Real-time dynamics and screening

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Introduction

- flavor non-singlet screening masses in the quark-gluon plasma at temperature T;
- in Matsubara formalism, 'turn your head to the side' and interpret the x_3 direction as Euclidean time; direction of x_0, of length $1/T$, now a finite 'spatial' extent.
- dimensional reduction $\Rightarrow$ quarkonium system in 2+1 dimensions
- $m_Q = (2n+1)\pi T$, $n = 0, 1, 2, \dots$

Largely based on "A relation between screening masses and real-time rates", JHEP 1405 (2014) 117: Brandt, Francis, Laine, HM.

Non-static screening masses in the high-temperature phase

Physics picture of flavor-non-singlet screening masses:

- fermions have an effective mass of at least $\pi T \Rightarrow$ dimensional reduction
- they form non-relativistic, 2+1d bound states of size $\mathrm{O}(m_E^{-1})$
  Laine, Vepsalainen hep-ph/0311268
- in general, there are several degenerate configurations at 0th order: in the sector $k_n = 6\pi T$, the momenta of the $\bar{Q}Q$ pair can be $(5\pi T,\, \pi T)$, $(3\pi T,\, 3\pi T)$ or $(\pi T,\, 5\pi T)$.

Consider the states coupling to V_0 (time component of the vector current):
△ For given p_n, let $w(z, \boldsymbol{y})$ be the screening correlator of a local current and a point-split current with separation $\boldsymbol{y}$ at the sink in the EFT. For $z > 0$,

$$\begin{aligned}
(\partial_z + \hat{H}^+) w(z, \boldsymbol{y}) &= 0\,, \qquad w(0, \boldsymbol{y}) = \delta^{(2)}(\boldsymbol{y}), \\
\hat{H}^+ &\equiv M_{\mathrm{cm}} - \frac{\nabla^2}{2M_{\mathrm{r}}} + V^+, \\
V^+_{\mathrm{LO}}(\boldsymbol{y}) &= \frac{g_E^2 C_F}{2\pi}\left[\ln\left(\frac{m_E y}{2}\right) + \gamma_{\mathrm{E}} + K_0(m_E y)\right].
\end{aligned}$$

△ The Fourier transform of $w(z, \boldsymbol{y})$ is closely related to the 'resolvent' g^+,

$$\left(\hat{H}^+ - \omega - i0^+\right) g^+(\omega, \boldsymbol{y}) = \delta^{(2)}(\boldsymbol{y}) \ .$$

△ Finally, we obtain for the screening spectral function

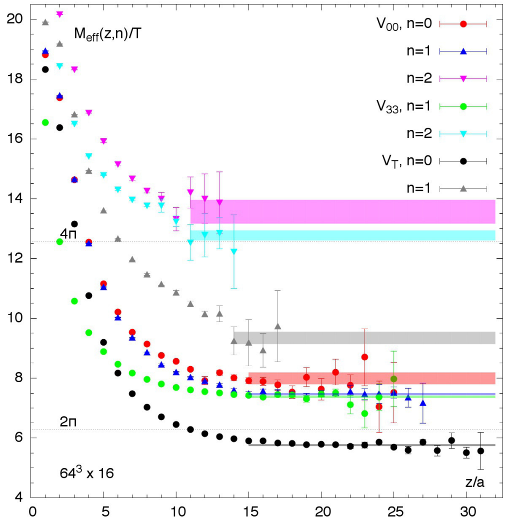
$$\rho_{00}^{(k_n)}(\omega) = - \sum_{0<p_n<k_n} 2N_c T \lim_{\boldsymbol{y}\to\boldsymbol{0}} \mathrm{Im}\, g^+(\omega, \boldsymbol{y}) \ .$$

△ Close resemblance with the corresponding equations appearing in the LPM resummation of longitudinal modes for dilepton production
(same potential V^+, looking for an s-wave bound state)
[Aurenche, Gelis, Moore, Zakaret hep-ph/0211036].

△ Potential V^+ can be defined non-perturbatively using a (modified) Wilson loop and has been computed in 3d lattice simulations [Caron-Huot 0811.1603; Panero, Rummukainen, Schäfer 1307.5850]

△ Idea: test the predictions for non-static screening masses resulting from solving the Schrödinger equation for the 2+1d potential V^+.

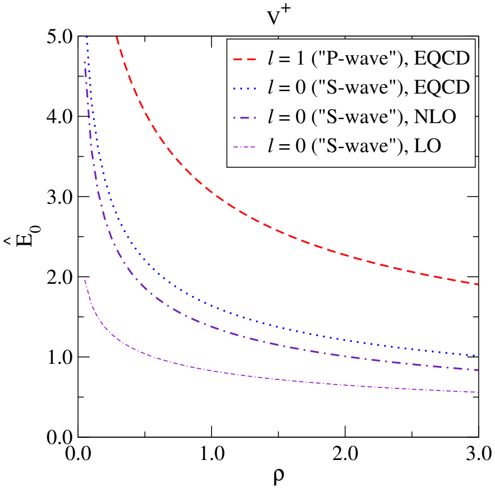
Effective mass plot of screening correlators at $T = 254$ MeV



Two-flavor QCD $\quad m_\pi(T=0) = 270$ MeV $\quad 16 \times 64^3$.

$\bar{Q}Q$ Binding energy in the EFT

$$g^+(\omega,\boldsymbol{y}) = \sum_{i=0}^{\infty} \frac{\psi_i(\boldsymbol{y})\psi_i^*(\boldsymbol{0})}{E_i-\omega-i0^+}\,, \qquad \rho_{00}^{(k_n)}(\omega) = -2\pi N_c T \sum_{0<p_n<k_n} \sum_{i=0}^{\infty} \delta(E_i-\omega)|\psi_i(\boldsymbol{0})|^2$$

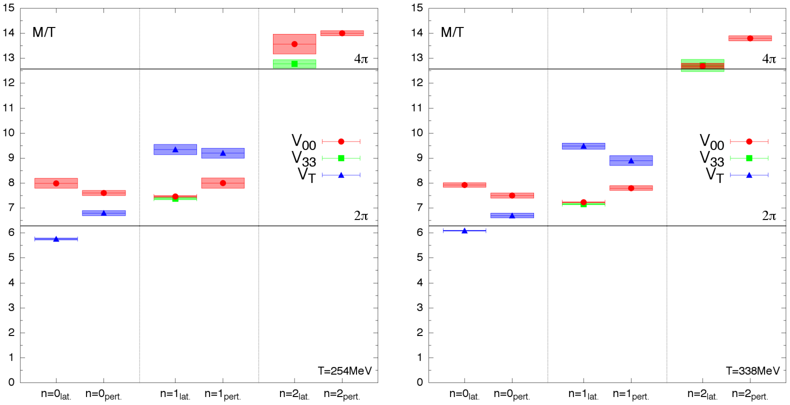


$$\rho = \frac{g_E^2 C_F M_{\rm r}}{\pi m_{\rm E}^2}$$

$$E_{\rm full} = M_{\rm cm} + \frac{g_E^2 C_F}{2\pi}\hat{E}$$

$$(m_{\rm E} = {\rm O}(gT), \quad g_E^2 = g^2 T + \dots)$$

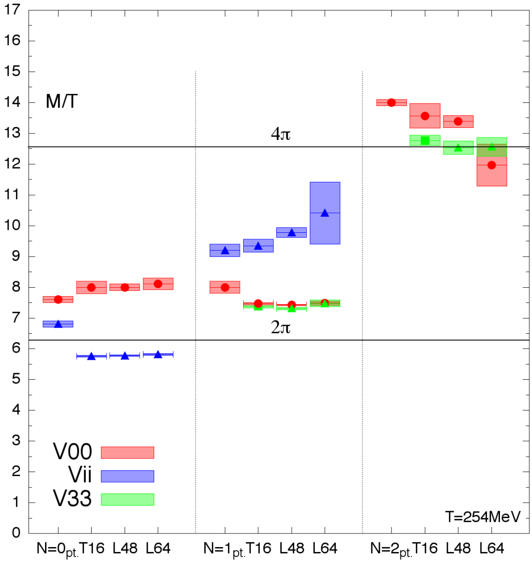
Vector screening masses: lattice vs. EFT



$T = 254$ MeV $\qquad$ $T = 340$ MeV

Satisfactory agreement between lattice QCD and the EFT predictions.

Checking for systematics at $T = 254$MeV and $m_\pi(T = 0) = 270$MeV



EFT vs. 16×48^3 vs. 12×48^3 vs. 12×64^3 Francis et al. Preliminary

Screening masses and transport coefficients

Linear response along with a constitutive equation for the current $\boldsymbol{V}$ $\Rightarrow$

$$G_R(\omega,k) \overset{\omega,k\to 0}{=\!=\!=} \frac{\chi_s D k^2}{-i\omega + Dk^2}, \qquad \chi_s \equiv \int d^4x \, \langle V_0(x) V_0(0)\rangle.$$

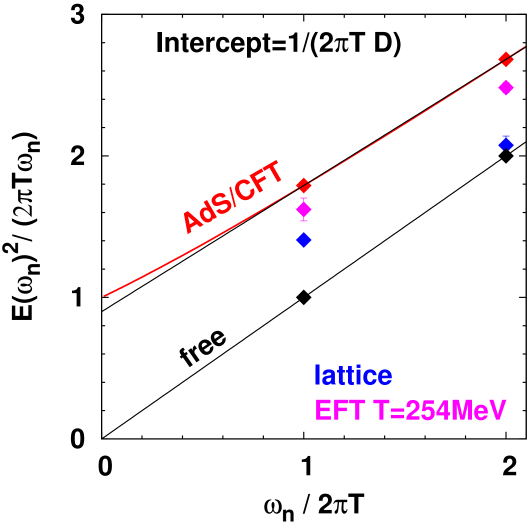
- in terms of the frequency ω: pole at $\omega = -iDk^2$ in the lower half of the complex plane ($D =$ diffusion constant).
- continuing to imaginary frequencies, we obtain the Euclidean correlator, $G_E(\omega_n,k) = G_R(i\omega_n,k)$ ($k \neq 0$; $\omega_n = 2\pi T n > 0$).
- poles in k of $G_E(\omega_n,k)$ are equal to i times a screening 'mass' corresponding to a fixed Matsubara frequency sector ω_n.
- continuing the screening mass as a function of the Matsubara frequency, Eq. (9) suggests that for $\omega_n \to 0$, we should find a k-pole at $k^2 = -\frac{\omega_n}{D}$ $\Rightarrow$

One screening mass $E(\omega_n)$ continues to

$$E(\omega_n)^2 \overset{\omega_n\to 0}{\sim} \frac{\omega_n}{D}.$$

[1408.5917 Brandt, Francis, Laine, HM]

Testing the idea



A note on inverse (generalized) Laplace transforms

- $\bar{\rho}(\omega) \equiv \rho_{ii}(\omega, T)\tanh(\omega\beta/2)/\omega^2$ is finite when $\omega \to 0$ and $\omega \to \infty$.
- $\Rightarrow$ rescale the spectral function and the kernel,

$$G(t) = \int_0^\infty d\omega\, K(t,\omega)\bar{\rho}(\omega), \qquad K(t,\omega) = \frac{\omega^2 \cosh \omega(\beta/2 - t)}{\sinh(\omega\beta/2)\tanh(\omega\beta/2)}.$$

- let $\widehat{\rho}(\omega)$ be the result of the procedure to reconstruct $\bar{\rho}(\omega)$.
- for any linear method, $\exists$ functions $\{g_i(\omega)\}$ such that

$$\widehat{\rho}(\omega) = \sum_{i=1}^{n} g_i(\omega)\, G(t_i).$$

- the resolution function [HM, PRL 100 (2008) 162001]

$$\delta(\omega, \omega') \equiv \sum_{i=1}^{n} g_i(\omega)\, K(t_i, \omega')$$

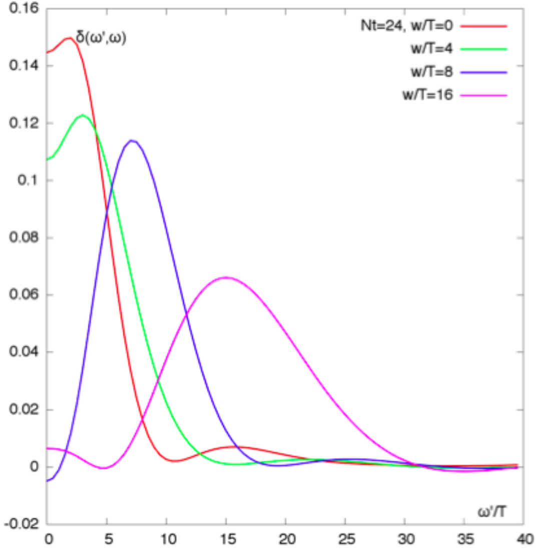
  has the property that

$$\widehat{\rho}(\omega) \equiv \int_0^\infty d\omega'\, \delta(\omega, \omega')\, \bar{\rho}(\omega', T)$$

- Backus-Gilbert: the $g_i(\omega)$ can be optimized to minimize the width of the resolution function,

$$\Gamma_\omega \equiv \int_0^\infty d\omega'\, (\omega - \omega')^2\, \delta(\omega, \omega')^2, \quad \text{constraint}: \quad \int_0^\infty d\omega' \delta(\omega, \omega') = 1.$$

Realistic resolution functions

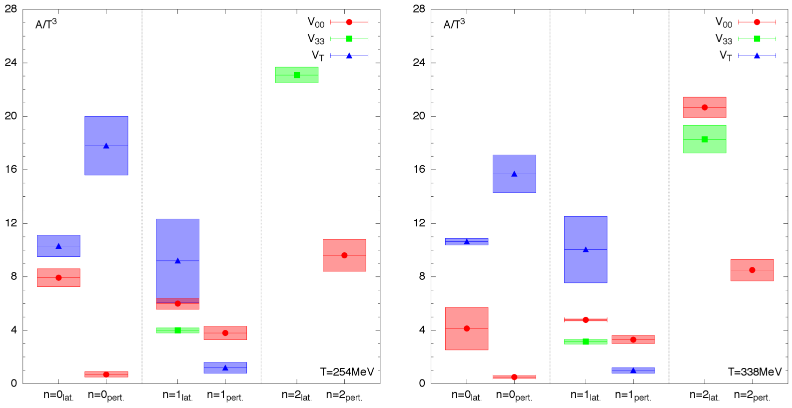


Outlook

- 'Integrating out' the non-static gauge modes perturbatively seems to be a decent approximation even at $T = 250$ MeV.
- Using a non-perturbative potential V^+ improves the predictions for the non-static screening masses.
- Interesting interplay between screening masses and real-time phenomena.

Backup slides

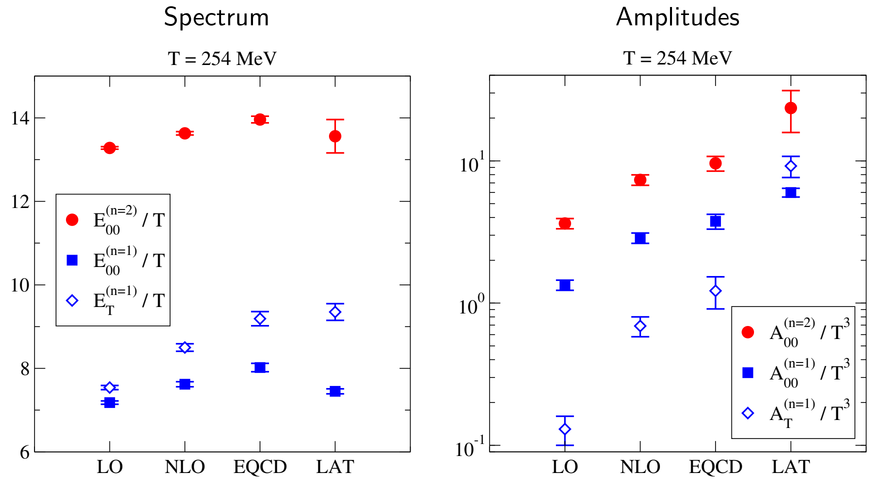
Amplitudes of vector screening states: lattice vs. EFT



$T = 250$ MeV $\qquad\qquad$ $T = 340$ MeV

Prediction for the amplitude $\langle B|V_0|0\rangle$ is harder to get; better with non-pert. potential.

Perturbative vs. non-perturbative potential



Indication that the non-perturbative potential leads to better agreement with the results of full QCD simulations.

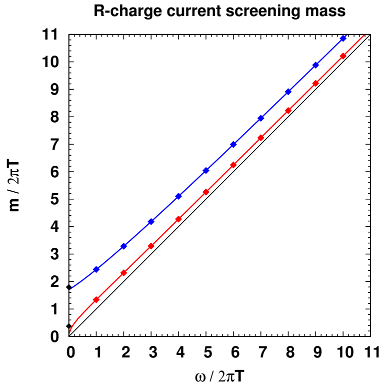
Screening masses in AdS/CFT

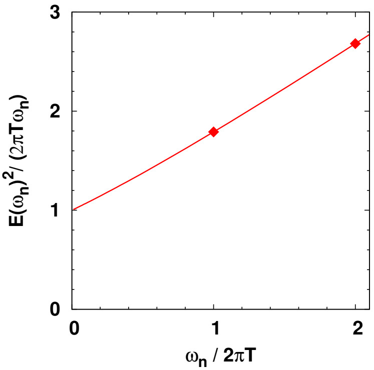
In the vector channel, we have to solve starting from
[Kovtun, Starinets hep-th/0506184]

$$\mathcal{E}_z''(u) + \frac{\hat{\omega}^2 f'(u)}{(\hat{\omega}^2 - \hat{q}^2 f(u))f(u)}\, \mathcal{E}_z'(u) + \frac{\hat{\omega}^2 - \hat{q}^2 f(u)}{u f^2(u)}\, \mathcal{E}_z(u) = 0 \qquad (1)$$

- $\mathcal{E}_z$ can be interpreted as a component of the electric field to which the conserved vector current is coupled
- $\hat{q} = \frac{q}{2\pi T}$, $\hat{\omega} = \frac{\omega}{2\pi T}$
- set $\hat{\omega} \doteq i\hat{\omega}_n$, $\hat{q} \doteq i\hat{E}(\hat{\omega}_n)$ in Eq. (1) and choose the following boundary conditions:
- at $u = 0$ (the boundary), the solution $\mathcal{E}_z(u)$ must be normalizable, hence go to 0;
- choose $\mathcal{E}_z(u)$ regular at the horizon $u = 1$, i.e. $u^{+\hat{\omega}_n/2}$ for $\omega_n > 0$.
- as $\omega_n \to 0$, we observe the behavior $E \sim \sqrt{\omega_n/D}$ with the known result $D = 1/(2\pi T)$ Policastro, Son, Starinets hep-th/0205052.

Longit. Vector Screening masses in AdS/CFT





$$E(\omega_n) \overset{\omega_n \to 0}{\sim} \sqrt{\omega_n / D} \qquad \text{with} \qquad D = 1/(2\pi T).$$

Computational setup for vector screening masses

$$G^{(k_n)}_{\mu\nu}(z) \equiv \int_0^{1/T} \mathrm{d}\tau\, e^{ik_n\tau} \int_{\boldsymbol{x}} \Big\langle V_\mu(\tau,\boldsymbol{x},z)\; V_\nu(0)\Big\rangle_{\mathrm{c}}$$
$$\stackrel{\mu=\nu}{=} \int_0^\infty \frac{\mathrm{d}\omega}{\pi}\, e^{-\omega|z|}\, \rho^{(k_n)}_{\mu\nu}(\omega)\;, \qquad k_n \equiv 2\pi n T.$$

Dimensional reduction:
Keep only the Matsubara zero modes of the SU(3) gauge fields in the covariant derivatives $D_\mu = \partial_\mu - igA_\mu$.

With $\psi = \frac{1}{\sqrt{T}}\begin{pmatrix}\chi\\ \phi\end{pmatrix}$, leads to (at treelevel, in a certain repres. of the γ_μ's)

$$S_0 = \sum_{\{p_n\}} \int_{\boldsymbol{x},z} \Bigg[i\chi^\dagger_{p_n}\Big(p_n - gA_0 + D_3 - \frac{D_iD_i + i\sigma_3\epsilon_{ij}D_iD_j}{2p_n}\Big)\chi_{p_n}$$
$$+ i\phi^\dagger_{p_n}\Big(p_n - gA_0 - D_3 - \frac{D_iD_i + i\sigma_3\epsilon_{ij}D_iD_j}{2p_n}\Big)\phi_{p_n} + \mathrm{O}\Big(\frac{1}{p_n^2}\Big)\Bigg].$$

△ Free propagators:

$$\langle\chi_{p_n}(z_1)\chi^\dagger_{p_n}(z_2)\rangle \simeq \int_{p_3} e^{ip_3(z_1-z_2)}\frac{-i}{p_n+ip_3}\,,$$
$$\langle\phi_{p_n}(z_1)\phi^\dagger_{p_n}(z_2)\rangle \simeq \int_{p_3} e^{ip_3(z_1-z_2)}\frac{-i}{p_n-ip_3}\,.$$

Forward-propagating mesons are $\phi^\dagger_{p_n}\chi_{p'_n}$ and $\phi^\dagger_{p_n}\phi_{-p'_n}$ with $p_n, p'_n > 0$.

△ For instance, consider for $k_n > 0$

$$V_0^{(k_n)} = \sum_{0<p_n<k_n}\left(\chi^\dagger_{p_n}\chi_{p_n-k_n} + \phi^\dagger_{p_n}\phi_{p_n-k_n}\right).$$

△ For given p_n, let $w(z,\boldsymbol{y})$ be the screening correlator of a local current and a point-split current with separation $\boldsymbol{y}$ at the sink in the EFT. For $z > 0$,

$$(\partial_z + \hat{H}^+)w(z,\boldsymbol{y}) = 0\,, \qquad w(0,\boldsymbol{y}) = \delta^{(2)}(\boldsymbol{y}),$$
$$\hat{H}^+ \equiv M_{\mathsf{cm}} - \frac{\nabla^2}{2M_{\mathsf{r}}} + V^+,$$
$$V^+_{\mathsf{LO}}(\boldsymbol{y}) = \frac{g_E^2 C_F}{2\pi}\left[\ln\left(\frac{m_E y}{2}\right) + \gamma_{\mathrm{E}} + K_0(m_E y)\right].$$