Charm quark equilibration in hot QCD

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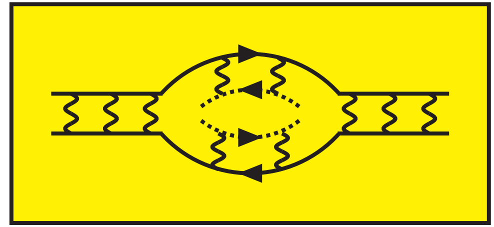
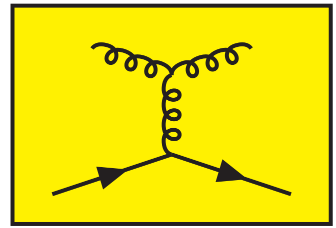
What is it?

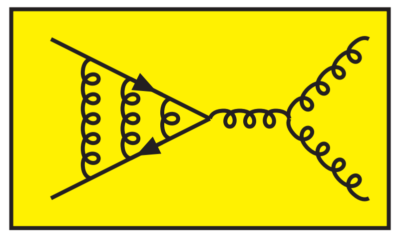
Melting /
recombination:



Kinetic equilibration:
(both bound and open charm)



Chemical equilibration:
(in either direction;
dominated by open charm)

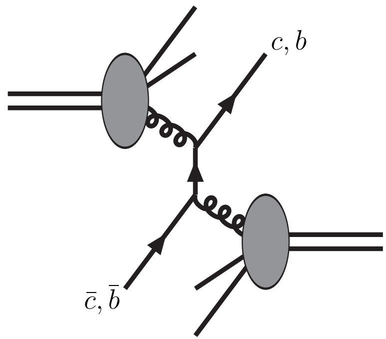


Should charm be included in the equation of state?

What is the current understanding?

(i) Initial production

Initial state is out-of-equilibrium, with a non-thermal abundance of heavy quarks with hard momenta:[1]



If nothing happens afterwards, heavy quarks and antiquarks constitute separate conserved charges.[2]

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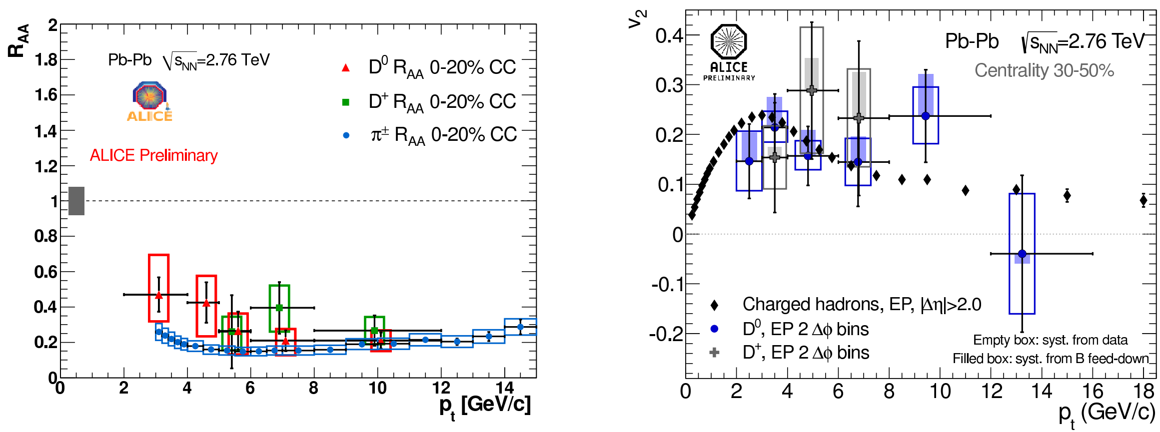
[1] e.g. M. Cacciari *et al*, Phys. Rev. Lett. 95 (2005) 122001 [hep-ph/0502203].

[2] e.g. A. Andronic *et al*, Nucl. Phys. A 789 (2007) 334 [nucl-th/0611023].

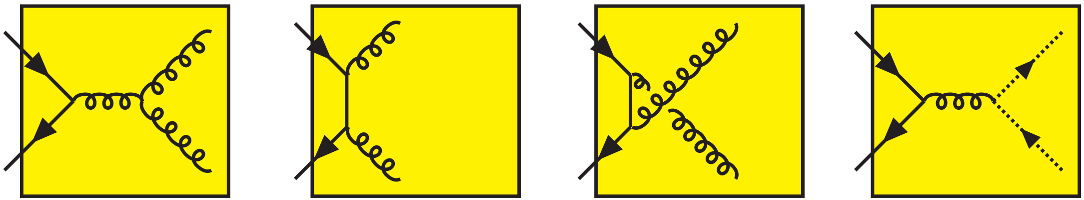
(ii) Kinetic equilibration

Charm (and even bottom) do equilibrate kinetically: jets get quenched,[3] quarks adjust their velocities to hydrodynamic flow.[4]



3 e.g. A. Dainese [ALICE Collaboration], 1106.4042.
4 e.g. G. Ortona [ALICE Collaboration], 1207.7239.

(iii) Chemical equilibration: how fast does pair creation or annihilation take place?



The computation is in principle the same as for strangeness,[5] and near equilibrium the answer can be expressed as:

$$\Gamma_{\text{chem}} = \frac{g^4 C_F}{8\pi M^2}\left(N_{\rm f} + 2C_F - \frac{N_{\rm c}}{2}\right)\left(\frac{TM}{2\pi}\right)^{\frac{3}{2}} e^{-M/T} \;.$$

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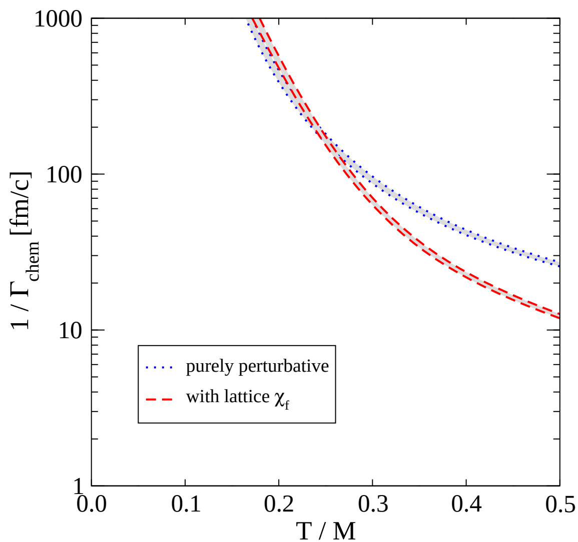
[5] T.S. Biró and J. Zimányi, Phys. Lett. B 113 (1982) 6; J. Rafelski and B. Müller, Phys. Rev. Lett. 48 (1982) 1066 [Erratum-ibid. 56 (1986) 2334]; T. Matsui, B. Svetitsky and L.D. McLerran, Phys. Rev. D 34 (1986) 783 [Erratum-ibid. D 37 (1988) 844].

Numerical estimates: [6]

$$\Gamma_{\rm chem} \;\simeq\; \frac{2\pi\alpha_s^2 T^3}{9M^2}\left(\frac{7}{6}+N_{\rm f}\right)\frac{\chi_f}{\chi_0}\;,$$

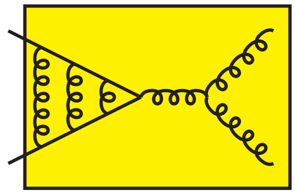


[6] ML and K. Sohrabi, *Charm contribution to bulk viscosity*, 1410.6583.

Issues with perturbation theory (staying non-relativistic)

Sommerfeld effect (i)

Pair-annihilating particles have strong "initial state" interactions; pair-created particles have strong "final state" interactions.



The methods have been elucidated in cosmology, where the "Sommerfeld effect" may also play an important role.[7]

[7] J. Hisano, S. Matsumoto, M. Nagai, O. Saito and M. Senami, *Non-perturbative effect on thermal relic abundance of dark matter,* Phys. Lett. B 646 (2007) 34 [hep-ph/0610249]; J.L. Feng, M. Kaplinghat and H.-B. Yu, *Sommerfeld Enhancements for Thermal Relic Dark Matter,* Phys. Rev. D 82 (2010) 083525 [1005.4678]; A. Hryczuk, R. Iengo and P. Ullio, *Relic densities including Sommerfeld enhancements in the MSSM,* JHEP 03 (2011) 069 [1010.2172]; A. Strumia, *Sommerfeld corrections to type-II and III leptogenesis,* Nucl. Phys. B 809 (2009) 308 [0806.1630].

Sommerfeld effect (ii)

Consider two heavy particles of mass M, interacting through an attractive Coulomb-like potential

$$V(r) = -\frac{g^2 C_{\rm F}}{4\pi r} \; ,$$

where $r = |\mathbf{r}_1 - \mathbf{r}_2|$ is the relative distance. Recalling that the reduced mass is $M/2$, and denoting by v the velocity with respect to the center-of-mass frame ($v = v_{\rm rel}/2$), the stationary Schrödinger equation takes the form

$$\left(-\frac{\nabla^2}{M} + V(r)\right)\psi = M v^2 \psi \; .$$

The probability that the two particles meet, allowing them to co-annihilate, is proportional to $|\psi|^2(0)$.

Sommerfeld effect (iii)

Now, we could first solve the problem with free particles, obtaining a plane-wave solution, and an r-independent $|\psi|^2_{(g^0)}$.

However, because of the attractive force, there is an increased probability for the particles to meet.

This increase constitutes the **Sommerfeld effect**, and is characterized by the coefficient

$$S_1 \equiv \frac{|\psi|^2_{(g^2)}(0)}{|\psi|^2_{(g^0)}(0)} \; .$$

[This can be defined separately for s-wave, p-wave, ...]

Sommerfeld effect (iv)

Remarkably, the value of S_1 can be determined in closed form for the s-wave case:[8]

$$S_1 = \frac{X_1}{1 - e^{-X_1}} \; , \quad X_1 = \frac{g^2 C_\mathrm{F}}{4v} \; .$$

If we then consider a thermal environment, the factor needs to be averaged over the thermal ensemble:

$$\bar{S}_1 \equiv \frac{4}{\sqrt{\pi}} \left(\frac{M}{T} \right)^{3/2} \int_0^\infty \mathrm{d}v \, v^2 e^{-Mv^2/T} \, S_1 \; .$$

8 L.D. Landau and E.M. Lifshitz, *Quantum Mechanics, Non-Relativistic Theory,* Third Edition, §136; V. Fadin, V. Khoze and T. Sjöstrand, *On the threshold behavior of heavy top production,* Z. Phys. C 48 (1990) 613.

Sommerfeld effect (v)

As it happens, in pQCD the process splits up into two parts, the "colour-singlet" discussed here as well as a "colour-octet" one,[9] in which case the interaction is repulsive, and $\bar{S}_8 < 1$.

$$\Gamma_{\text{chem}} = \frac{g^4 C_{\rm F}}{8\pi M^2} \left(\frac{MT}{2\pi}\right)^{3/2} e^{-M/T} \times \left[\frac{1}{N_{\rm c}} \bar{S}_1 + \left(\frac{N_{\rm c}^2 - 4}{2N_{\rm c}} + N_{\rm f} \right) \bar{S}_8 \right] .$$

The colour-octet channel is weighted more than the colour-singlet channel (with $\bar{S}_1 \simeq 3.4$). So, accidentally, the numerical effect on charm equilibration in QCD is small.

[9] Virtuality $\sim MT \gg k^0 \times$ (width for colour decoherence) $\sim M \times g^2 T/\pi$.

Beyond perturbation theory?

Recall scales:

Extent of imaginary time coordinate: $\frac{1}{T}$.

Expected physical time scales:

$$\frac{1}{\Gamma_{\text{kin}}} \sim \frac{M}{T^2} \gg \frac{1}{T}.$$

$$\frac{1}{\Gamma_{\text{chem}}} \sim \frac{M^{1/2}}{T^{3/2}}\, e^{M/T} \gg \frac{1}{T}.$$

So *even if* managed to shift away the exponential factor, the dynamical time scale is still much larger than the lattice extent, and naive Wick rotation is insufficient.

The ideal theoretical probe for charm is the trace anomaly.[10]

$$T^{\mu}{}_{\mu} = \underbrace{c_\theta\, g_\mathrm{B}^2 F^{a\mu\nu} F^a_{\mu\nu}}_{\equiv\,\theta} + \underbrace{\bar\psi M_\mathrm{B} \psi}_{\equiv\, S}\,, \quad c_\theta = -\frac{b_0}{2} - \frac{b_1 g^2}{4} + \ldots\ .$$

The contribution from S should be small (i) in the chiral limit $M \ll T$, and (ii) for $M \gg T$ when the charm decouples.

Is the relevant comparison $M \leftrightarrow T$, $M \leftrightarrow 3T$, $M \leftrightarrow 2\pi T$, and which mass to use for M (pole, $\overline{\mathrm{MS}}$, D^0)?

[10] Y. Burnier and ML, *Charm mass effects in bulk channel correlations*, JHEP 11 (2013) 012 [1309.1573].

Physics is in the 2-point correlator.

Trace of the energy-momentum tensor yields "bulk viscosity":

$$\zeta \;=\; \frac{1}{9}\lim_{\omega\to 0^+}\left\{\frac{1}{\omega}\int_{\mathcal{X}} e^{i\omega t}\left\langle \frac{1}{2}\left[T^{\mu}{}_{\mu}(\mathcal{X}),T^{\mu}{}_{\mu}(0)\right]\right\rangle_T\right\}\,.$$

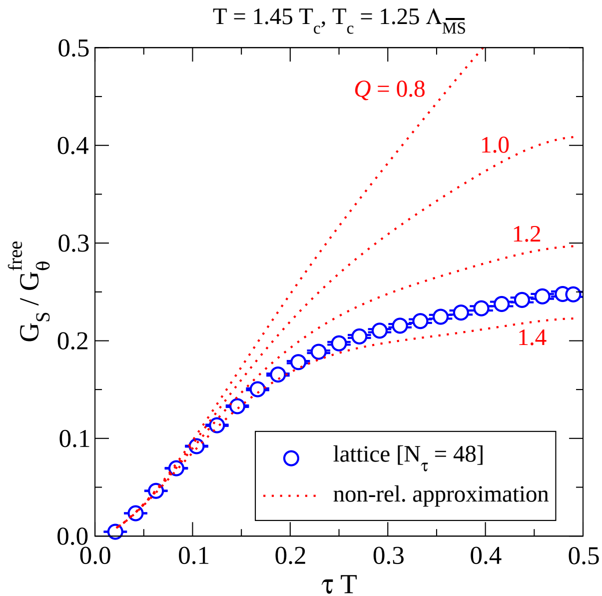
Heavy-quark contribution:

$$\delta\zeta \;=\; \frac{1}{18T}\lim_{\omega\to 0^+}\left\{\frac{2M^2\chi_f\Gamma_{\rm chem}}{\omega^2+\Gamma^2_{\rm chem}}\right\} \;=\; \frac{M^2\chi_f}{9T\Gamma_{\rm chem}}\,.$$

Measure:

$$G_{\rm S}(\tau) = \left\langle \int_{\mathbf{x}} S(\tau,\mathbf{x})\,S(0)\right\rangle_T\,.$$

Current status[11]



⇒ Here charm has 25-30% influence even at $T \sim 300$ MeV.

[11] H.-T. Ding *et al*, 1204.4945 (quenched). In the simulations, $m_c(\bar{\mu}_{\text{ref}}) \approx 0.97$ GeV. In the plot, $Q \sim M/m_c(\bar{\mu}_{\text{ref}}) = 1 + 4g^2(\bar{\mu}_{\text{ref}})C_F/(4\pi)^2 + \mathcal{O}(g^4) \simeq 1.2$.