Quarkonium spectral functions in medium at next-to-leading order for any quark mass

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Talk based on:
[YB, arXiv:1410.1304 [hep-ph]
YB, M. Laine, JHEP **1311**, 012 (2013)
YB, M. Laine, JHEP **1211** (2012) 086]

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Chicheley Hall

Outline

1. Introduction
2. Euclidean correlators and comparison to lattice
   - NLO Calculation
   - Results and comparison to the lattice
   - Fermionic contribution to bulk viscosity
3. Spectral functions
   - Vector spectral function
   - Transport physics
4. Conclusion

1. Introduction
Definitions

Current correlator

$$G_a(\tau) = \int_x \langle \left(\bar{\Psi}\Gamma_a\Psi\right)(\tau, x)\left(\bar{\Psi}\Gamma^a\Psi\right)(0,0)\rangle_T$$

Channels

- $\Gamma_a = \gamma_\mu$: Vector $\qquad$ $\Gamma_a = \gamma_\mu\gamma_5$ Axial Vector
- $\Gamma_a = 1$: Scalar $\qquad$ $\Gamma_a = \gamma_5$: Pseudo Scalar

Susceptibilities

$$\chi_a = \beta G_a(\tau) = \int_x \langle \left(\bar{\Psi}\gamma_0\Gamma_a\Psi\right)(\tau, x)\left(\bar{\Psi}\gamma_0\Gamma_a\Psi\right)(0,0)\rangle_T$$

Channels:

- $\Gamma_a = 1$: susceptibility
- $\Gamma_a = \gamma_5$: chiral susceptibility

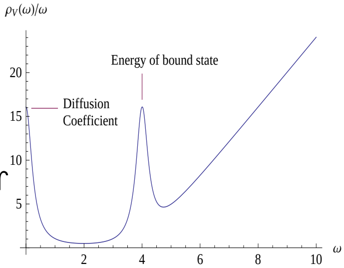
Physics from Euclidean correlators?

Physical quantities we are interested in are defined in Minkowski space. They can be fitted from the correlator's spectrum

$$\rho(\omega) = \mathrm{Disc}[G(-i\omega)], \quad \text{or} \quad G(\tau) = \int d\omega K(\omega,\tau)\rho(\omega).$$

Physics:

- Vector channel:
  - Heavy quark diffusion
  - Quarkonium bound state $J/\Psi, \Upsilon$
- Scalar channel:
  - Part of the energy momentum tensor
  - Fermionic contribution to the bulk viscosity
  - Quarkonium bound state $\chi_{c,b0}$



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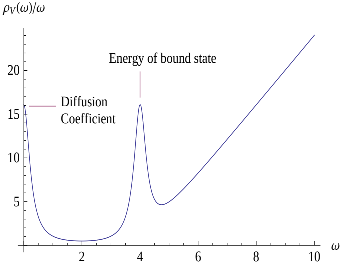
$$\rho(\omega) = \mathrm{Disc}[G(-i\omega)], \quad \text{or} \quad G(\tau) = \int d\omega K(\omega,\tau)\rho(\omega)$$

Physics:

- Axial Vector channel:
  - Quarkonium bound state $\chi_{c,b1}$
- Pseudo Scalar channel:
  - No constant part.
  - Quarkonium bound state $\eta_{c,b}$



Why perturbation theory?

Perturbation theory: minuses

- QCD has a large coupling
- Breaks down for small frequencies (or close to the bound state)

Lattice:

- Contains all the physics
- Bound to Euclidean
- Need analytic continuation to Minkowski space...

Analytic continuation is ill defined in general, here $G(\tau)$ is not even analytic ... That's were perturbation theory comes back:

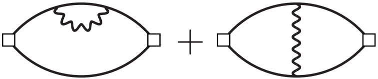
- Handle Euclidean and Minkowski
- Large frequency limit is fine
- Could be used to define the analytic continuation
- Resummations exists for bound state (and small) energies

2. Euclidean correlators
NLO Calculation: Diagrams and renormalization

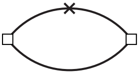
We calculated the thermal NLO correlators for any M, T:

[YB, M.Laine; V, χ: (2012), S: (2013), PS, AV, χ_5: (2015)]

- The "genuine" 2-loop graphs for G are



- Mass counterterm: δM^2



- Pole mass scheme $\delta M^2 = -\frac{6g^2 C_\mathrm{F} M^2}{(4\pi)^2}\left(\frac{1}{\varepsilon} + \ln\frac{\bar{\mu}^2}{M^2} + \frac{4}{3}\right)$

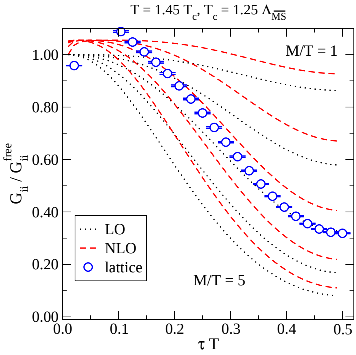
$\Rightarrow$ $\chi, \chi_5, G_V, G_{AV}$ UV and IR finite after mass renormalization

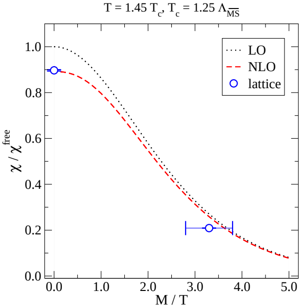
$\Rightarrow$ $M^2_{bare} G_S, M^2_{bare} G_{PS}$ UV, IR finite after mass renormalization

Results and comparison to the lattice

To compare the curves, scale:

- G_{ii} and χ to the free, M=0 result:
  - $G_{ii}^{free} = 2N_c T^3 \left(\frac{1}{6} + \frac{2\cos(2\pi\tau T)}{\sin^2(2\pi\tau T)} + \pi(1-2\tau T)\frac{1+\cos^2(2\pi\tau T)}{\sin^3(2\pi\tau T)} \right)$
  - $\chi^{free} = \frac{N_c T^2}{3}$





[Ding, Francis, Kaczmarek, Karsch, Satz, Soeldner 1204.4945; Ding, Francis, Kaczmarek, Karsch, Laermann, Soeldner 1012.4963]

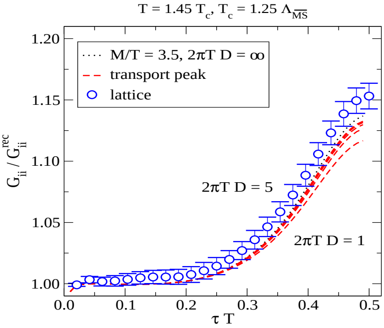
Addition of a transport peak

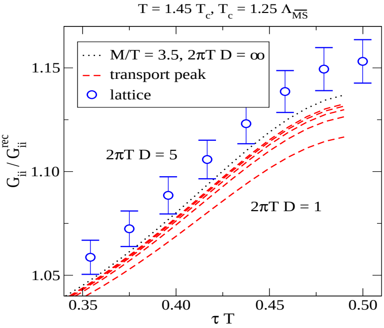
The spectral function ρ_{ii}/ω has a $\omega\delta(\omega)$ transport peak.

- In the full result one expects a Lorentzian shape.
- Replace in G_{ii} the constant part by a τ-dependent part from

$$\rho_{ii}^{(\mathrm{L})}(\omega \sim 0) \;\approx\; 3D\chi \, \frac{\omega\eta^2}{\omega^2+\eta^2}$$

- We vary D tuning η to keep the area under $\rho(\omega)/\omega$ constant

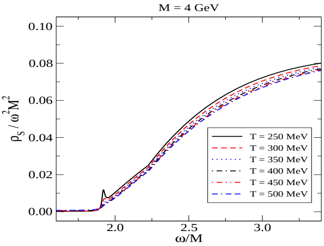


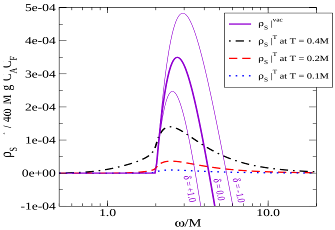


Scalar channel: Renormalization

G_S Not renormalizable through a redefinition of the mass!

- But $M^2_{bare} G_S(\tau)$ is.
- Using pole mass scheme $M^2_{bare} = M^2 - \frac{6g^2 C_F M^2}{(4\pi)^2} \left(\frac{1}{\varepsilon} + \ln \frac{\bar{\mu}^2}{M^2} + \frac{4}{3} \right)$
- $\Rightarrow$ Negative contribution to the spectrum dominating at $\omega \gg M$
- $\Rightarrow$ Euclidean correlator decreases at small τ.





[YB, Laine, Vepsäläinen, JHEP 02 (2009) 008]

Mass dependence and comparison with lattice

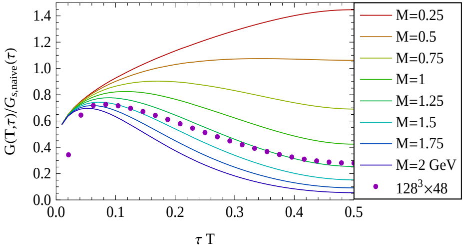
Unphysical behavior in the pole mass scheme

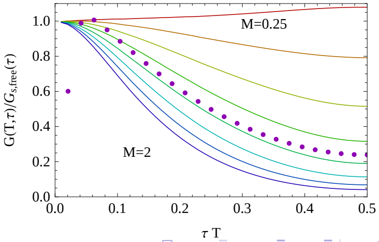
- $\overline{MS}$ scheme $\rightarrow \delta M^2 = -\frac{6g^2 C_F M^2}{(4\pi)^2}\left(\frac{1}{\varepsilon}\right)$
- Now $M \rightarrow m(\mu)$ and we refer to $M = m(\mu = 2GeV)$.
- Renormalization scale $\mu \propto \omega, 1/\tau$

$\Rightarrow$ Plot the results in the $\overline{MS}$ scheme:

At small τ, $M^2_{bare} G_S^{\mathrm{NLO}}(\tau)$

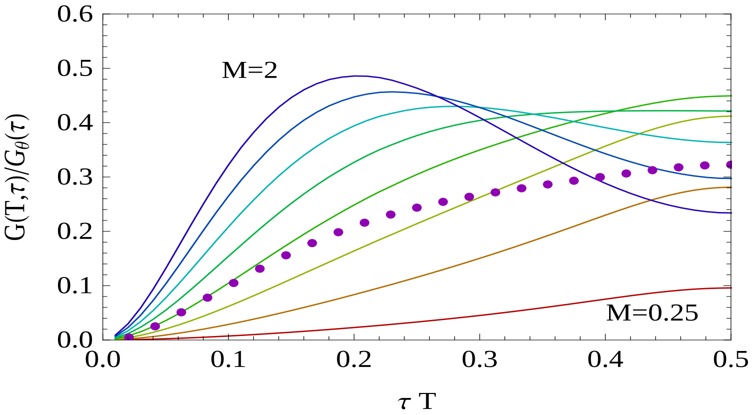
- Does not match $M^2_{bare} G_{\mathrm{free}}(\tau)$
- Matches $m(\mu(\tau))^2 G_{\mathrm{free}}(\tau)$





Fermionic contribution to bulk viscosity

Even if this should be extracted for the spectral function, we can compare $G_S(\tau)$ to the gluon part of the trace anomaly $G_\theta(\tau)$.



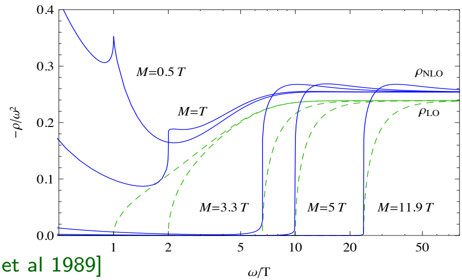
⇒ Charm quark might give a large contribution to bulk viscosity!

3. Spectral functions
Vector spectral functions and mass shift

The calculation goes as before and we extract the discontinuity.

- After mass renormalization ⇒ spectrum is finite except at threshold.
- This divergence can be reabsorbed by redefining $M^2 \rightarrow M^2 + \delta M_T^2$.
- δM_T^2 is the usual thermal mass shift (for $M_{T=0} = 0$):
  $\delta M_T^2 = g^2 C_{\rm F} \int_0^\infty \frac{dk}{\pi^2} k(n_{\rm B}(k) + n_{\rm F}(k)) = \frac{g^2 C_{\rm F} T^2}{4}$

Spectrum, for different masses at T=1.5T_c



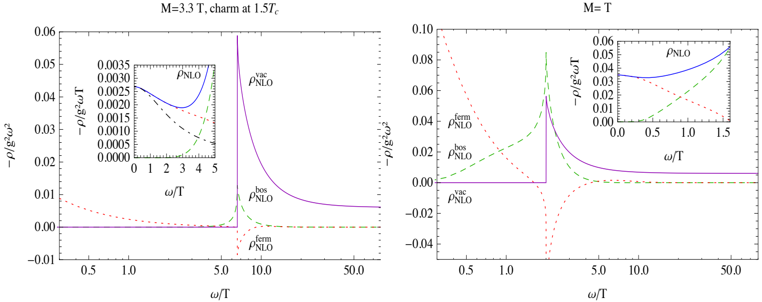
[YB 2014, Altherr et al 1989]

Unlike in old NLO calculations with $M_{T=0} = 0$, the limit $\omega \rightarrow 0$ is finite

Vector spectral functions and transport physics

NLO corrections and $\omega \to 0$ limit for charm and light flavors.

'bos' thermal corrections are $\propto n_{\rm B}(k)$ and the 'ferm' $\propto n_{\rm F}(\sqrt{M^2+p^2})$.



With thermal mass shift and all contributions: 'well defined' transport peak for any $M_{T=0}$ (with D: diffusion and η_D: drag coefficients).

$$-\frac{\rho^V(\omega)}{\omega} \overset{0<\omega\lesssim\omega_{UV}}{\approx} 3\chi D \frac{\eta_D^2}{\eta_D^2+\omega^2}$$

Results for transport coefficients

Diffusion coefficient: $D = -\frac{1}{3\chi}\lim_{\omega\to 0}\frac{\rho^V(\omega)}{\omega}$. If the onset of non-transport physics is well separated from the transport peak:

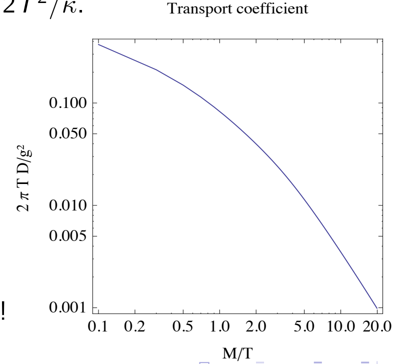
$$\kappa = 2M_{kin}T\eta_D \approx -\frac{2M_{kin}^2\omega^2}{3\chi}\frac{\rho^V(\omega)}{\omega}\Big|_{\eta_D\ll\omega\ll\omega_{UV}}$$

Momentum diffusion coeft. [Caron-Huot et al, 2009]

Fluctuation dissipation theorem: $D = 2T^2/\kappa$.

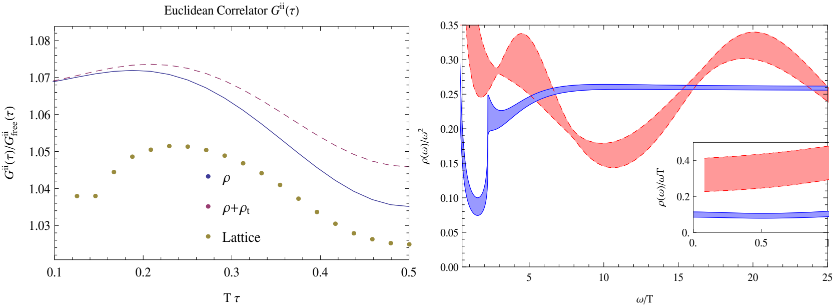
Charm at $T = 1.45Tc$:

- Here: $2\pi TD \approx 0.1$.
- Lattice results $2\pi TD \approx 2$ [Ding et all, 2010]
- Perturbative Heavy quark limit $2\pi TD \approx 10$ [CaronHuot, 2007].
- Here via fluct.-diss: $2\pi TD \sim 0.8$!



Comparison to lattice: Massless fermion

Euclidean corr. from [Ding et al 2010] and spectrum [Aarts et al 2014]
$(n_F = 0,\ T = 1.46\,Tc)$ $(n_F = 3,\ T = 201\text{MeV})$



- Here: $2\pi TD \sim 0.3$
- Perturbative resummation from [Arnold et al 2000, 2003] $2\pi TD \sim 25$
- Lattice results ranges from $2\pi TD \sim 0.5 - 6$
- To fit the Euclidean lattice data an additional: $2\pi T\bar{D} = 0.05$

4. Conclusion

On renormalization:

- V, AV are renormalizable through a mass redefinition.
- The $M^2_{bare}\times$Scalar and $M^2_{bare}\times$Pseudo Scalar as well.
  - For the latter, use the $\overline{MS}$ scheme.
  - They scale to $m(\mu(\tau))^2 G_{\mathrm{free}}(\tau)$ at small τ.

On physical properties:

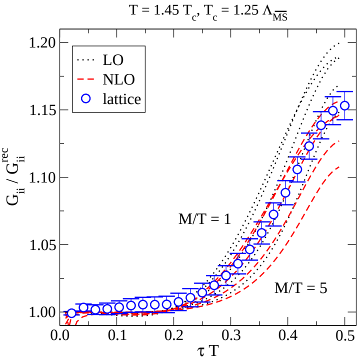
- The determination of transport and bound state properties requires very precise lattice data.
- The charm quark could be important for the bulk viscosity.
- The value of the transport coefficient is method dependent.
- Transport peak not well separated from other physics.
- Thermal mass corrections might be an important ingredient.

Backup Slides

Reconstructed correlators and comparison to the lattice

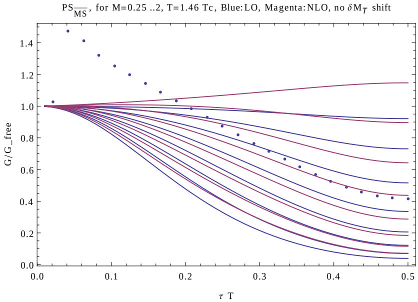
To compare the curves, scale:

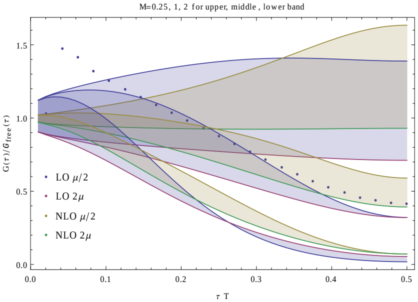
- G_{ii} to the reconstructed correlator
  - NLO: $G_{ii}^{\mathrm{NLO},rec}(\tau, T) = \int \frac{d\omega}{2\pi}(-\rho_V^{\mathrm{NLO},vac})K(\tau,\omega)$
  - Lattice: $G_{ii}^{\mathrm{lattice},rec}(\tau, T, T') = \sum_{\tau'(T,T')} G(\tau', T')$



Pseudo scalar

Compare $M_b^2 G_{PS}$ with $m(\mu(\tau))^2 G_{PS\,\mathrm{free}}(\tau)$:

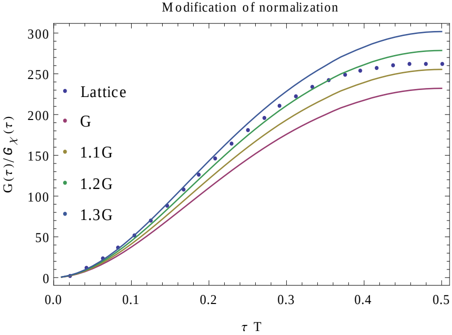




Normalization of the lattice data doesn't match even if we

- Add bound state peaks, transport peaks
- Add the NNLO vacuum part
- Take the thermal mass shift into account

Pseudo scalar



Scaled here to the anomalous contribution to the non-conservation of the pseudo scalar current

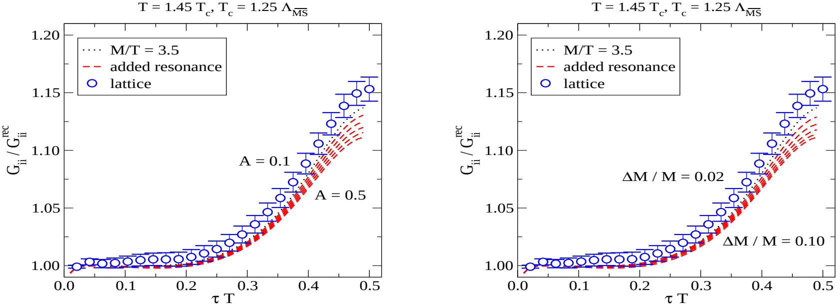
Addition of a bound state peak

In the temperature range of interest $\rightarrow$ at most one peak.

- Slightly to the left from the free quark-antiquark threshold
- Model this by a skewed Breit-Wigner shape added to NLO

$$\rho_{ii}^{(\mathrm{BW})}(\omega \sim 2M) \approx \frac{A\omega^2\gamma^2}{(\omega - 2M + \Delta M)^2 + \gamma^2}$$

- Set $\Delta M \equiv 2\gamma$, add to the thermal NLO result
- Add to vacuum result with $A \rightarrow 5A$, $\gamma \rightarrow \gamma/5$

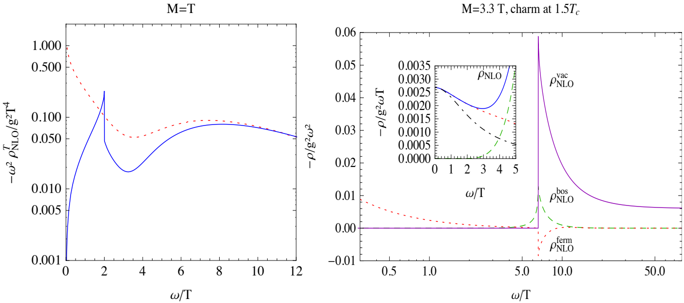


Spectral functions for all channels at NLO*

Comparison with old calculations

Thermal correction with/out thermal mass shift ($M_{T=0} = 0$):

[Altherr, Aurenche 1989, YB 2014]



Difference lies in the way the mass shift is performed:

- Altherr → only in LO ($M = 0$ was set in NLO from the start)
- Here → in both LO and NLO