Overview of Heavy Quark Spectroscopy from a Lattice Perspective

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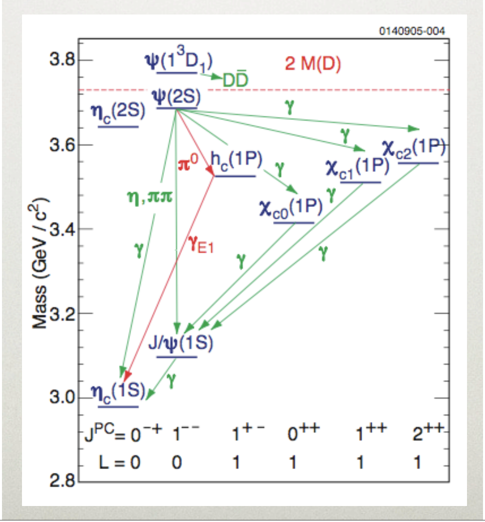
Purpose of this talk

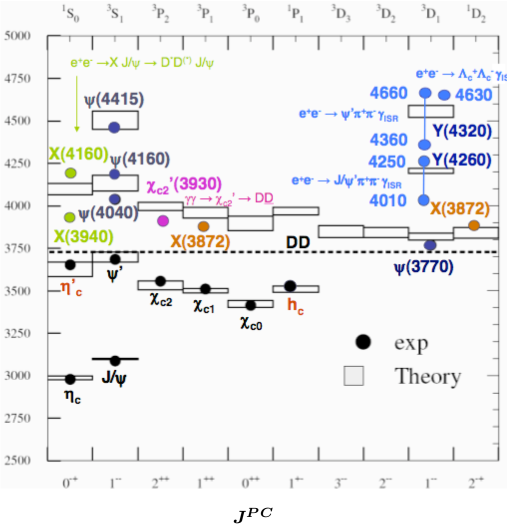
- Provide an overview of $T = 0$ heavy-quark spectroscopy
- Review current lattice methods: possibilities and limitations
- Not a review of results
- See talks by Prelovsek, Thomas
- First, the challenge

Charmonium spectroscopy before the B-factories



Mariana Nielsen (CHARM 2010)

Charmonium spectroscopy after the B-factories



Mariana Nielsen (CHARM 2010)

Theoretical challenges

- Test of our ability/methodology for solving QCD.
- Precision spectroscopy of the low levels.
- Characterize and predict states.
  - Spin exotics: (J^{PC} inaccessible through $\bar{Q}Q$).
  - Hybrids: $\bar{Q}Q + glue$
  - Tetraquarks: $\bar{Q}Q\bar{q}q$ as in the Z_c^+ (?)
  - Molecules: deuteron-like $\bar{D} - D$

Lattice QCD

- Our only *ab initio* method
- Limitations
  - Including multihadronic states is difficult.
  - Real-time behavior is only indirectly accessible (see next talk)
- To extend its reach: supplement with models, effective field theory.

Lattice Spectroscopy Methods

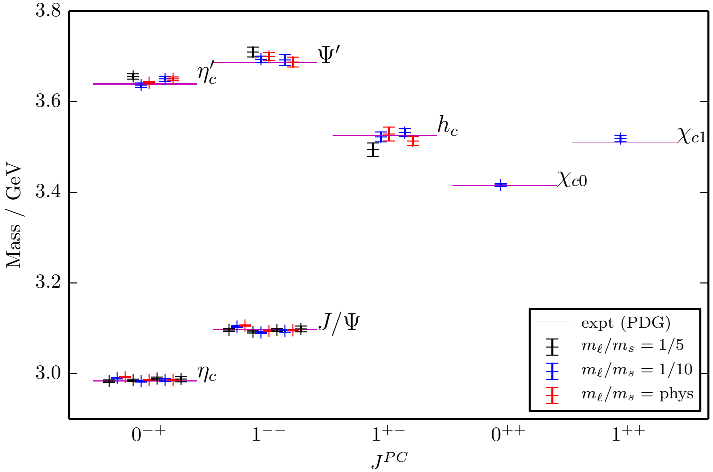
- Hadronic correlator. Hermitian interpolating operators $\mathcal{O}_i$.

$$C_{ij}(t) = \langle 0|\mathcal{O}_i(t)\mathcal{O}_j^\dagger(0)|0\rangle$$

- Spectral decomposition

$$C_{ij}(t) = \sum_n \langle 0|\mathcal{O}_i(0)|n\rangle e^{-E_n t} \langle n|\mathcal{O}_j^\dagger(0)|0\rangle$$

Low lying charmonium spectrum



- HPQCD Galloway *et al.* [1411.1318]
- Two lattice spacings, three sea-quark mass sets, two smearings
- Bayesian multiexponential fit (up to 9 exponentials)

Variational Method

- Spectral decomposition

$$C_{ij}(t) = \sum_n \langle 0|\mathcal{O}_i(0)|n\rangle e^{-E_n t} \langle n|\mathcal{O}_j^\dagger(0)|0\rangle$$

- Matrix form

$$C(t) = Z T^t Z^\dagger \,,$$

- Effective transfer matrix and overlap matrix

$$T = \mathrm{diag}\exp(-E_n) \quad Z_{i,n} = \langle 0|\mathcal{O}_i(0)|n\rangle$$

- Choose a large basis set of interpolating operators.
- Solve the generalized eigenvalue problem (reference time t_0)

$$C(t)u_n = \lambda_n(t,t_0)C(t_0)u_n$$

- If we have N linearly independent interpolating operators, and exactly N energy levels, we get, exactly,

$$\lambda_n(t,t_0) = \exp[-E_n(t-t_0)].$$

- Otherwise, there are corrections that die exponentially as $t_0 > t - t_0 \to \infty$.
- Low-lying levels are more reliable, higher levels are difficult.

Example for low-lying charmonium

Bilinear $\bar{c}\mathrm{Operator}c$. Here operators for the T_1^{PC} irrep are shown. In the notation below, ∇_i generates a discrete covariant difference in direction i, $\mathbb{D}_k = |\varepsilon_{ijk}|\nabla_i\nabla_j$, and $\mathbb{B}_i = \varepsilon_{ijk}\nabla_i\nabla_j$.

T_1^{--}	T_1^{+-}	T_1^{-+}	T_1^{++}
γ_i	$\gamma_4\gamma_5\gamma_i$	$\gamma_4\nabla_i$	$\gamma_5\gamma_i$
$\gamma_4\gamma_i$	$\gamma_5\nabla_i$	$\varepsilon_{ijk}\gamma_4\gamma_5\gamma_j\nabla_k$	$\varepsilon_{ijk}\gamma_j\nabla_k$
∇_i	$\gamma_4\gamma_5\nabla_i$	$\varepsilon_{ijk}\gamma_j\mathbb{B}_k$	$\varepsilon_{ijk}\gamma_4\gamma_j\nabla_k$
$\varepsilon_{ijk}\gamma_5\gamma_j\nabla_k$	$\lvert\varepsilon_{ijk}\rvert\gamma_4\gamma_5\gamma_j\mathbb{D}_k$	$\varepsilon_{ijk}\gamma_4\gamma_j\mathbb{B}_k$	$\lvert\varepsilon_{ijk}\rvert\gamma_5\gamma_j\mathbb{D}_k$
$\lvert\varepsilon_{ijk}\rvert\gamma_j\mathbb{D}_k$	$\mathbb{B}_i$		$\gamma_4\mathbb{B}_i$
$\lvert\varepsilon_{ijk}\rvert\gamma_4\gamma_j\mathbb{D}_k$	—		$\varepsilon_{ijk}\gamma_4\gamma_5\gamma_j\mathbb{B}_k$
$\gamma_5\mathbb{B}_i$			
$\gamma_4\gamma_5\mathbb{B}_i$			

Low-lying charmonium: FNAL/MILC result

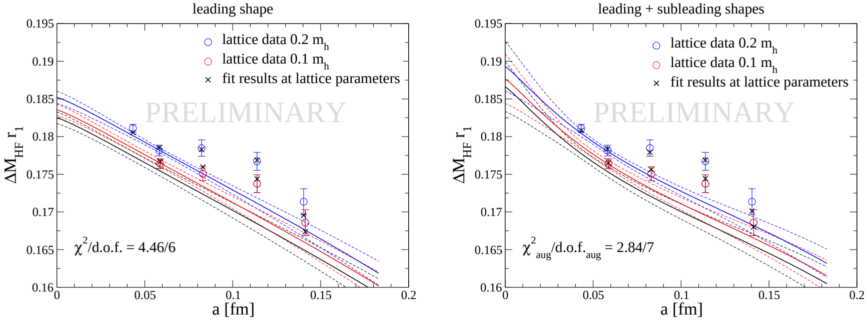
- 5 lattice spacings, 2 sea quark masses. Can take physical limit.
- Spin-averaged 1P and 1S masses:

$$M_{\overline{1P}} = (M_{\chi_{c0}} + 3M_{\chi_{c1}} + 5M_{\chi_{c2}})/9$$
$$M_{\overline{1S}} = (M_{\eta_c} + 3M_{J/\psi})/4$$

- Splittings

$$\text{1S hyperfine} = M_{J/\psi} - M_{\eta_c}$$
$$\text{1P} - \text{1S splitting} = M_{\overline{1P}} - M_{\overline{1S}}$$
$$\text{1P spin} - \text{orbit} = (5M_{\chi_{c2}} - 3M_{\chi_{c1}} - 2M_{\chi_{c0}})/9$$
$$\text{1P tensor} = (3M_{\chi_{c1}} - M_{\chi_{c2}} - 2M_{\chi_{c0}})/9$$
$$\text{1P hyperfine} = \overline{\text{1P}} - M(h_c)$$

Low-lying charmonium: FNAL/MILC result

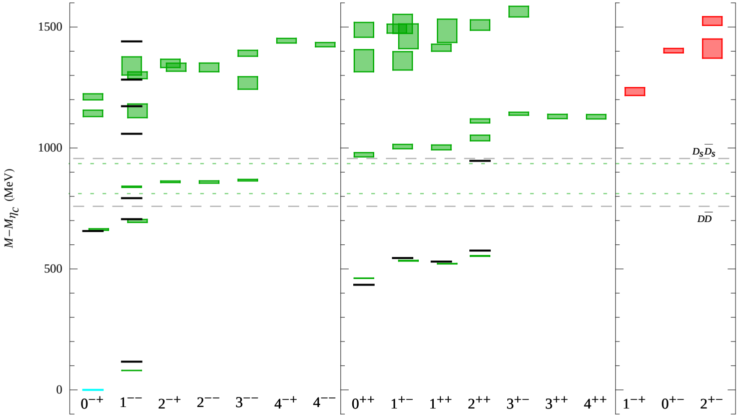


- Mohler, Lattice 2014 [1412.1057]
- Fit to shapes derived from NRQCD power counting.
- Experiment $\Delta M_{HF} r_1 = 0.178$.
- Note width of η_c is 32 MeV!

Low-lying charmonium: FNAL/MILC result

Mass difference	This analysis [MeV]	Experiment [MeV]
1P-1S splitting	457.3 ± 3.6	457.5 ± 0.3
1S hyperfine	$118.1 \pm 2.1^{-1.5}_{-4.0}$	113.2 ± 0.7
1P spin-orbit	49.5 ± 2.5	46.6 ± 0.1
1P tensor	17.3 ± 2.9	16.25 ± 0.07
1P hyperfine	-6.2 ± 4.1	-0.10 ± 0.22

Higher-lying lattice cc spectroscopy



Hadron Spectrum Collaboration 1204.5425 (2012)

Problems

- Not safe to omit open charm (multihadronic states).
- Only one lattice spacing ($a_s = 0.12$ fm, $a_t \approx 0.032$ fm)
- Heavy up/down quarks: 400 MeV pion
- Got charmonium HFS = 80 MeV *vs* 113.2(7) MeV (expt).

Optimized interpolating operators

- Variationally optimized state vector

$$
\begin{aligned}
Z_{i,n} &= \langle 0|\mathcal{O}_i(0)|n\rangle \\
|\Psi_n\rangle &= \sum_i Z^{-1}_{n,i}\mathcal{O}_i\langle 0|
\end{aligned}
$$

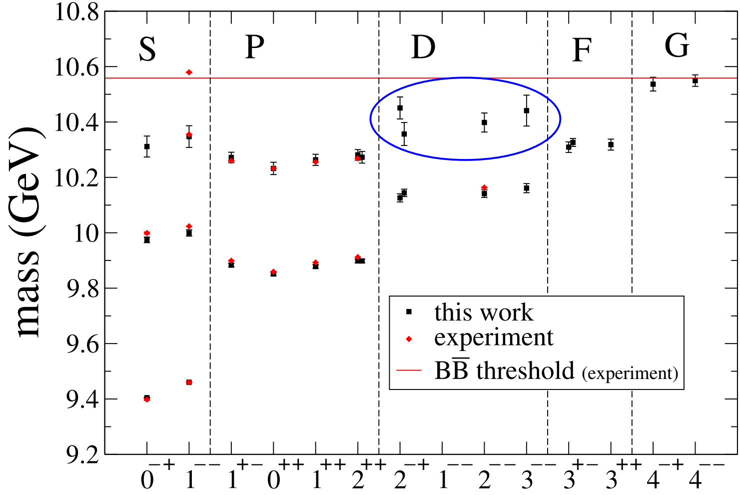
- $|\Psi_n\rangle$ is an approximate eigenstate with eigenvalue E_n.
- Could be helpful.

Optimized interpolating operators

- Customized smearing.
- von Hippel *et al* [1306.1440]: "Free form" interpolating operator
- Gauge invariant form giving complete control over relative wave function
- Replace Gaussian smearing with a more realistic wave function.
- e.g. Mark Wurtz *et al* [1409.7103] for first excited D-wave bottomonium.

$$\psi_{ij}(x) = \sin(2\pi x_i/L)\sin(2\pi x_j/L)(r-b)\exp(r/a)$$

Crafting interpolating operators



Including two-hadron states

- Discrete scattering states at finite volume, center of mass.

$$E_n = 2\sqrt{m^2 + p^2} \quad \mathbf{p} = (2\pi/L)\mathbf{n} \quad \text{(Noninteracting, equal mass, two-body)}$$

- Lüscher method. With interaction, E_n is shifted.
- As long as the box is larger than the size of the interaction region $L/2 > R$, and scattering is only elastic, the shift in energy carries information about the elastic scattering phase shift.
- Replace $p = (2\pi/L)q$ from (noninteracting $q = n$).

$$E = 2\sqrt{m^2 + (2\pi/L)^2 q^2}$$

- Then we get the phase shift at momentum $p = (2\pi/L)q$ from

$$p \cot \delta(p) = \frac{2Z_{00}(1; q^2)}{\sqrt{\pi} L}$$

- This is for S-wave. Generalizations apply to higher angular momentum and moving frames.

Near-threshold bound states

- Partial wave scattering amplitude.

$$T = 1/[p \cot \delta(p) - ip]$$

- Effective range formula

$$p \cot \delta(p) = 1/a_0 + \frac{1}{2} r_0^2 p^2$$

- So use lattice result at discrete p to determine a_0 and r_0^2. Then do analytic continuation.
- Poles in T represent bound states ($a < 0$) or resonances.
- For example, Lang *et al* [1301.7670] use this method to conclude that the D_{s1} is a $D^* K$ bound state and the D_{s0}^* is a DK bound state.
- Also applied to charmonium excitations (Prelovsek talk.)
- For a recent example, Lang *et al.* [1501.01646] [1403.8103]

Perturbative mixing model

- Treat light quark annihilation/production perturbatively

$$H = H_0 + V$$

- Unperturbed states H_0: DD^* and cc
- Mixing term V connects these two sectors.
- Use Euclidean lattice correlators to compute matrix elements of V.
- Compute level shifts in conventional real-time perturbation theory.

Application to X(3872)

- Model: The X(3872) is the result of mixing the $\chi_{c1}(2P)$ with DD^* scattering states.
- Second order energy shift (continuum) of the unperturbed $\chi_{c1}(2P)$:

$$E = E^{(0)}(\chi_{c1}(2P)) + \int d^3p \frac{x(p)^2}{E - E(p)}$$

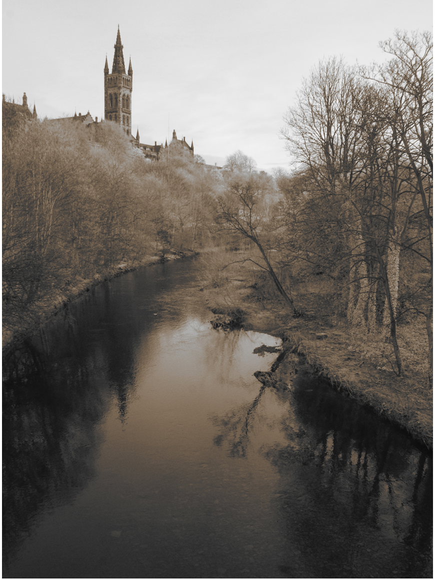
where $x(p)$ is the transition hamiltonian matrix element taking the $\chi_{c1}(2P)$ to the DD^* scattering state with momentum p, and $E(p)$ is the energy of the DD^* scattering state.

- Measure $x(p)$ and $E^{(0)}(\chi_{c1}(2P))$ on the lattice. (Work in progress)

Conclusions

- Lattice QCD is needed to characterize, predict states
- State of the art
  - Multiple interpolating operators, including multihadronic states where appropriate
  - Multiple lattice spacings permitting a continuum extrapolation
  - Ability to reach physical quark masses
  - Sufficiently large volume
  - Careful tuning of the heavy quark masses.
- Variational method
- Elastic scattering phase shift determination
- Effective range approximation for states near threshold
- Lattice-inspired phenomenology

Thank you



University of Glasgow above the River Kelvin, photo by Laurel Casjens