# Bridging the gap from b to c with Lattice QCD

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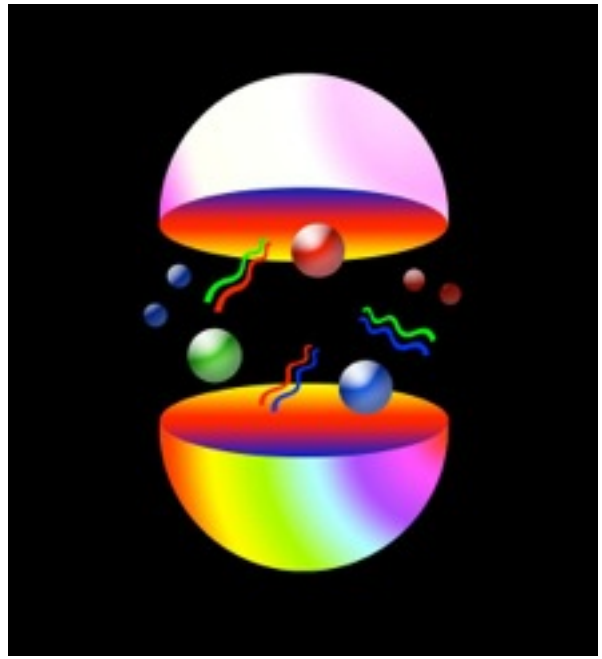
Lattice QCD calculations at T=0 with realistic sea quarks now achieving accurate results for hadron masses and simple decay rates.



C.Davies et al, HPQCD/MILC/FNAL: hep-lat/0304004

? how much of this accuracy can be translated into calculations for T>0 ?

Bottom to charm quark region particularly useful for a QGP 'thermometer'

Key lattice QCD issue for heavy b and c quarks is that of discretisation errors and control of these using understanding of their non relativistic nature.

## HPQCD has developed two very different techniques:

- Improved NonRelativistic QCD - accurate through $\alpha_s v^4$

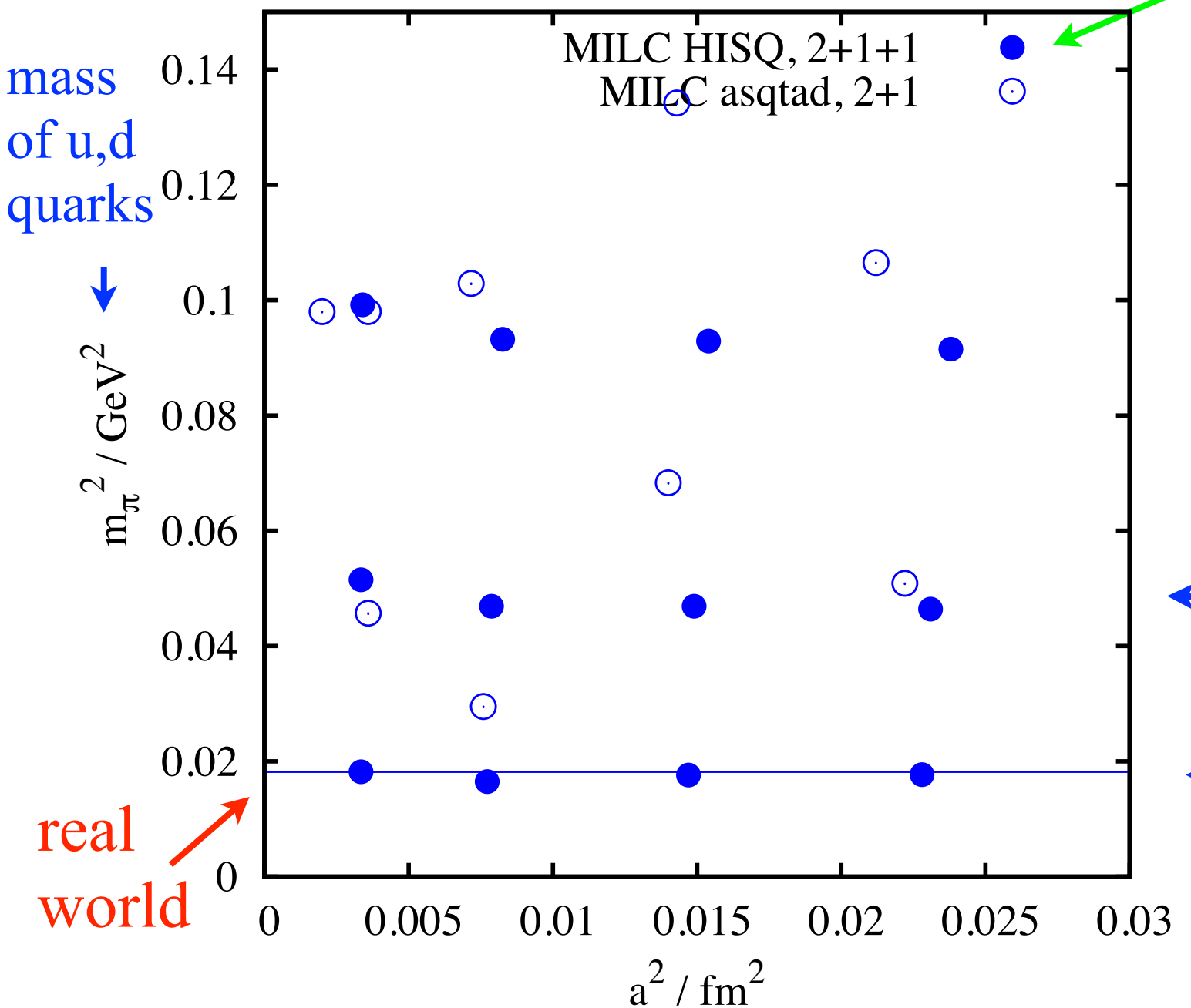
along with discretisation corrections (through $\alpha_s v^2 a^2 p^2$)
am > 1 allows calculations for b quarks on lattices with a=0.15fm - 0.06fm; down to $m_b/2$ on a=0.15fm
$v^2 \sim 0.1$ bottomonium; $v^2 \sim 0.3$ charmonium

- Highly Improved Staggered Quarks (HISQ) is a relativistic action with discretisation errors starting at:

$\alpha_s a^2 \qquad a^4$

Good for u, d, s and c at all lattice spacings. Can reach b on 0.045fm lattices.
Note: most $a^2$ effects are suppressed by powers of v

# Example parameters for calculations now being done with 'staggered' quarks in sea.




2nd generation u,d,s,c in sea Highly Improved Staggered Quarks

E.Follana et al, HPQCD, hep-lat/0610092.

Volume: $m_\pi L > 3$ and ensemble sizes ~1000

$m_{u,d} \approx m_s/10$

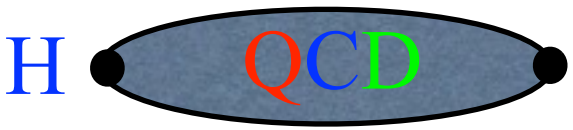
$m_{u,d} \approx m_s/27$

* physical u/d *

MILC:1212.4768

Hadron correlation functions ('2point functions') give masses and decay constants.

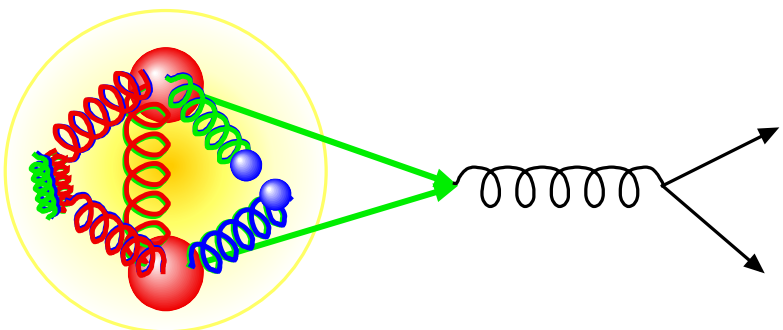
$$\langle 0|H^\dagger(T)H(0)|0\rangle = \sum_n A_n e^{-m_n T} \overset{T \text{ large}}{\rightarrow} A_0 e^{-m_0 T}$$




masses of all hadrons with quantum numbers of H

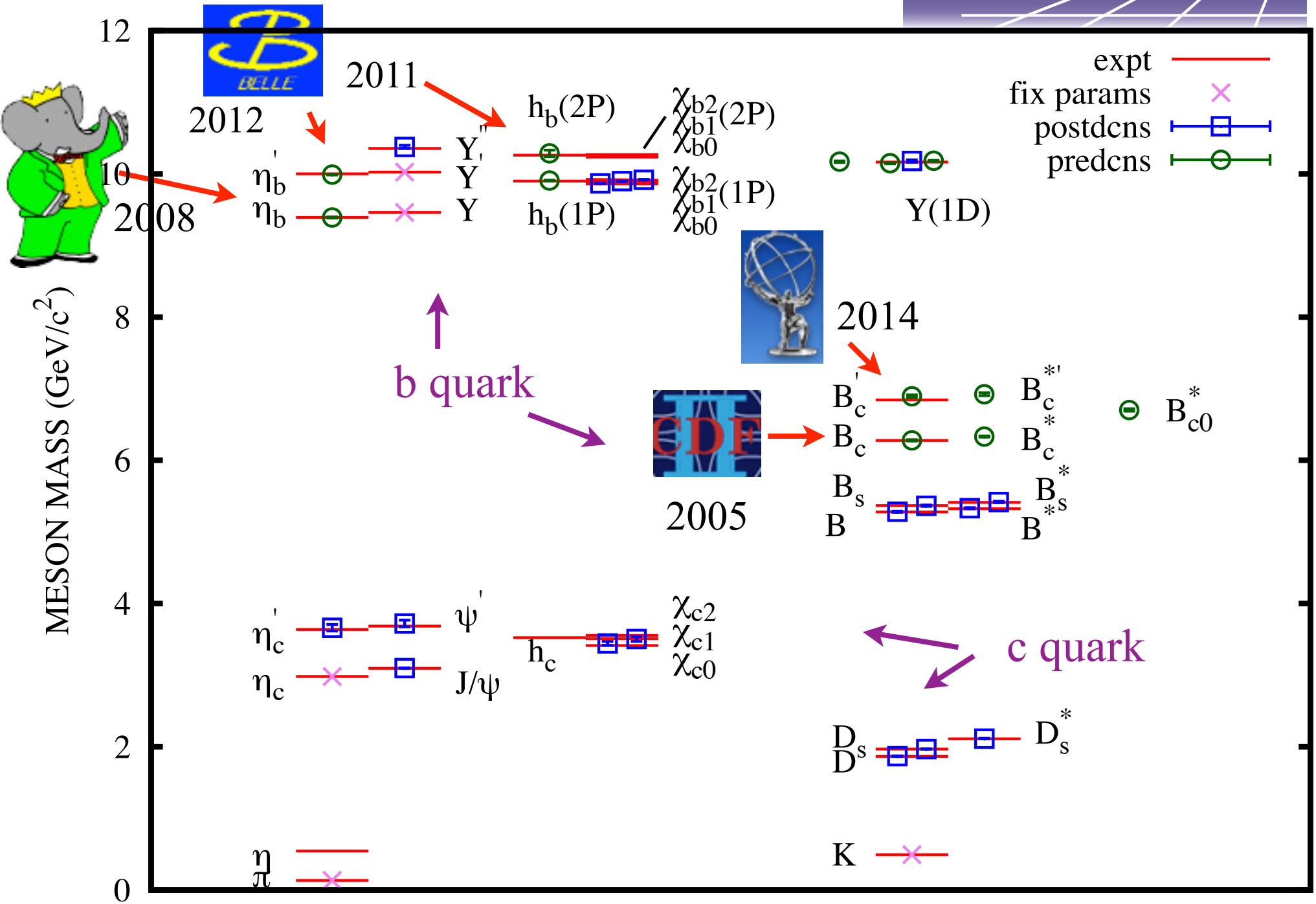
$$A_n = \frac{|\langle 0|H|n\rangle|^2}{2m_n} = \frac{f_n^2 m_n}{2}$$

decay constant parameterises amplitude to annihilate - a property of the meson calculable in QCD. Relate to experimental decay rate. Must normalise H.



Accurate experimental info. for f and m for gold-plated mesons
Compare formalisms to test uncertainties.

# The gold-plated meson spectrum

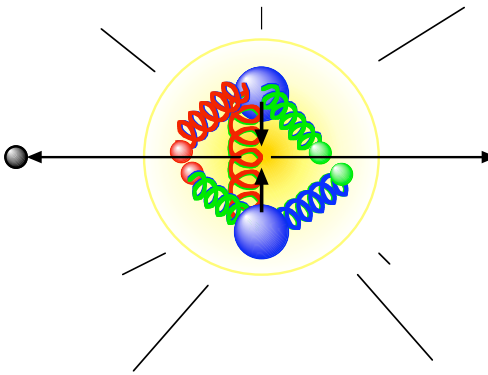


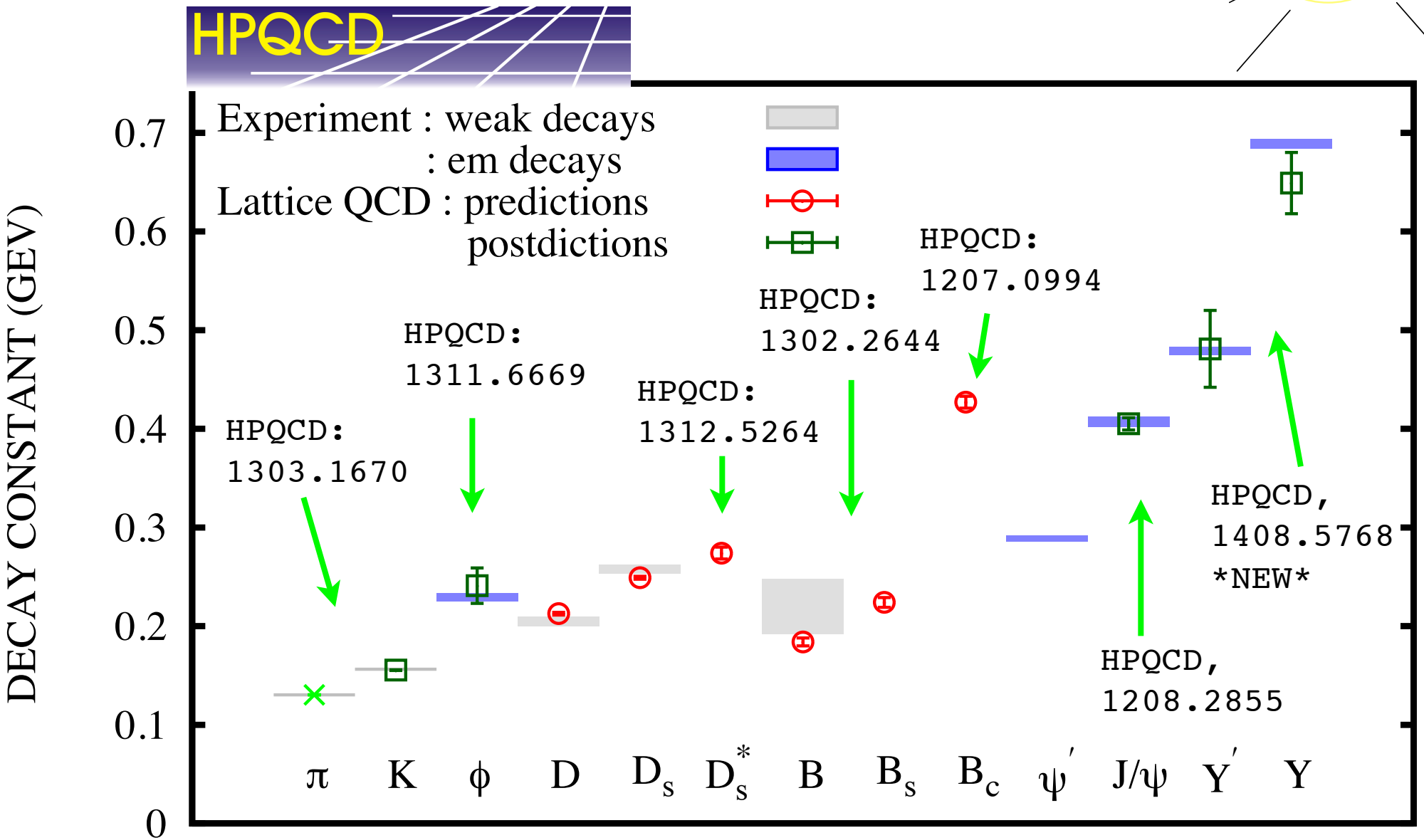

HPQCD recent: 1110.6887, 1112.2590, 1207.5149, 1208.2855, 1302.2644. 1312.5264 …

# Summary of Meson decay constants

Parameterises hadronic information needed for annihilation rate to W or photon:

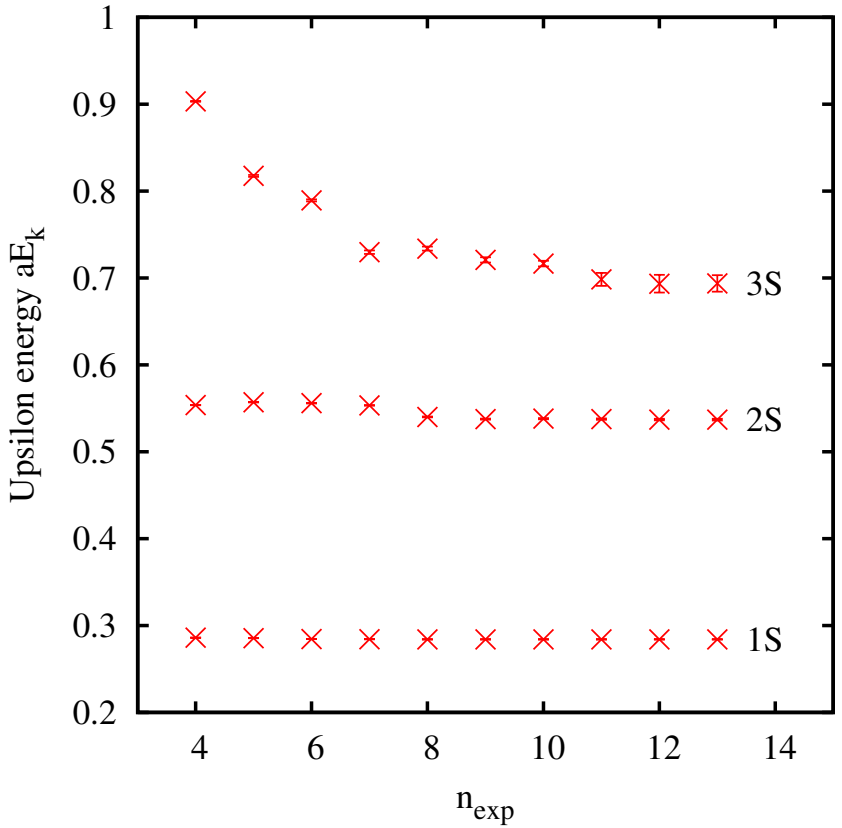
$$\Gamma \propto f^2$$

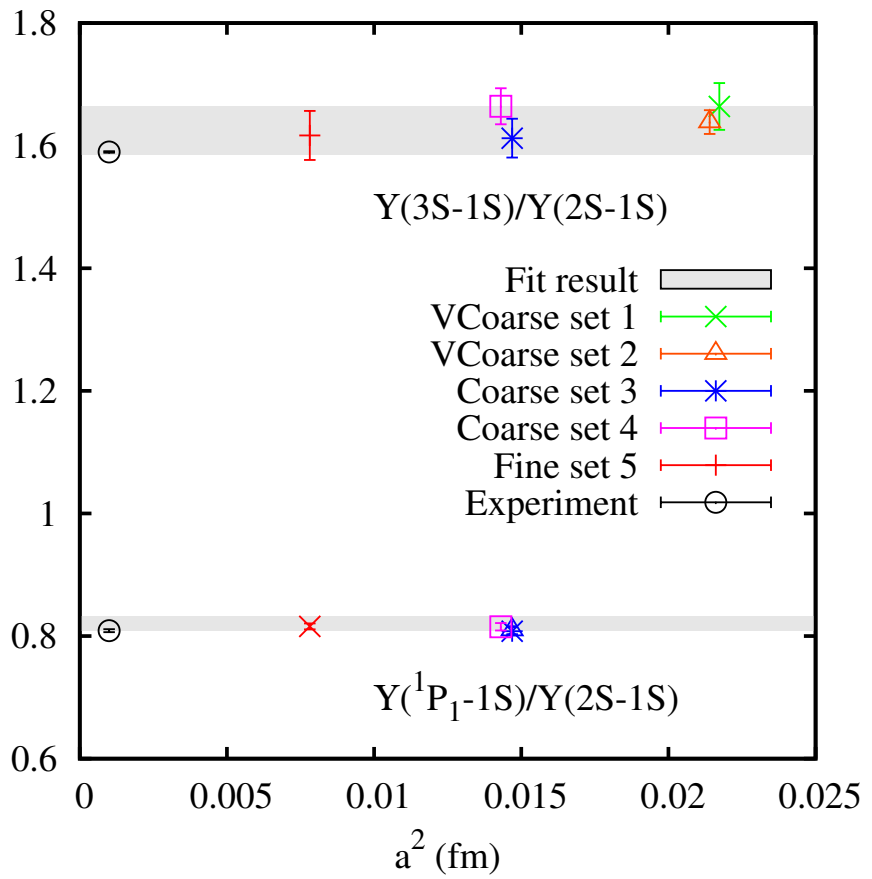





Note range of values for f much smaller than that for m.

# Example results from calculations of masses:

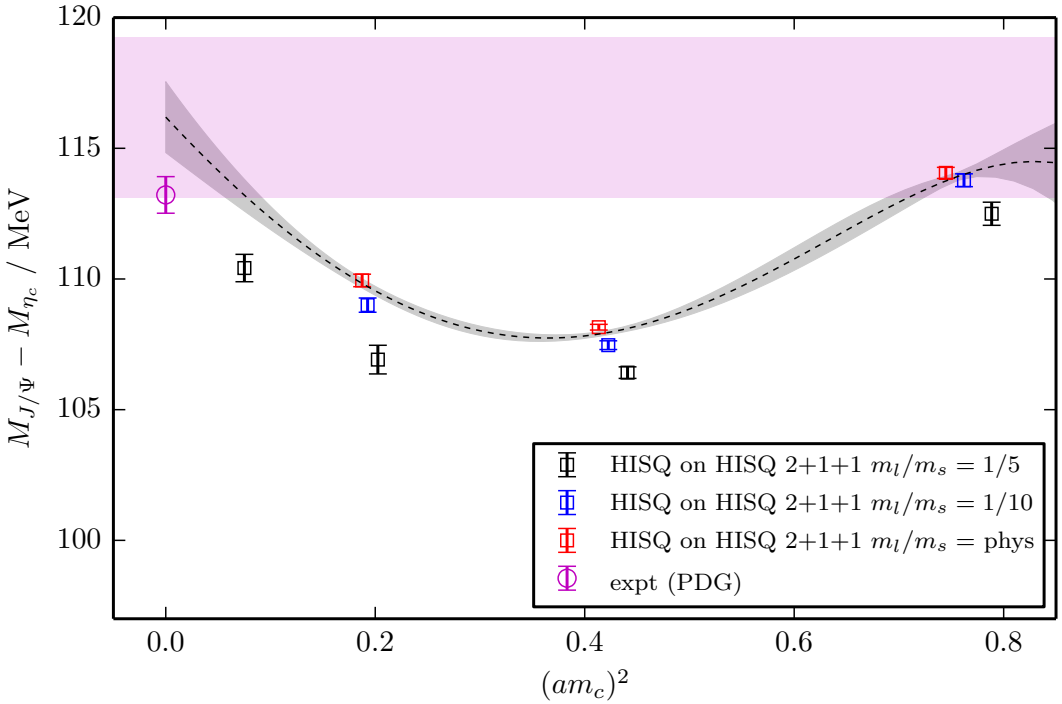


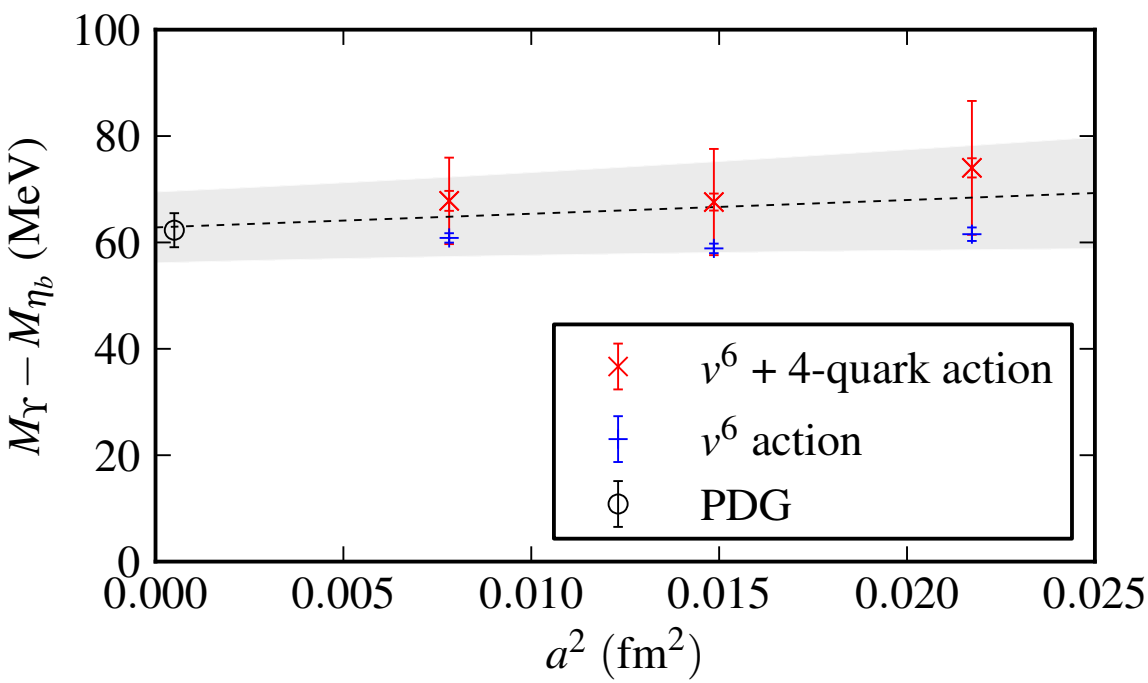



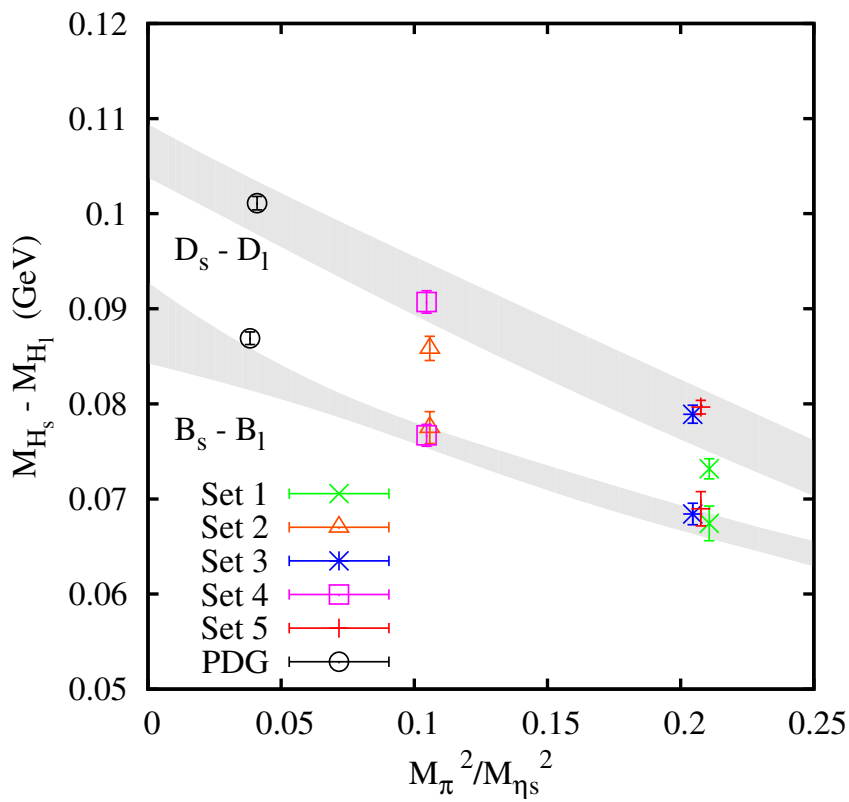

Radial excitations in Upsilon system with NRQCD - use multiple smearings and constrained multi-exponential fit

HPQCD, R. Dowdall et al 1110.6887

# Further examples on details of the spectrum - NRQCD b and HISQ u/d, s and c

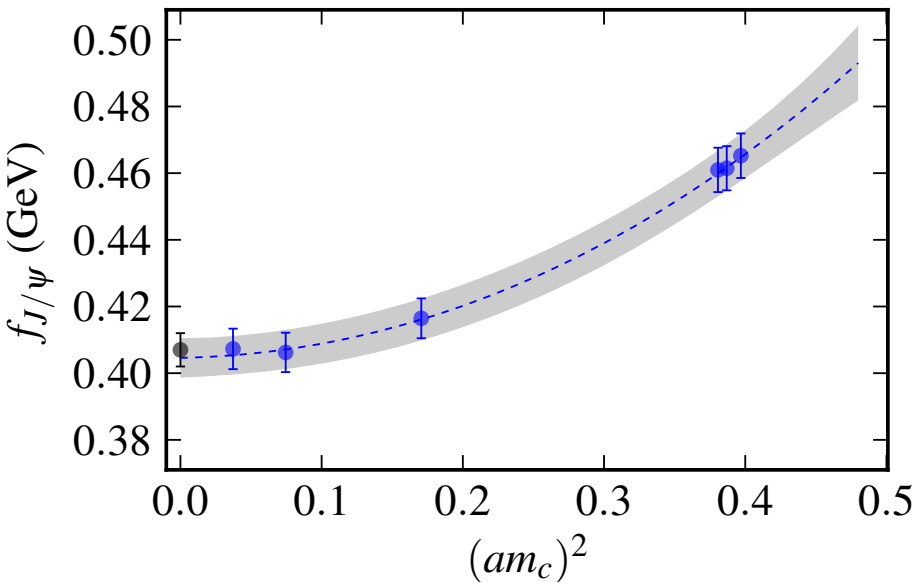


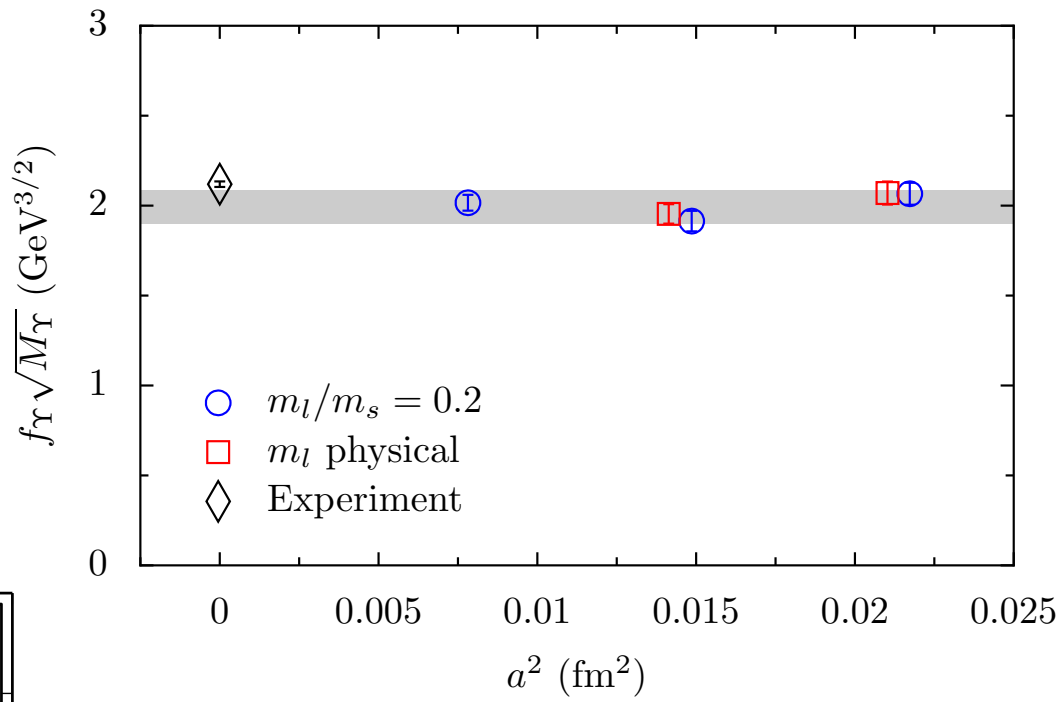



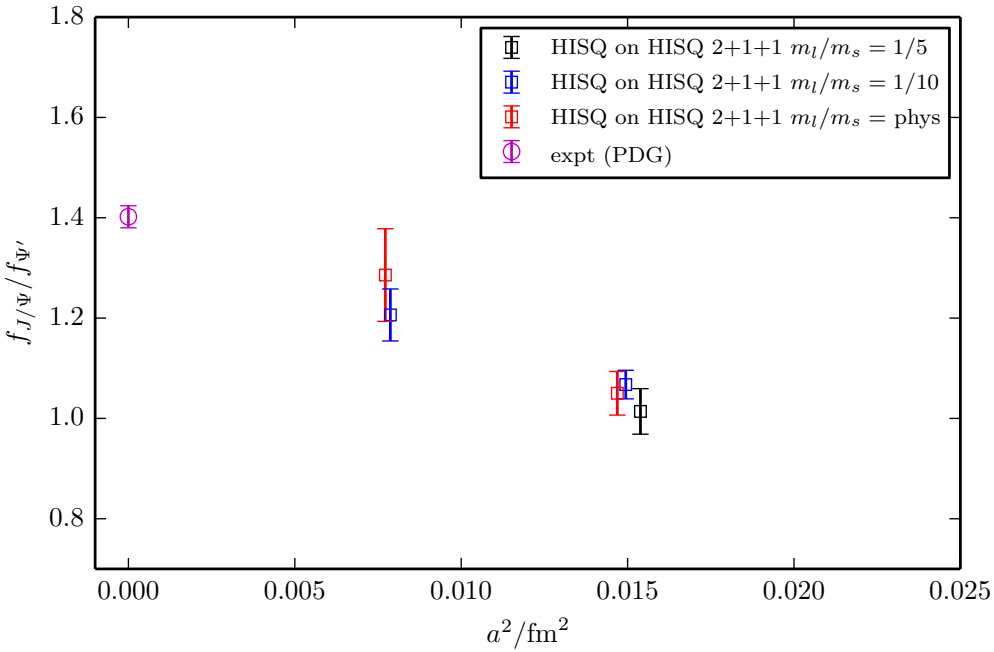




HPQCD, B. Galloway et al, LAT14; R. Dowdall et al 1207.5149, 1309.5797

# Decay constants for leptonic decay



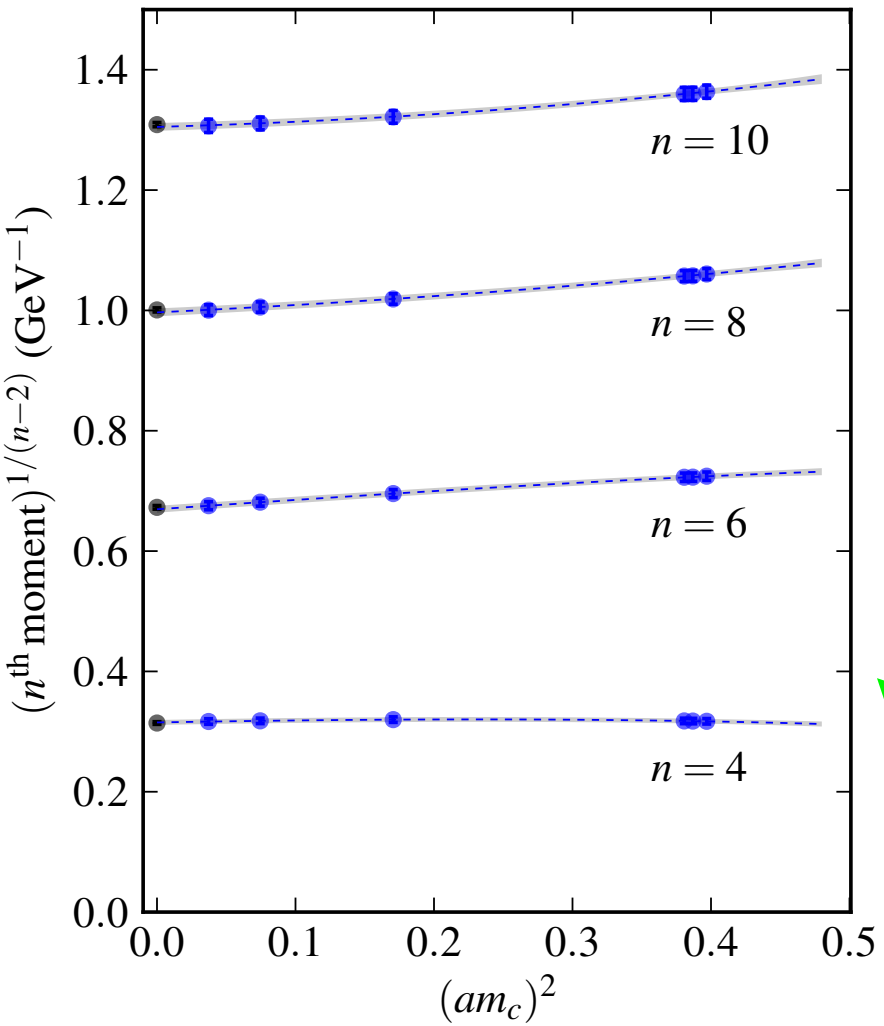



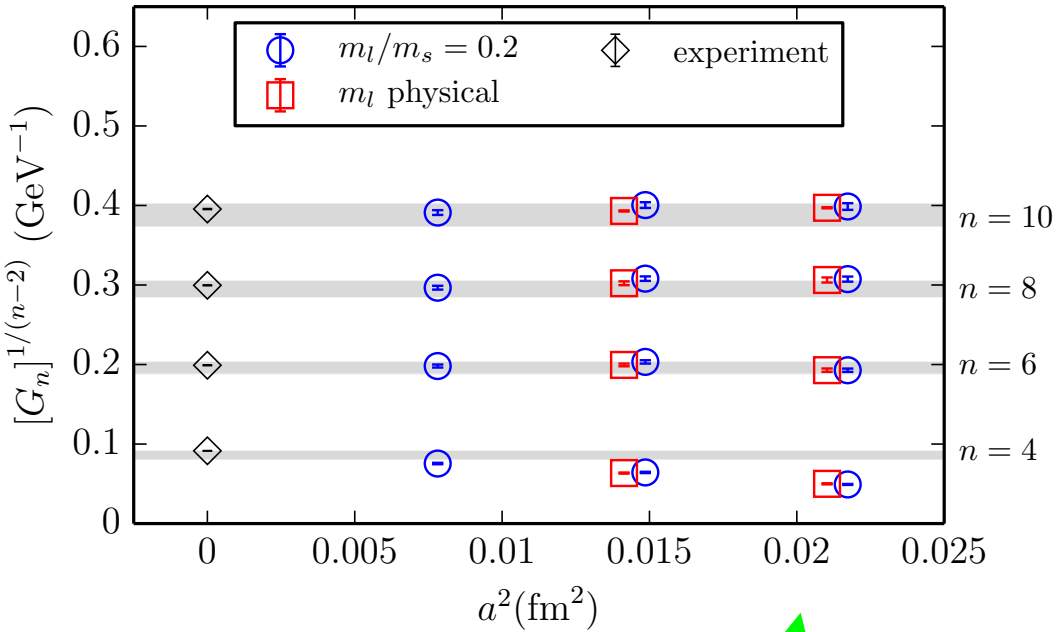




NRQCD b; HISQ c nonpert renorm of current operator

HPQCD, G. Donald et al, 1208.2855; B. Galloway et al, LAT14; B. Colquhoun et al 1408.5768

# Time-moments of current-current correlators




$$M_n = \sum_t t^n G(t)$$

HISQ c; NRQCD b

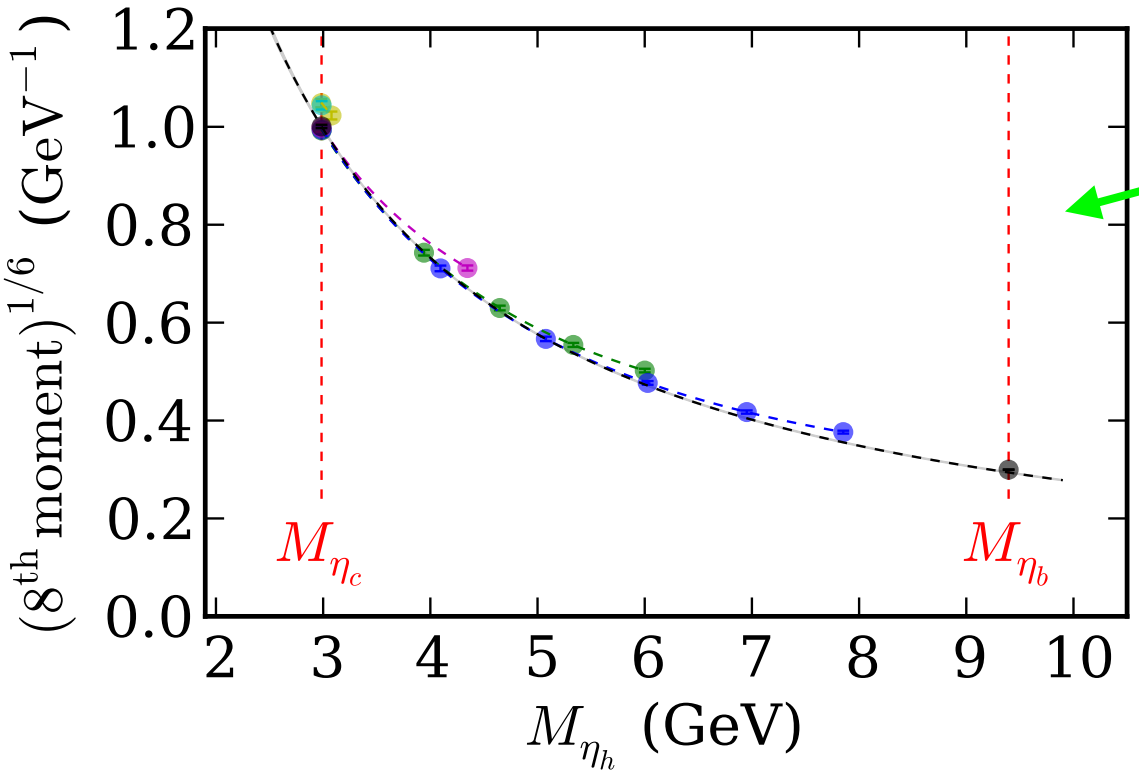
compare to experiment

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma_{\text{pt}}}$$

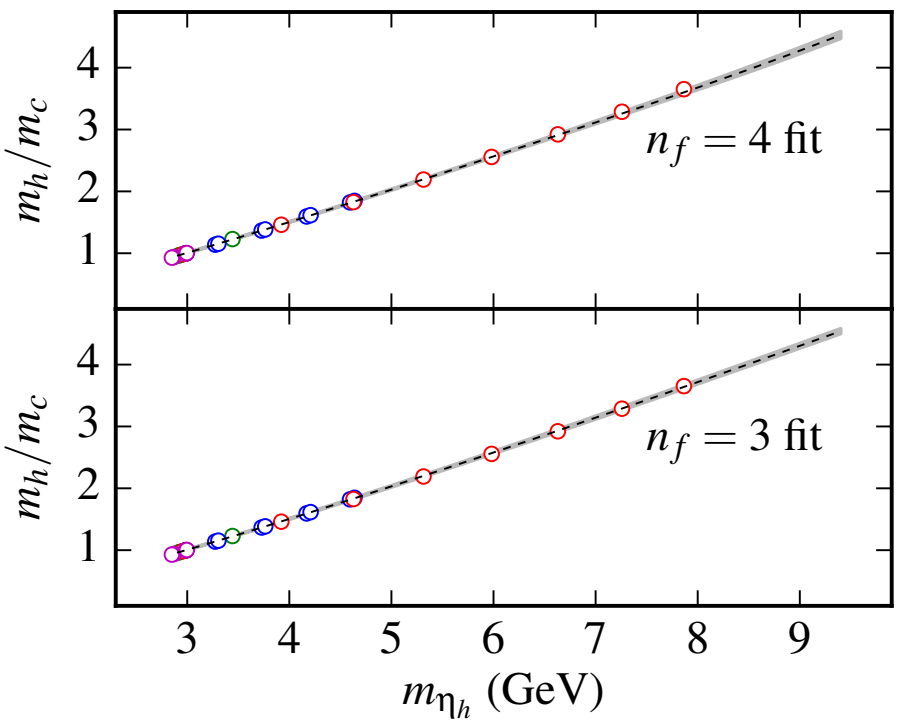
Useful also at T>0 ?

HPQCD, G. Donald et al, 1208.2855; B. Colquhoun et al 1408.5768

# Using HISQ to span range from c to b




Moments for $m_h > m_c$ and for a=0.15fm to 0.045fm ($n_f$=2+1)



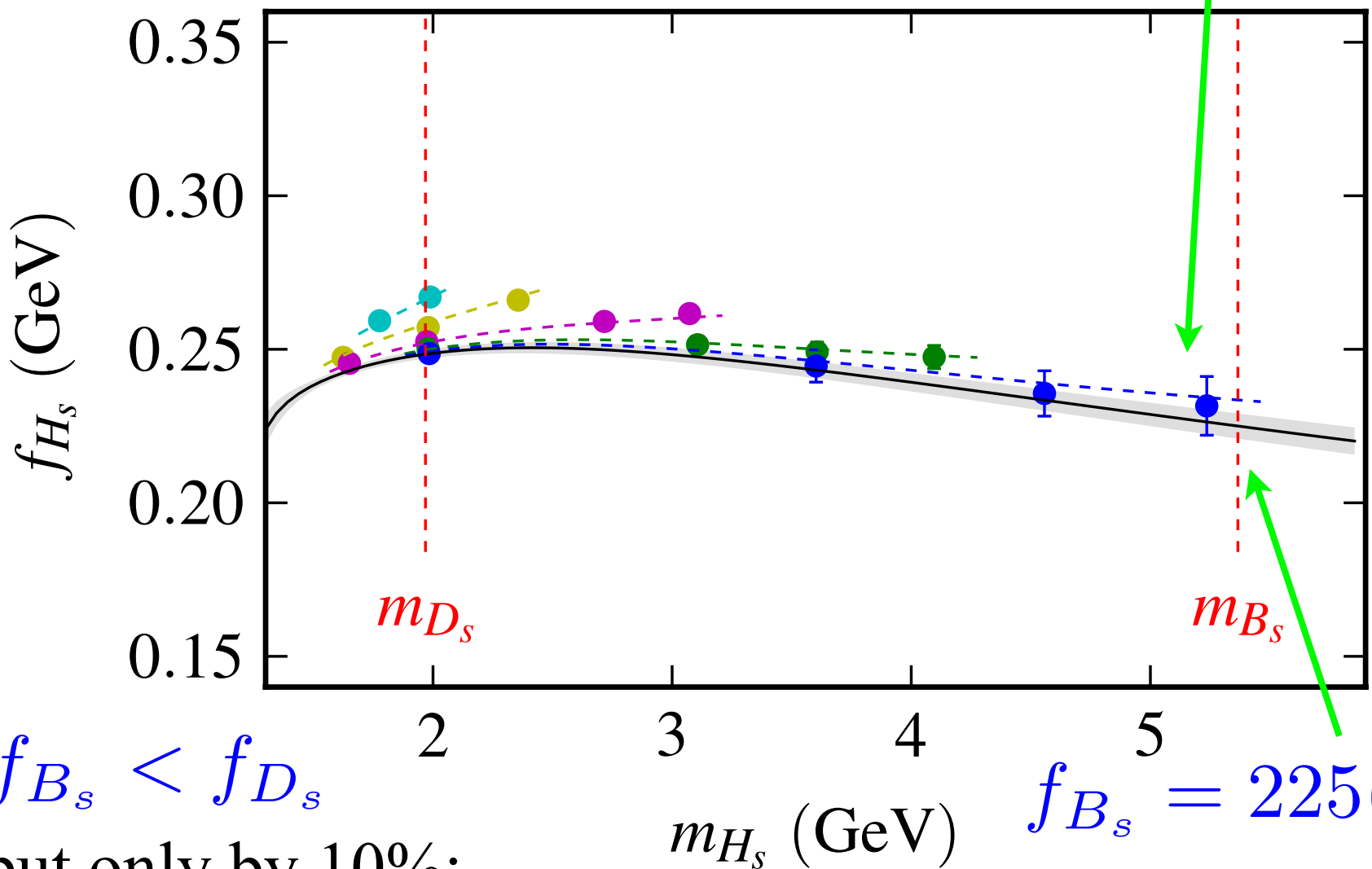

Quark mass ratios for a =0.15fm to 0.06fm ($n_f$=2+1+1) compared to $n_f$=2+1

$$m_b/m_c = 4.53(5)$$

HPQCD, in prep; B. Chakraborty et al 1408.4169

# Mapping out dependence of heavy-light physics on heavy quark mass using HISQ ...

uses HISQ and multiple $m_h$ and a. Finest: a=0.045fm




Tests HQET:

$f \propto 1/\sqrt{m_h}$ + corrections

$f_{B_s} < f_{D_s}$

$f_{B_s} = 225(4)\text{MeV}$

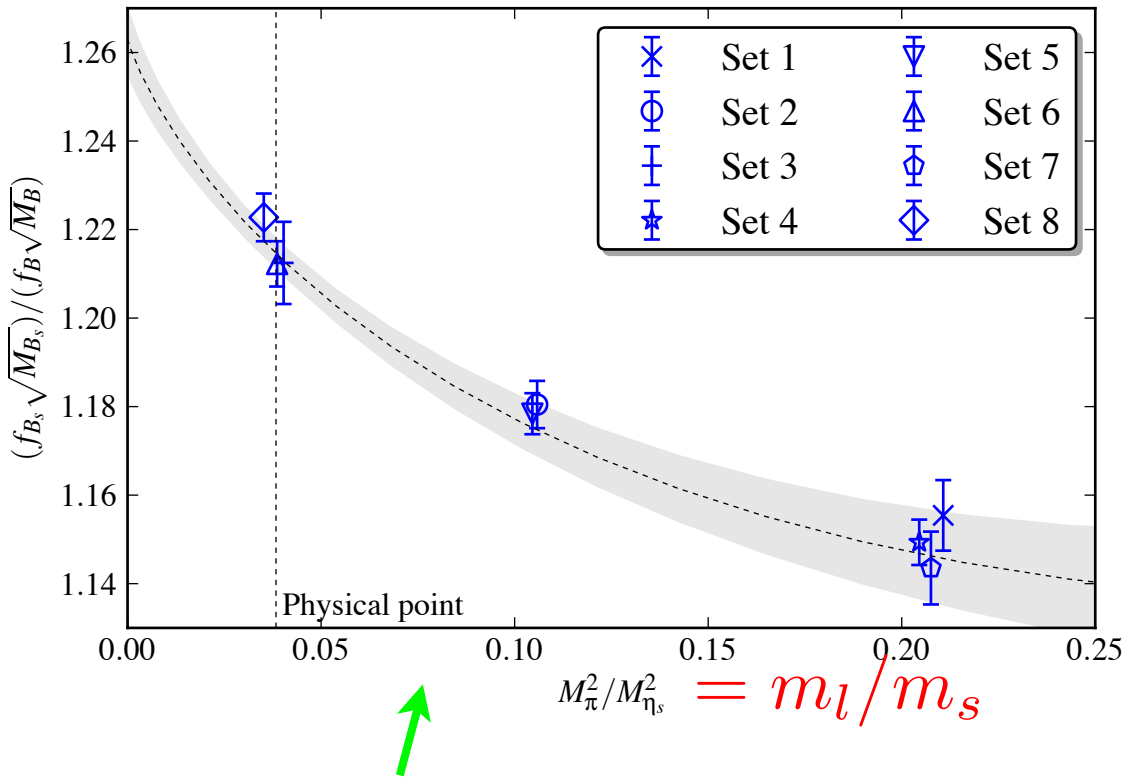
but only by 10%:

$f_{B_s}/f_{D_s} = 0.906(14)$

HPQCD: C. Davies et al, 1008.4010;
C McNeile et al,1110.4510.

# State-of-the-art B and Bs meson decay constants using physical u/d quarks and NRQCD b

HPQCD: R Dowdall et al, 1302.2644.



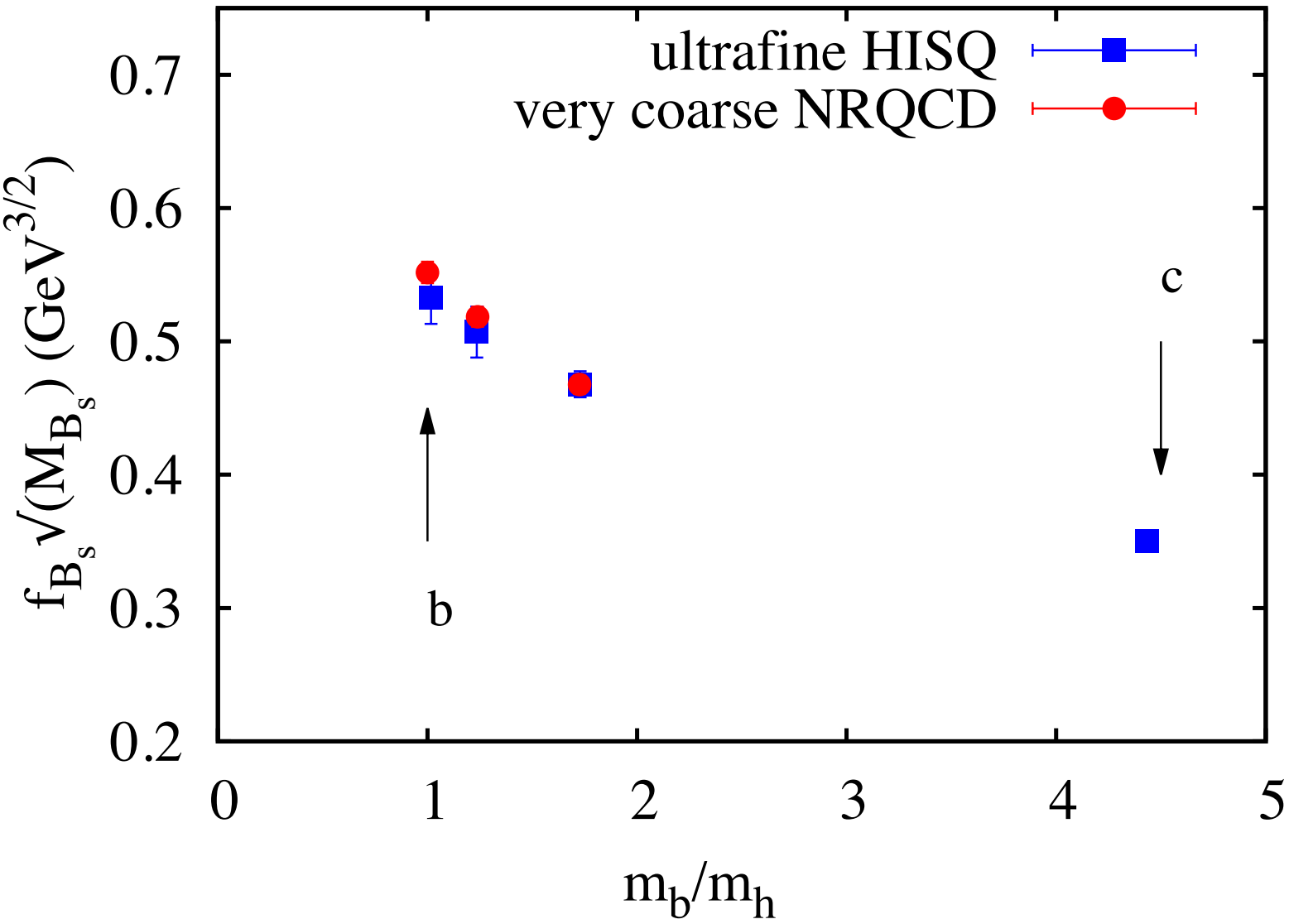

MILC HISQ 2+1+1 configs with u/d down to physical values + improved NRQCD for b

$B_s$ to B decay constant ratio accurate to 0.6% - since Z factors cancel. Separate decay constants to 2%

$$f_{B_s} = 224(5)\text{MeV}$$

$$f_B = 186(4)\text{MeV}$$

# Compare NRQCD and HISQ for lighter-than-b quarks




Good agreement, despite very different formalisms, where they overlap.

# Conclusion

- Lattice QCD results for gold-plated hadron masses and decay constants now providing stringent tests of QCD/SM at zero temperature.
- Gives QCD parameters and some CKM elements to 1%, tests of sum rules/HQET etc.
- sets of '2nd generation' gluon configs now being made:

$m_{u,d}$ at physical value (so no extrapoln) *or*

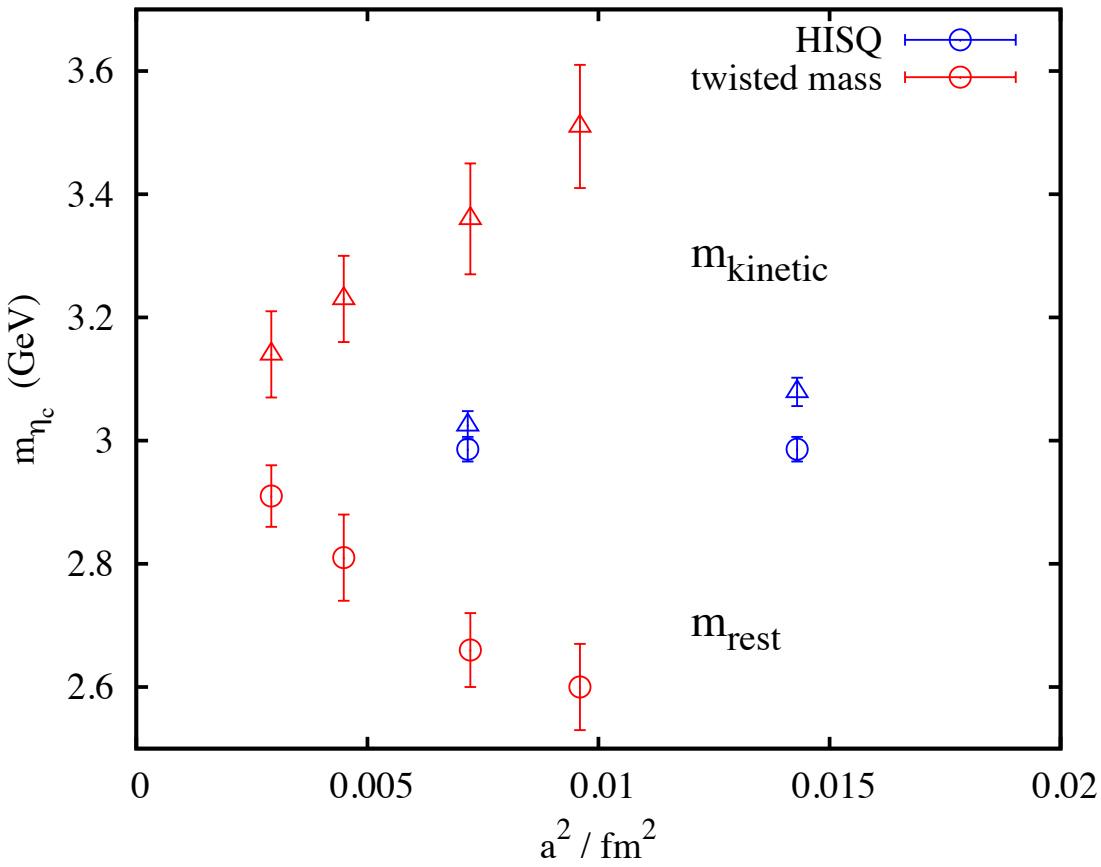
$a$ down to 0.05fm (so b quarks are 'light') *or*

*much* higher statistics (which will help with e.g. multi-hadron calculations)

- using relativistic b will become routine for some calculations. Nonrelativistic methods will continue to be useful when very high statistics is key requirement.

# Spares

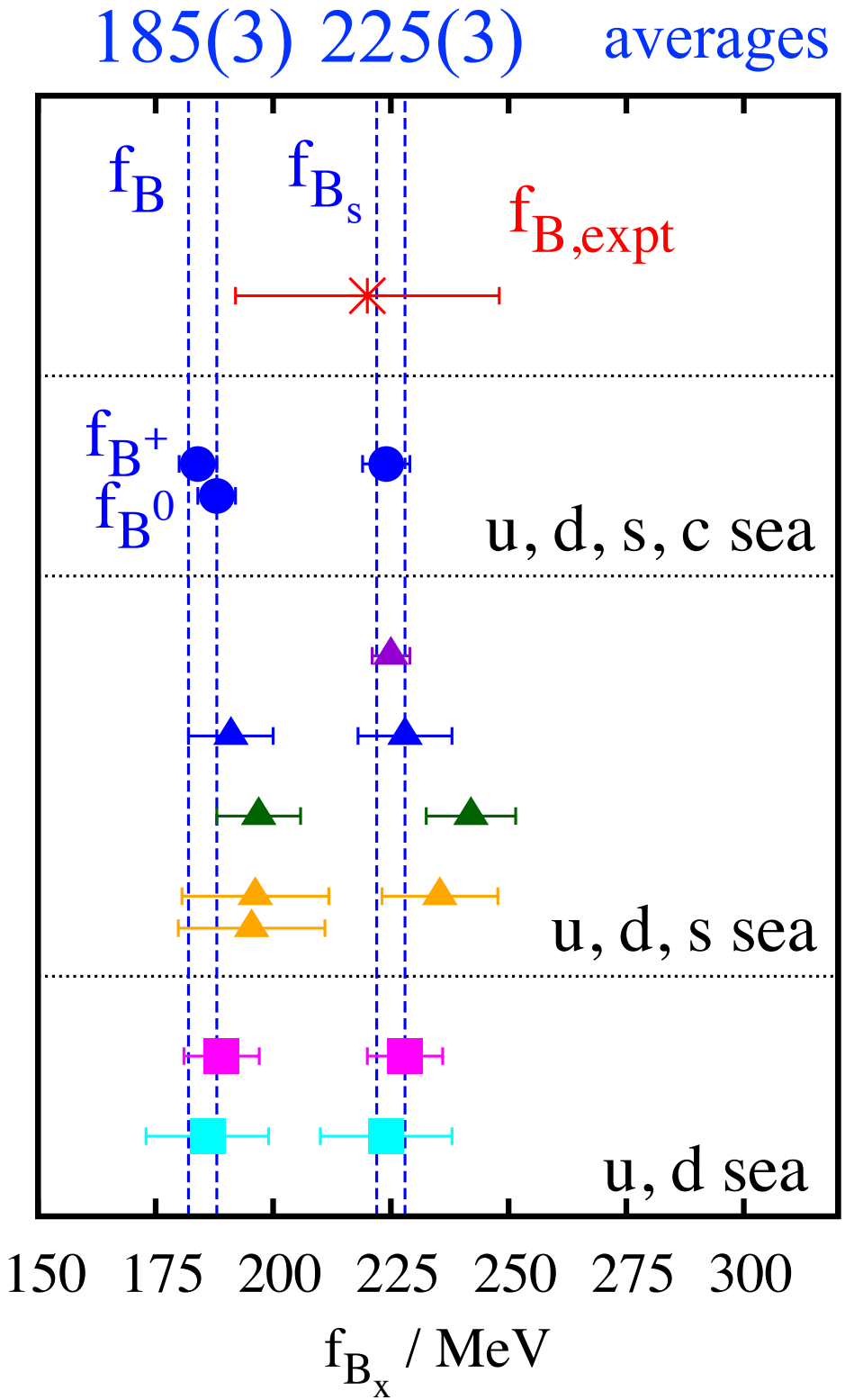
# Comparison of discretisation errors between HISQ and twisted mass (which has tree-level $a^2$ errors)




Plot shows mismatch of kinetic mass and static mass, equivalent to a test of the 'speed of light'.

HPQCD, G. Donald et al, 1208.2855

# B, $B_s$ decay constant summary 2014




PDG av. branching fraction
+ unitarity Vub

HPQCD NRQCD
1302.2644

HPQCD HISQ 1110.4510

HPQCD NRQCD
1202.4914

FNAL/MILC 1112.3051

RBC/UKQCD 1404.4670

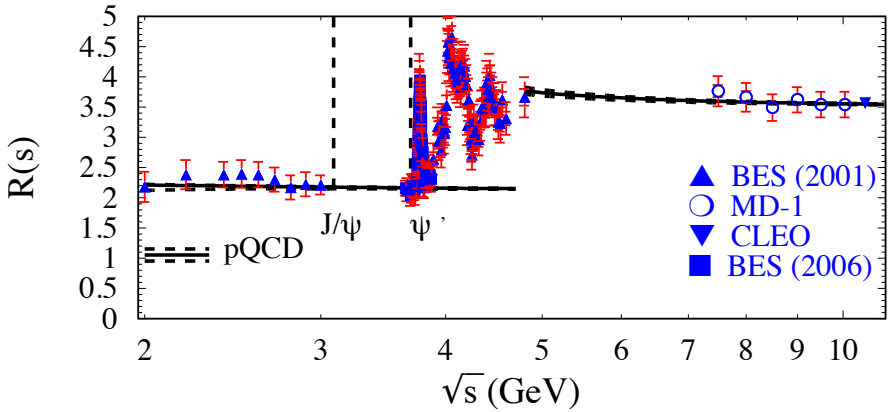
ETMC 1308.1851

ALPHA 1404.3590

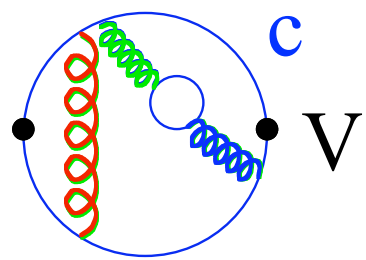
Different lattice results in good agreement

# Charm contribution to

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma_{\text{pt}}}$$




Subtract u,d,s using pert. th. to get contribn of charm vector correlator



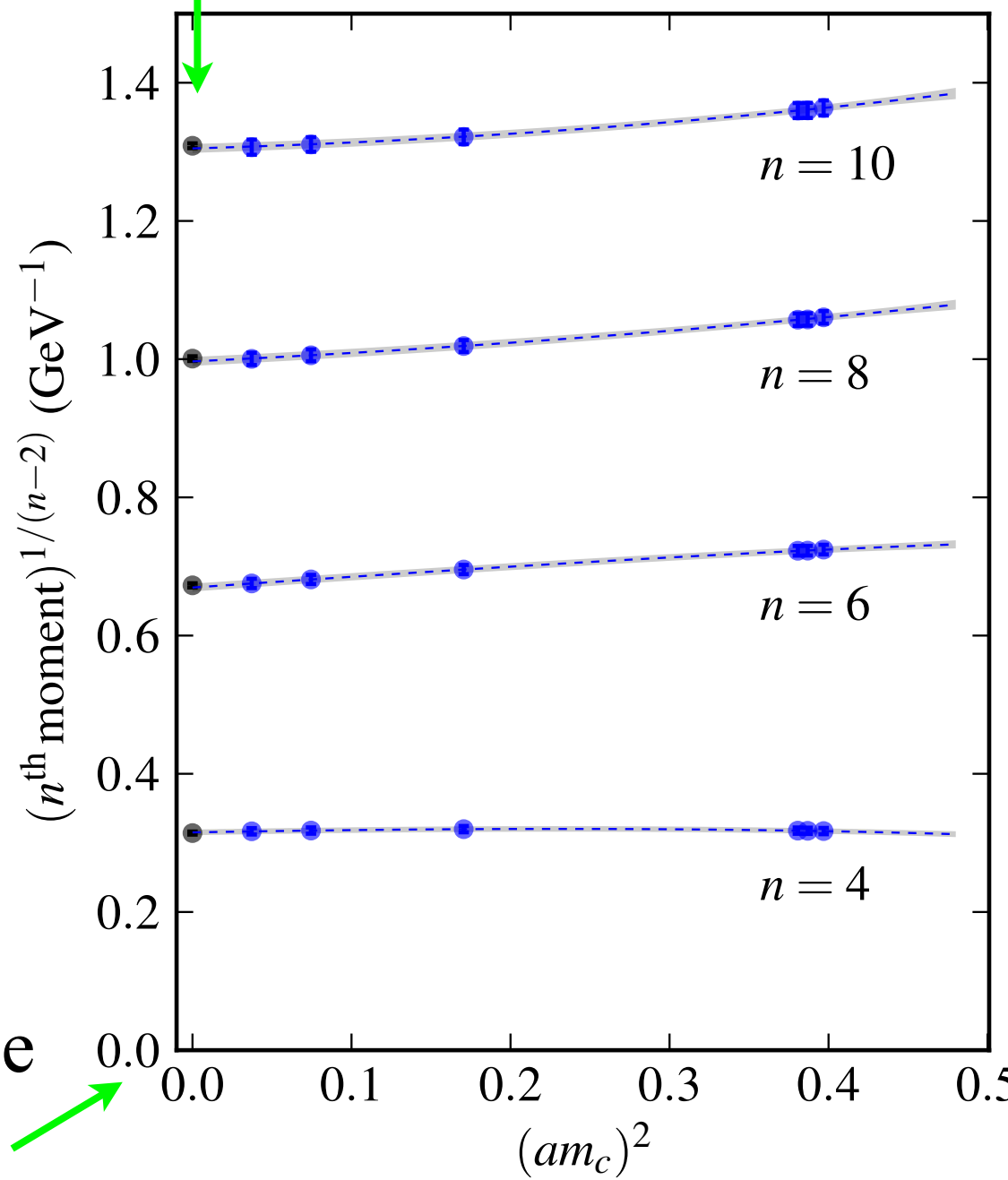

$$\mathcal{M}_n \equiv \int \frac{ds}{s^{n+1}} R_c(s)$$

Lattice calcln:

$$M_n = \sum t^n G(t)$$

Agree to 1.5%

'expt' ← from J. Kuhn et al, hep-ph/0702103




G. Donald et al, HPQCD,1208.2855 - b physics calculation underway

Look at error budgets to see how things will improve in future ...

1302.2644: calculation of B, $B_s$ masses and decay constants

errors divided into extrapolation and other systematics:

| Error % | $\Phi_{B_s}/\Phi_B$ | $M_{B_s} - M_B$ | $\Phi_{B_s}$ | $\Phi_B$ |
|---|---|---|---|---|
| EM: | 0.0 | 1.2 | 0.0 | 0.0 |
| $a$ dependence: | 0.01 | 0.9 | 0.7 | 0.7 |
| chiral: | 0.01 | 0.2 | 0.05 | 0.05 |
| g: | 0.01 | 0.1 | 0.0 | 0.0 |
| stat/scale: | 0.30 | 1.2 | 1.1 | 1.1 |
| operator: | 0.0 | 0.0 | 1.4 | 1.4 |
| relativistic: | 0.5 | 0.5 | 1.0 | 1.0 |
| total: | 0.6 | 2.0 | 2.0 | 2.1 |

for different quantities different systematics are important