

# Non-Relativistic QCD at $T > 0$

Seyong Kim

Sejong University

# Outline

- 1 Systematics
- 2  $\bar{b}b$  and  $\bar{c}c$  at  $T > 0$
- 3 Discussion

# Systematics of lattice NRQCD at $T = 0$

- $O(v^2)$  :  $\sim 0.3$  for  $J/\psi$  and  $\sim 0.1$  for  $\Upsilon$
- $O(a^2 p^2)$  and  $O(aK)$
- radiative corrections for the coefficients in NRQCD lagrangian
- finite volume effect
- vacuum polarization

(cf. G.P. Lepage et al, Phys. Rev. D46 (1992) 4052 )

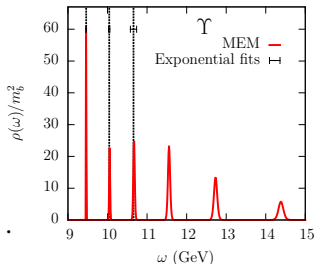
Systematics of lattice NRQCD at  $T = 0$ 

$$\begin{aligned}
 G_\Lambda(\tau) &= \sum_{\vec{x}} \langle \bar{\psi}(\tau, \vec{x}) \Lambda \psi(\tau, \vec{x}) \bar{\psi}(0, \vec{0}) \Lambda \psi(0, \vec{0}) \rangle \\
 &= \int \frac{d^3 p}{(2\pi)^3} \int_0^\infty \frac{d\omega}{2\pi} K(\tau, \omega) \rho_\Lambda(\omega, \vec{p}) \quad (1)
 \end{aligned}$$

$$G(\tau) = \sum_n e^{-E_n \tau} |\langle 0 | \phi(0) | n \rangle|^2 \quad (2)$$

- if the states are well defined stationary states,

$$\rightarrow G(\tau) \sim a_0 e^{-E_0 \tau} + a_1 e^{-E_1 \tau} + a_2 e^{-E_2 \tau} + \dots \quad (3)$$



Systematics of lattice NRQCD at  $T > 0$ 

- expect no new UV complication
- temperature,  $T = \frac{1}{N_\tau a_\tau}$

- for consistent lattice NRQCD,

$$Ma_\tau \sim 1 \rightarrow N_\tau \sim \frac{M}{T}$$

- for  $M = M_b (\sim 4.5\text{GeV})$  and  $T \sim 2T_c (\sim 2 \times 150\text{MeV})$

$$N_\tau = O(10)$$

- at  $T > 0$ , spectral function is more complex while available lattice data points is smaller ( $\rightarrow$  reliability of the method is important)

Systematics of lattice NRQCD at  $T > 0$ 

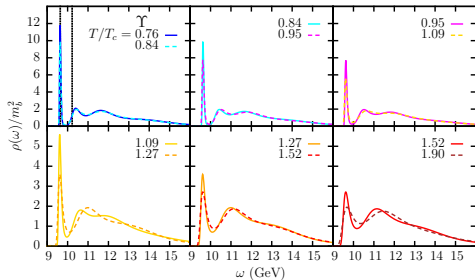
$$G_{\Lambda}(\tau) = \sum_{\vec{x}} \langle \bar{\Psi}(\tau, \vec{x}) \Lambda \Psi(\tau, \vec{x}) \bar{\Psi}(0, \vec{0}) \Lambda \Psi(0, \vec{0}) \rangle$$

$$= \int \frac{d^3 p}{(2\pi)^3} \int_0^{\infty} \frac{d\omega}{2\pi} K(\tau, \omega) \rho_{\Lambda}(\omega, \vec{p}) \quad (4)$$

$$K(\tau, \omega) = \frac{\cosh[\omega(\tau - 1/2T)]}{\sinh(\omega/2T)} \quad (5)$$

In NRQCD,

$$K(\tau, \omega) \sim e^{-\omega\tau} \quad (6)$$



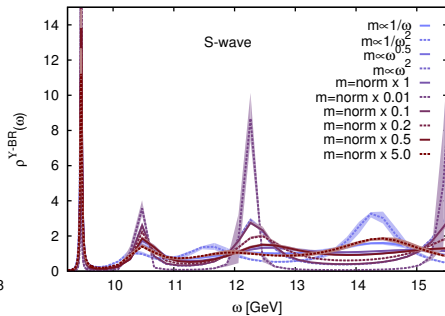
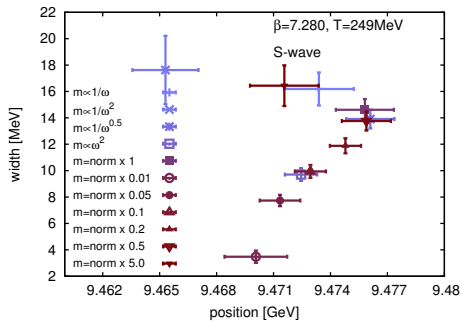
# Reliable reconstruction of the spectral function

Bayesian method is essential and sources of systematics are

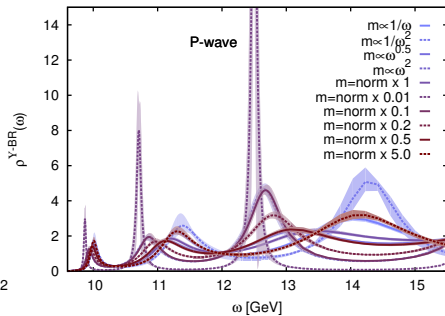
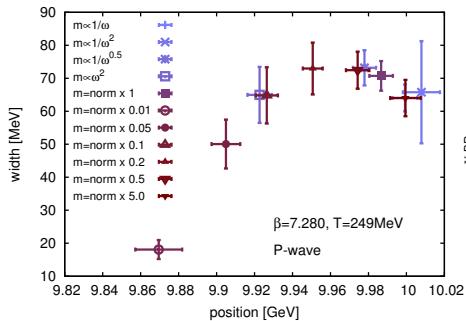
- $\omega_{\min}, \omega_{\max}$  range
- $\tau$  range dependency
- default model dependency
- statistical error dependency
- comparison with free NRQCD spectral function
- high momentum stability

(cf. SK, P. Petreczky and AR, arxiv:1409.3630)

# Reliable reconstruction of the spectral function



# Reliable reconstruction of the spectral function



## charmonium and bottomonium using HotQCD configurations

$\beta$	T	$T/T_c$	$a(\text{fm})$	$u_0$	$M_{ba}$	$M_{ca}$
6.664	140	0.911	0.117	0.87025	2.759	0.7566
6.700	145	0.944	0.113	0.87151	2.667	0.7314
6.740	151	0.980	0.109	0.87288	2.566	0.7035
6.770	155	1.01	0.106	0.87388	2.495	0.6841
6.800	160	1.04	0.103	0.87485	2.424	0.6657
6.840	166	1.08	0.0989	0.87612	2.335	0.6403
6.880	172	1.12	0.0953	0.87736	2.249	0.6167
6.910	177	1.15	0.0926	0.87827	2.187	0.5996
6.950	184	1.19	0.0893	0.87945	2.107	0.5776
6.990	191	1.24	0.086	0.88060	2.030	0.5566
7.030	198	1.29	0.0829	0.88173	1.956	0.5364
7.100	211	1.37	0.0777	0.88363	1.835	0.5030
7.150	221	1.44	0.0743	0.88493	1.753	0.4806
7.280	249	1.61	0.0660	0.88817	1.559	0.4274

Table: summary for  $N_f = 2 + 1$  HotQCD  $48^3 \times 12$  lattice (A.Bazavov et al, PRD85 (2012) 054503)

## charmonium and bottomonium using HotQCD configurations

- is  $Ma \sim 0.4$  feasible for NRQCD?

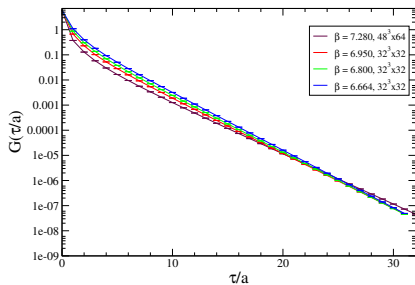
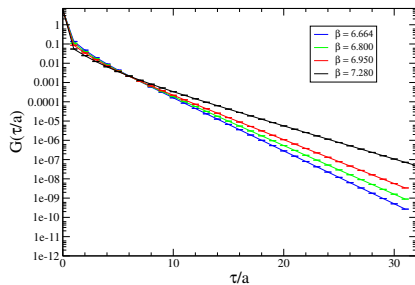
radiative correction (perturbatively  $u_0 = 1 - 0.083g^2 + \dots$ )

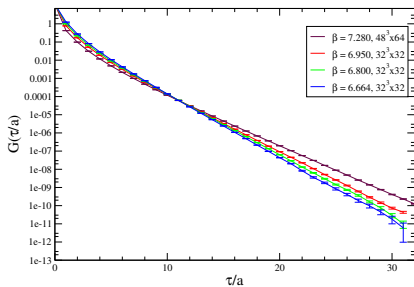
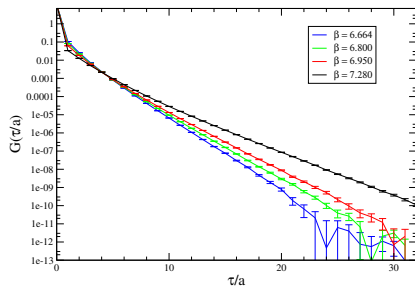
relativistic correction ( $v^2 \sim 0.3$ )

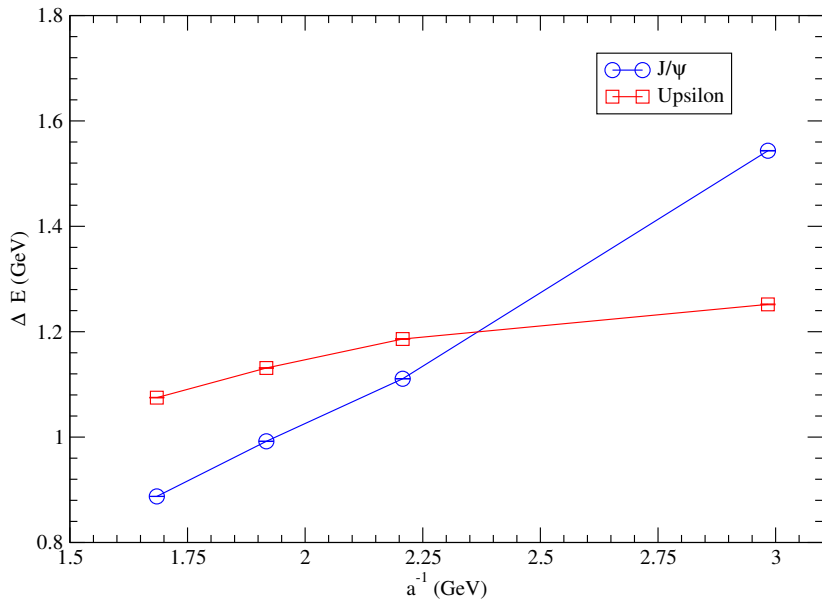
- melting temperature for  $J/\psi$  is lower than that for  $\Upsilon$ . Can we see the melting for charmonium system?

$T = 0$  comparison

Upsilon channel

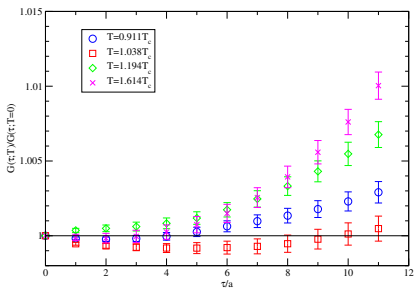
 $J/\psi$  channel

$T = 0$  comparison $\chi_{b1}$  channel $\chi_{c1}$  channel

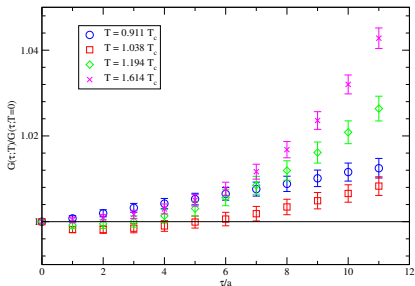
$T = 0$  comparison

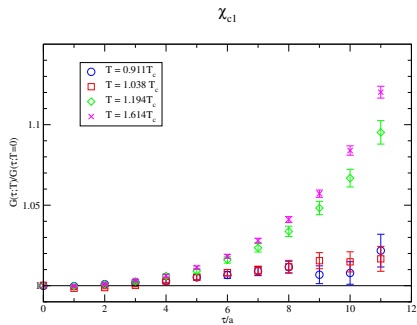
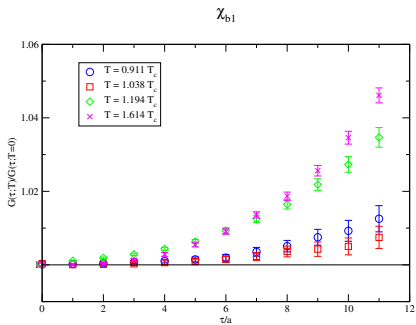
$T \neq 0$  comparison

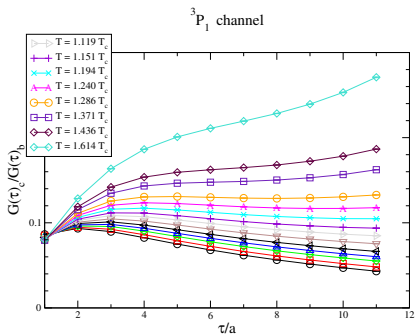
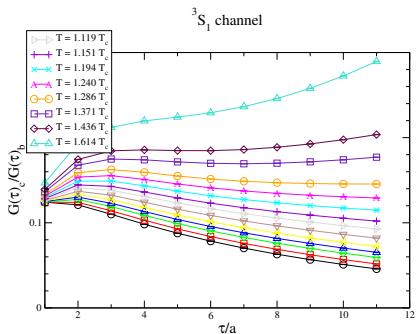
Upsilon



J/psi



$T \neq 0$  comparison

$T \neq 0$  comparison

# Conclusion

- Bayesian reconstruction of spectral function + NRQCD offers a systematic approach to the study of quarkonium at non-zero temperature.
- Most sources of the systematic errors associated with Bayesian reconstruction are under control.
- Despite the large relativistic effect, a tentative investigation on charmonium at non-zero temperature is under way.
- Larger temperature effect is seen for charmonium system than bottomonium system