bar{c}\gamma_j u(0) \bar{d}\gamma_l c(0), \\
 \mathcal{O}^{D^*(1)D^*(-1)} &= \sum_{e_k=\pm e_{x,y,z}} \epsilon_{ijl} \bar{c}\gamma_j u(e_k) \bar{d}\gamma_l c(-e_k), \\
 \mathcal{O}_1^{4q} &\approx [\bar{c} C \gamma_5 \bar{d}]_{3_c} [c \gamma_i C u]_{\bar{3}_c} \\
 \mathcal{O}_2^{4q} &\approx [\bar{c} C \bar{d}]_{3_c} [c \gamma_i \gamma_5 C u]_{\bar{3}_c}
 \end{aligned}$$

$$\begin{aligned}
 J^{PC} &= 1^{+-} \\
 &\left(\begin{array}{c} \bar{c} \quad u \\ c \quad \bar{d} \end{array} \right)
 \end{aligned}$$

Is Z_c related to coupled-channel effect?

- phenomenology [Swansen, 1409.3291]
- lattice (HALQCD method) $m_\pi \approx 410$ MeV, $N_f=2+1+1$



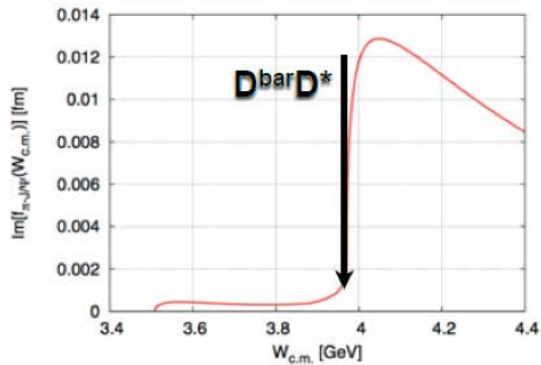
lattice (HALQCD method)

experiment

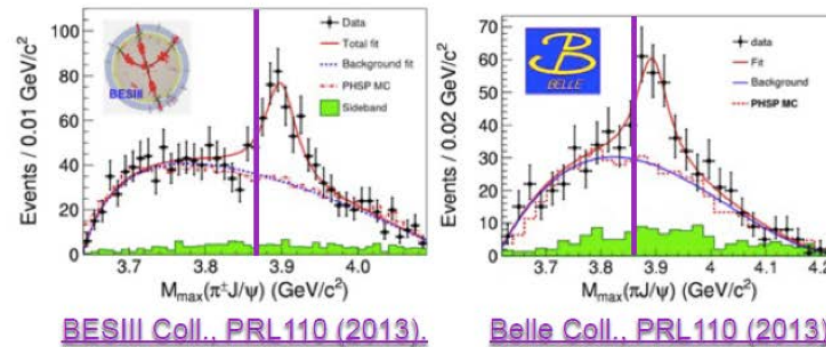
$$\sigma \propto \text{Im}[T]$$

$$J/\psi \pi$$

$\pi J/\psi$ invariant mass ($m_\pi=410$ MeV)



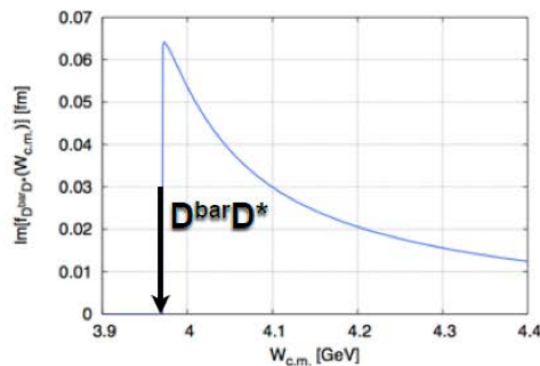
$e^+e^- \rightarrow \pi(\pi J/\psi)$ @ 4.26 GeV



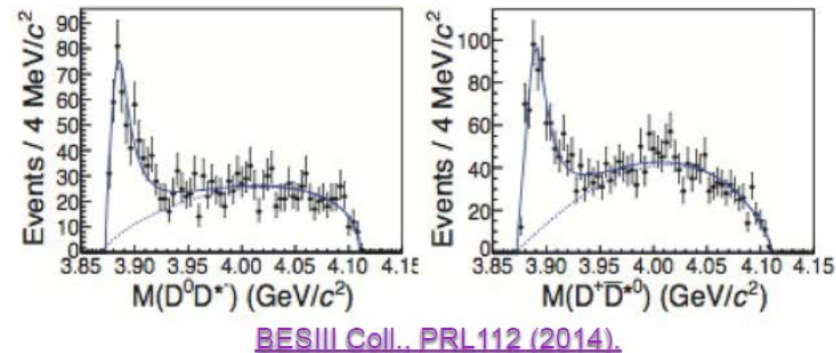
$D^{\bar{}} D^*$ invariant mass ($m_\pi=410$ MeV)

$$\sigma \propto \text{Im}[T]$$

$$D \bar{D}^*$$



$e^+e^- \rightarrow \pi^{+/-} (D^{\bar{}} D^*)^{+/-}$



HALQCD is investigating if extracted scattering matrix has a pole.

It needs to be investigated whether this scenario would lead to an additional energy level or not.

It is not clear yet whether this scenario could be compatible with absence of the Z_c energy level in our spectrum.²¹

Conclusions

- Evidence found for states with non-exotic flavor:
 - states well below th. : charmonium , bottomonium , ...
 - resonances via BW : ρ , K^* , $K_0^*(1430)$, K_2 , D_0^* , D_1 , a_1 , b_1 , $\Psi(3770)$
 - shallow bound states : D_{s0} , D_{s1} , B_{s0} , B_{s1} , $X(3872)$ with $I=0$

All these manifest themselves via an additional energy level !

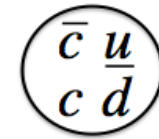
- No evidence for manifestly exotic states (yet) by searching an additional energy level
 - $X(3872)$ with $I=1$
 - $Z_c^+ = c\bar{c}u\bar{d}$
 - $c\bar{c}u\bar{d}$

Theory is facing a serious challenge to establish whether exotic states arise from QCD or not.

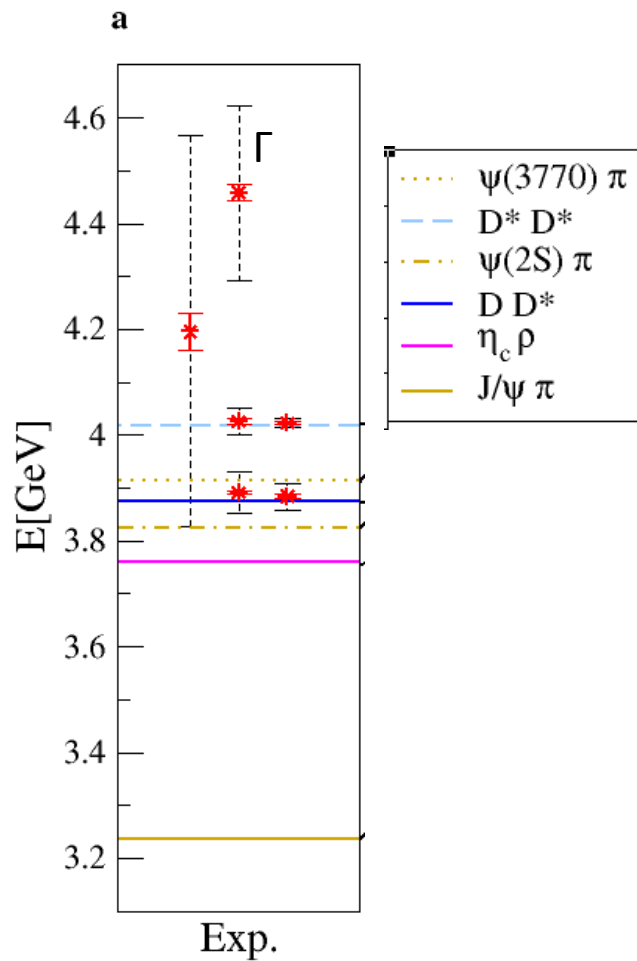
Only after this is settled, theory can claim structure (mesonic molecules, diquark antidiquark, coupled channel effects, ...)

Backup slides

Charged charmonium Z_c^+ : experimental status

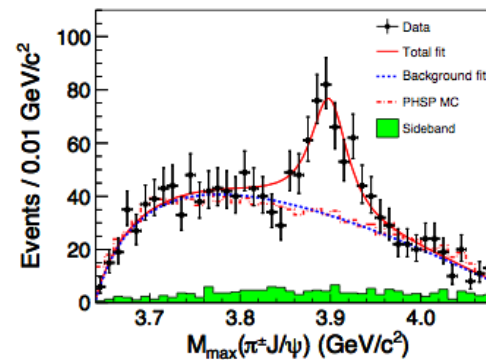


candidates with preferred
 $|G=1^+, J^{PC}=1^{+-}$

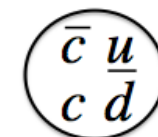
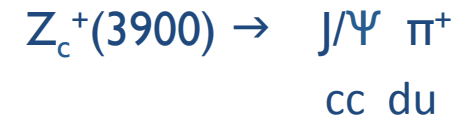


[review: Brambilla et al., 1404.3723]

particle	C	J ^P	decay	year	coll
$Z^+(4430)$	-	1+	$\psi(2S) \pi^+$	2008	Belle, BABAR, LHCb
$Z_c^+(3900)$	-	?	$J/\psi \pi^+$	2013	BESIII, Belle, CLEOc
$Z_c^+(3885)$	-	1+	$(DD^*)^+$	2013	BESIII
$Z_c^+(4020)$	-	?	$h_c(1P) \pi^+$	2013	BESIII
$Z_c^+(4025)$	-	?	$(D^* D^*)^+$	2013	BES III
$Z^+(4200)$	-	1+	$J/\psi \pi^+$	2014	Belle
$Z^+(4050)$	+	?	$\chi_{c1} \pi^+$	2008	Belle
$Z^+(4250)$	+	?	$\chi_{c1} \pi^+$	2008	Belle

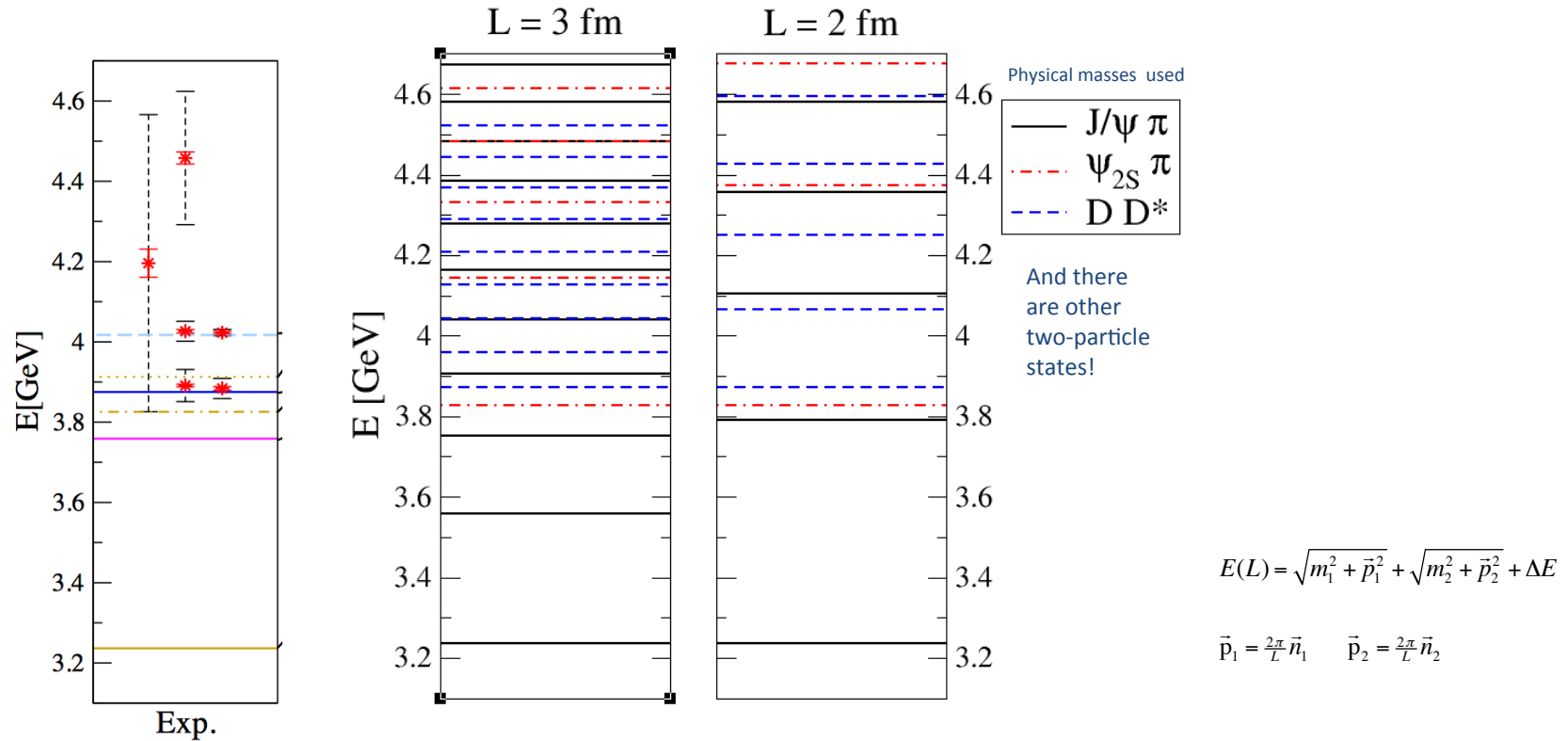


[BESIII, 2013, 1303.5949, PRL]



Challenge : precision simulation of Z_c^+

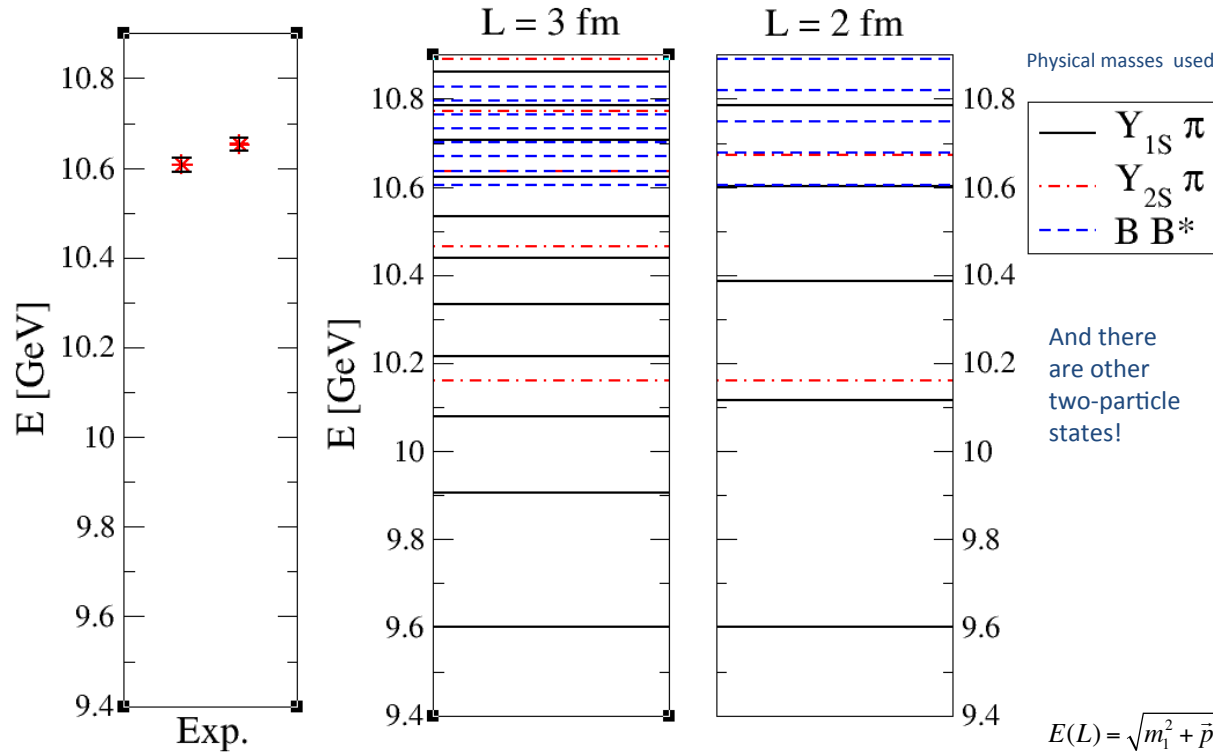
On larger volume: more two particle states



Rigorous treatment very challenging: at least 6 two-particle channels coupled !!

Another challenge: Z_b^+

On larger volume: more two-particle states

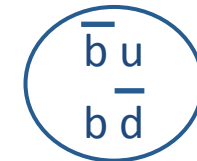
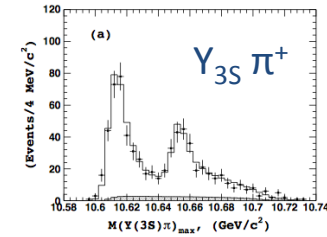
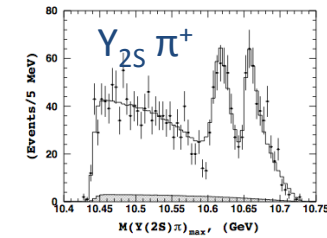
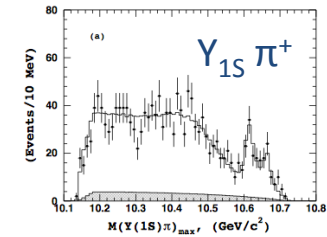


And there are other two-particle states!

$$E(L) = \sqrt{m_1^2 + \vec{p}_1^2} + \sqrt{m_2^2 + \vec{p}_2^2} + \Delta E$$

$$\vec{p}_1 = \frac{2\pi}{L} \vec{n}_1 \quad \vec{p}_2 = \frac{2\pi}{L} \vec{n}_2$$

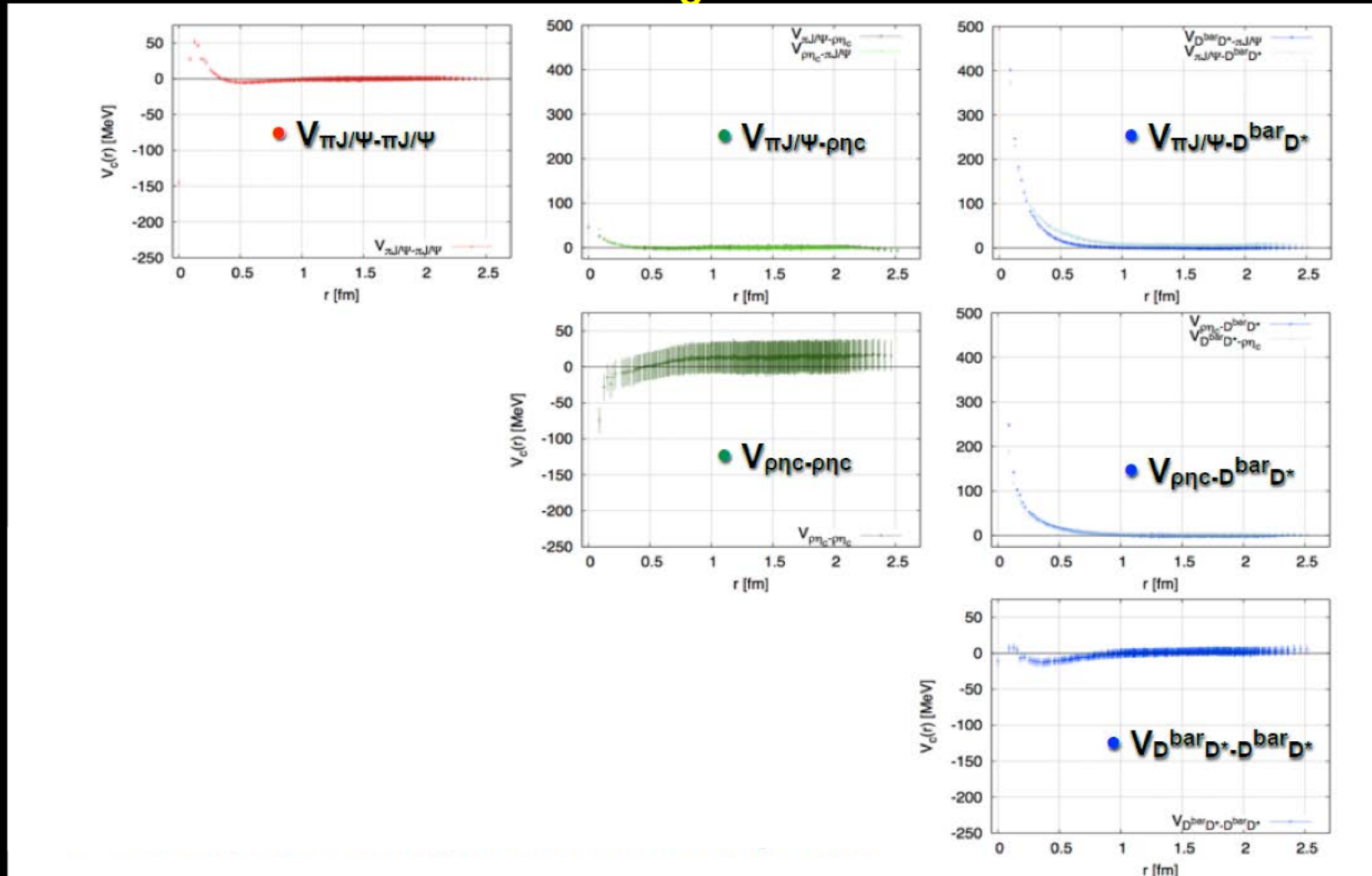
Belle 2011



Rigorous treatment very challenging: at least 6 two-particle channels coupled !!

Channel coupling ($\pi J/\Psi - \rho \eta_c - D^{\text{bar}} D^*$)

$$T = V + V G_0 T : 3 \times 3 \text{ matrix}$$



Ikeda+[HAL QCD Coll.] in preparation

Sasa Prelovsek, Royal Society meeting 2015

[talk by T. Doi]

D-meson resonances in $D\pi$ and $D^*\pi$

discussed by Daniel Mohler, plenary at Lattice 2012

$$\Gamma(E) \equiv g^2 \frac{P}{E^2}$$

g is compared to exp instead of Γ (Γ depends on phase sp. and m_π)

$J^P=0^+ : D \pi$

$J^P=1^+ : D^* \pi$

(analysis of spectrum in this case is based on an assumption given in paper below)

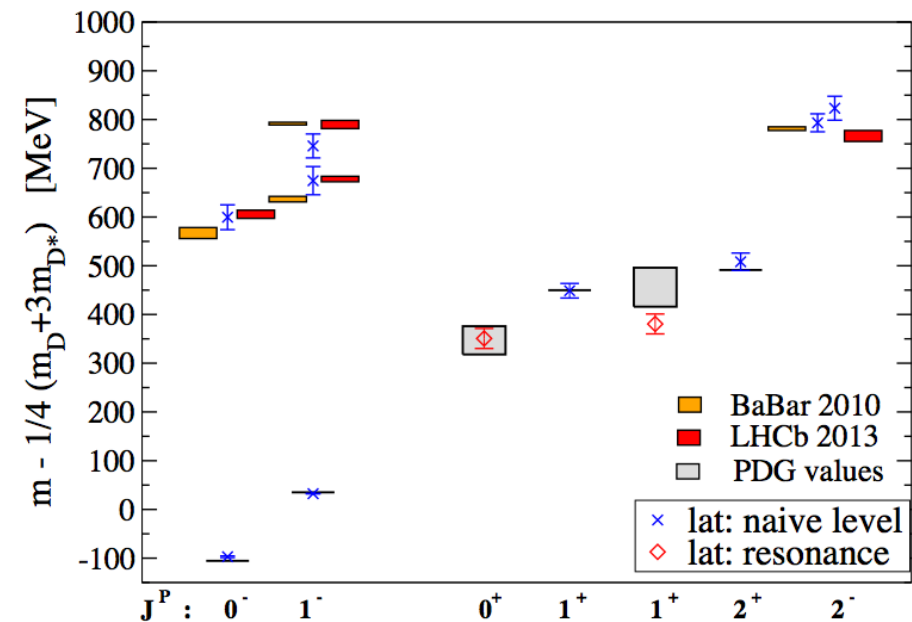
$D_0^*(2400)$	$m - 1/4(m_D+3m_{D^*})$	g
lat	351 ± 21 MeV	2.55 ± 0.21 GeV
exp	347 ± 29 MeV	1.92 ± 0.14 GeV

$D_1(2430)$	$m - 1/4(m_D+3m_{D^*})$	g
lat	381 ± 20 MeV	2.01 ± 0.15 GeV
exp	456 ± 40 MeV	2.50 ± 0.40 GeV

first lattice result for strong decay width of a hadron containing charm quark

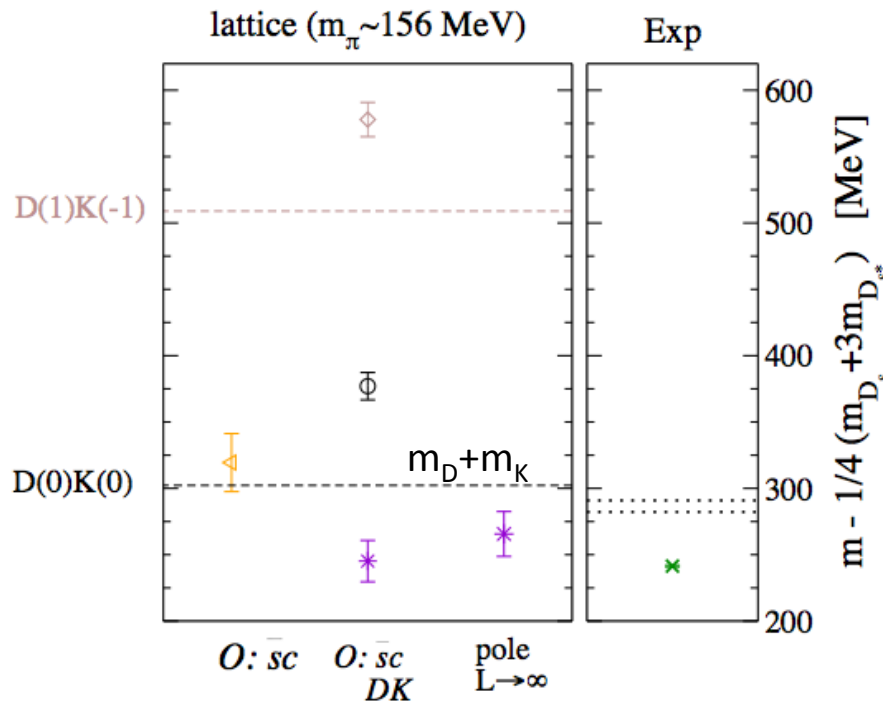
[D. Mohler, S.P., R. Woloshyn: 1208.4059, PRD]

- $m_\pi \approx 266$ MeV, $L \approx 2$ fm, $N_f=2$



$D_{s0}^*(2317)$: bound state below DK threshold, $J^P=0^+$

$$\mathcal{O} : \bar{s}c, \quad DK \approx [\bar{d}\gamma_5c] [\bar{s}\gamma_5d]$$

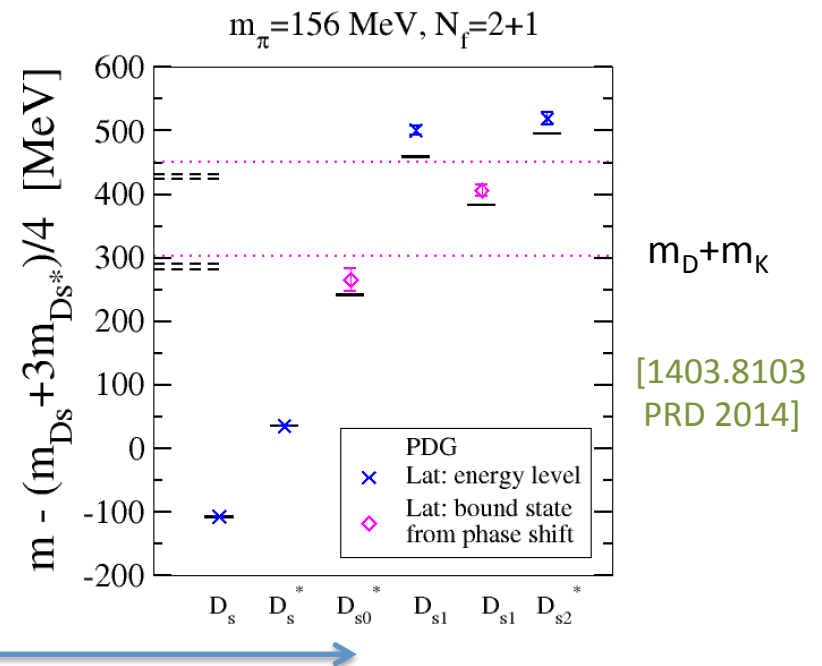


[D. Mohler, C. Lang, L. Leskovec, S.P., R. Woloshyn:
1308.3175, Phys. Rev. Lett 2013
1403.8103, PRD 2014]

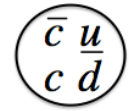
- δ for DK scattering in s-wave
extracted using Luscher's relation
- δ interpolated near threshold
- pole found in the scattering matrix

$$T \propto [\cot \delta - i]^{-1} = \infty, \quad \cot \delta(p_{BS}) = i$$

$$m_{D_{s0}^{lat, L \rightarrow \infty}} = E_D(p_{BS}) + E_K(p_{BS})$$



X(3872) channel: $I=1, J^{PC}=1^{++}$



Only expected two-particle states observed.
No candidate for X(3872) with $I=1$ found.

In agreement with experiment that does not find charged X either.

The simulation is done in the **isospin limit** $m_u=m_d$.
The absence of $I=1$ state for $m_u=m_d$ is in agreement with two interpretations:

$$(1) \quad X(3872) = a_{I=0} |DD^*\rangle_{I=0} + a_{I=1} |DD^*\rangle_{I=1}$$

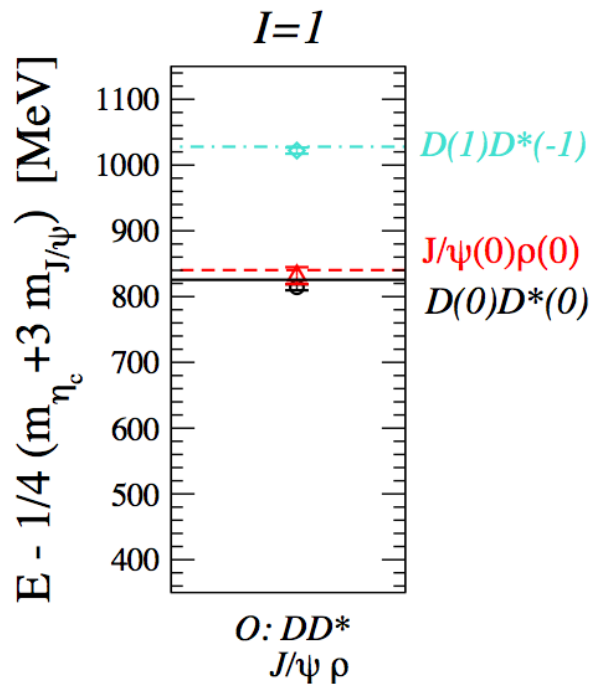
$$a_{I=1}(m_u = m_d) = 0$$

$$a_{I=1}(m_u \neq m_d) \ll a_{I=0}$$

(2) X(3872) pure $I=0$ state

isospin breaking decay $X(3872) \rightarrow J/\psi \rho$ ($I=1$)

is due to isospin splitting $D^0 D^{0*}, D^+ D^{-*}$



S. P. and L. Leskovec : 1307.5172

PRL 2013, $m_\pi \approx 266$ MeV, $N_f=2$

Puzzle from experiment: $Z_c^+(3900)$ seen only in Y decays

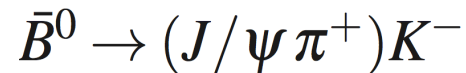
What about other channels in experiment and other experiments?

✧ $Z_c(3900)$ was found in $J/\psi \pi$ inv. mass only in

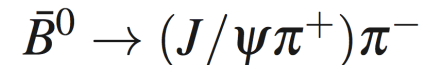
[BESIII, Belle, CLEOc, 2013] $Y(4260) \rightarrow (J/\psi \pi^+) \pi^-$

✧ $Z_c(3900)$ was NOT found in $J/\psi \pi$ inv. mass in

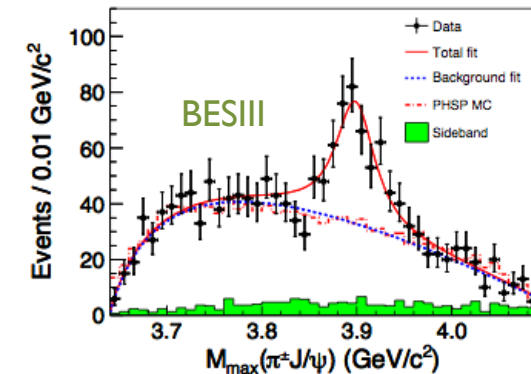
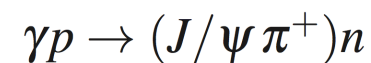
Belle 2014, 1408.6457



LHCb, 2014, 1404.5673



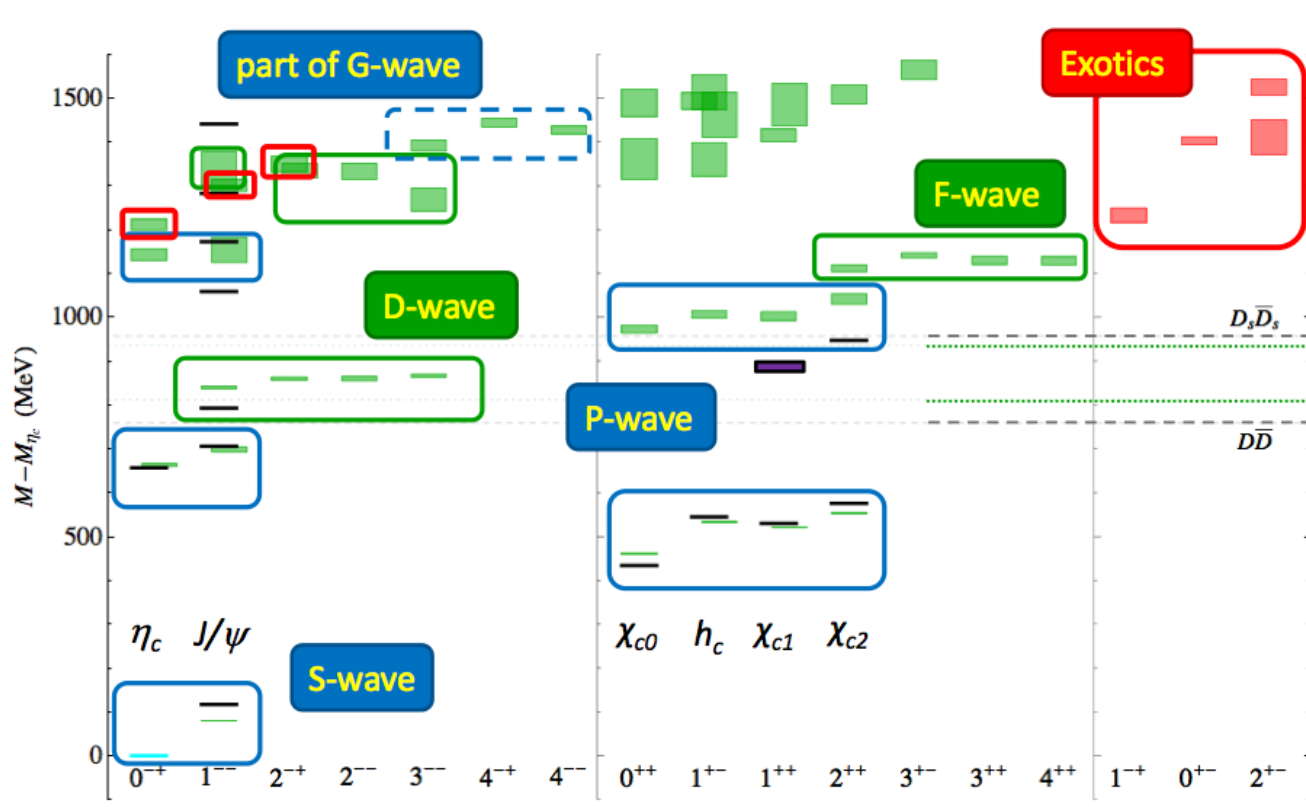
COMPASS, 2014, 1407.6186



Checking our implementation of Fermilab method: splittings on Ensemble (2)

Mass splitting	This work	Experiment
$m_{B^*} - m_B$	46.8(7.0)(0.7)	45.78(35)
$m_{B_s^*} - m_{B_s}$	47.1(1.5)(0.7)	$48.7_{-2.1}^{+2.3}$
$m_{B_s} - m_B$	81.5(4.1)(1.2)	87.35(23)
$m_Y - m_{\eta_b}$	44.2(0.3)(0.6)	62.3(3.2)
$2m_{\overline{B}} - m_{\overline{bb}}$	1190(11)(17)	1182.7(1.0)
$2m_{\overline{B_s}} - m_{\overline{bb}}$	1353(2)(19)	1361.7(3.4)
$2m_{B_c} - m_{\eta_b} - m_{\eta_c}$	169.4(0.4)(2.4)	167.3(4.9)

cc spectrum: single hadron approximation



[HSC , L. Liu et al: 1204.5425, JHEP]

- $m_\pi \approx 400$ MeV, $L \approx 2.9$ fm, $N_f = 2+1$
- reliable J^{PC} determination
- identification with $n^{2S+1}L_J$ multiplets using $\langle O | n \rangle$
- green: lat, black: exp

Hybrids:

some of them have exotic J^{PC}
large overlap with $O = \underline{q} F_{ij} q$

