Baryon chemical potential in large-N QCD

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University of Washington

- "Solving two-dimensional large-N QCD with a nonzero density of baryons and arbitrary quark mass"
  BB, arXiv:0901.4035

- "Volume dependence of two-dimensional large-N QCD with a nonzero density of baryons."
  BB, arXiv:0811.4141

- "The sign problem in two-dimensional large-N QCD"
  BB, in progress....

What got me interested ?

- Sign problem is due to

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- Sign problem is due to

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- Large-N QCD is simple (partly) thanks to

$$\text{quenched}(QCD) \overset{N=\infty}{=} QCD$$

$$\det \to 1$$

Trivial solution to sign problem ?

Probably not, but how ? after all

$\text{quenched}(QCD) \stackrel{N=\infty}{=} QCD$ **reflects a basic feature of large-N gauge theories**

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Given that it does not go away : can be made soft ?

$$\langle\, \text{phase}\,(\det\,(D+\gamma_0\mu+m))\,\rangle \sim e^{-\#V}$$

General arguments :
confined large-N gauge theories
are independent of **V**

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- Look for a gauge theory with
  - a sign problem at nonzero density
  - a smooth large-N limit.

But that can be solved exactly !

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Surprisingly, not much work done :

- Salcedo, Negele & Levit `92. $B = 1$
- Schon & Thies `00. vicinity of m=0, assume `t Hooft model = sine-Gordon.

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neglect certain type of quantum fluctuations

Goal of study :

Solve large-N `t Hooft model with arbitrary :

quark mass m, volume L, baryon number B, incorporating “0-modes”.

- Hope to understand general features of QCD at nonzero B :
  - Characteristics of phase diagram ?
  - Sign-problem ?
  - Large-N “quenched” confusions :
    are sea quark suppressed or not ?
    confusion resolved, but will not discuss it here :

  - While quarks do not back-react on gauge fields, they are not quenched.
  - Naive planar diagram analysis is miss-leading.

Outline

I. Formalism : Hamiltonian, gauge fixing, Gauss law.

II. Large-N coherent state : $H_{\rm quantum} \rightarrow \mathcal{H}_{\rm classical}$

Remarks on some features of $\mathcal{H}_{\rm classical}(L)$

III. Results from $\min\left(\mathcal{H}_{\rm classical}\right)$

IV. Numerical experiments with sign problem at large-N.

V. Conclusions + some ideas.

BB
arXiv:0901.4035
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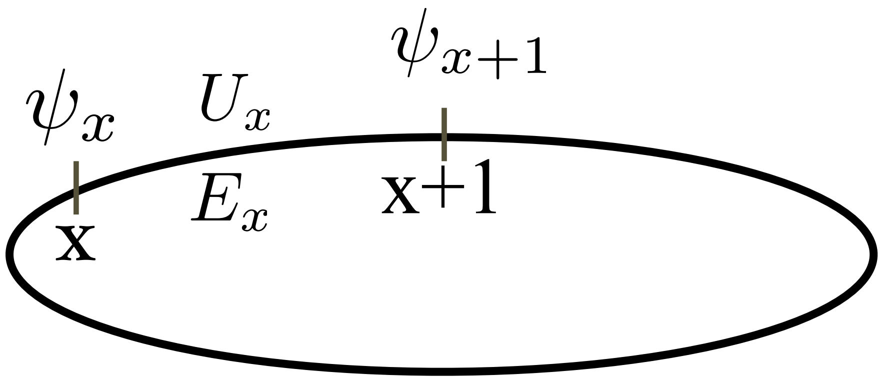
BB
in progress

Hamiltonian QCD in 1+1



$$U_x = U_{x+L_s}$$

$$\psi_x = \psi_{x+L_s}$$

Hamiltonian

$$H = \sum_{x=1}^{L_s} \left[g^2 \, \mathrm{Tr}\, E_x^2 + \left(-i\psi_x^\dagger \, U_x \, \psi_{x+1} + \mathrm{h.c.} \right) + m \, (-1)^x \, \psi_x^\dagger \psi_x \right]$$

Commutations

$$\left[E_x^{ab}, E_y^{cd} \right] = \left(E_x^{ad} \delta^{bc} - E_x^{bc} \delta^{cd} \right) \delta_{xy}$$

$$\left[E_x^{ab}, U_y^{cd} \right] = U_x^{ad} \delta^{bc} \delta_{xy}$$

Gauss Law

$$D\,E = \rho \qquad : \qquad E_x^{ab} - \left(U_{x-1} \, E_{x-1} \, U_{x-1}^\dagger \right)^{ab} = \rho_x^{ab}$$

“Fixing” Axial gauge

- Gauge away U_x , but get left with φ_a : $\mathrm{Eig}\left[\prod_x U_x\right] = e^{i\varphi_a}$

  - Hamiltonian :

$$H_F \to \sum_{x=1}^{L_s} \sum_a \left[\left(-i\psi^{\dagger a}_x \, e^{i\varphi_a/L_s} \, \psi^a_{x+1} + \text{h.c.} \right) + m\,(-1)^x \, \psi^{\dagger a}_x \psi^a_x \right]$$

  - Gauss law :

$$E^{ab}_x - \left(U_{x-1} \, E_{x-1} \, U^\dagger_{x-1} \right)^{ab} = \rho^{ab}_x \quad \Longrightarrow \quad E^{ab}_x - e^{i\varphi_a/L_s} \, E^{ab}_{x-1} \, e^{-i\varphi_b/L_s} = \rho^{ab}_x$$

$$E^{ab}_p \left(1 - e^{i(\varphi_a - \varphi_b)/L_s - ip} \right) = \rho^{ab}_p$$

Resolving Gauss law

$$E_p^{ab}\left(1 - e^{i(\varphi_a - \varphi_b)/L_s - ip}\right) = \rho_p^{ab}$$

generically

$$E_p^{ab} = \frac{\rho_p^{ab}}{\left(1 - e^{i(\varphi_a - \varphi_b)/L_s - ip}\right)}$$

$a = b, p = 0$

$$\forall a \quad : \quad \rho_{p=0}^{aa} = 0$$

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$a=b, p=0$

$$\forall a \quad : \quad \sum_x\left[\psi_x^{\dagger a}\psi_x^a-\frac{1}{N}\sum_c\psi_x^{\dagger c}\psi_x^c\right]=0$$

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N-1 $E_{p=0}^{aa}$ **unresolved**

$$g^2\ \mathrm{Tr}\,E_x^2=\frac{g^2}{L_s}\sum_{\substack{ab\\xyp}}{}^{'}\frac{\rho_x^{ab}\,\rho_y^{ba}\,e^{ip(x-y)}}{4\sin^2\left(\frac{(\varphi_a-\varphi_b)/L_s+p}{2}\right)}+g^2\sum_a\left(E_{p=0}^{aa}\right)^2$$

Axial gauge left us with

operators

Commutation relations

$$\rho^{ab} \sim \psi^{\dagger b} \psi^{a}$$

N-1 φ_a

N-1 $E^{aa}_{p=0}$

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operators	Commutation relations
$\rho^{ab} \sim \psi^{\dagger b}\psi^{a}$	$[\rho^{ab}, \rho^{cd}] \sim \rho^{ef}$
N-I φ_a	$[\varphi_a, \varphi_b] = 0$
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$$E^{aa}_{p=0} = -i\left(\frac{\delta}{\delta\varphi_a} - \frac{1}{N}\sum_c \frac{\delta}{\delta\varphi_c}\right) - \frac{i}{2}\left[\frac{\delta \log \Delta^2(\varphi)}{\delta\varphi_a} - \frac{1}{N}\sum_c \frac{\delta \log \Delta^2}{\delta\varphi_c}\right],$$

$$\Delta^2(\{\varphi\}) = \prod_{a<b} \sin^2\left(L_s \frac{\varphi_a - \varphi_b}{2}\right).$$

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“0-modes”

N-I $E^{aa}_{p=0}$

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Re-cap :

Hamiltonian :

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\begin{aligned}
H &= H_G + H_K + H_C, \\
H_G &= g^2 \sum_a \left(E^{aa}_{p=0}\right)^2 \\
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\end{aligned}
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H_C = \frac{g^2}{L_s} \sum_{\substack{ab\\ xyp}}{}' \frac{\rho^{ab}_x \, \rho^{ba}_y \, e^{ip(x-y)}}{4\sin^2\left(\frac{(\varphi_a - \varphi_b)/L_s + p}{2}\right)} \qquad \text{with} \qquad \rho^{ab}_x \sim \psi^{\dagger b}_x \psi^a_x
$$

Hilbert space :

$$
\sum_x \psi^{\dagger a}_x \psi^a_x = B
$$

Strategy for diagonalization : use large-N

Large-N QCD is simpler than 3-color QCD

mesons and glueballs do not interact, baryons are classical, no decays, no fluctuations, ...

Strategy for diagonalization : use large-N

The large-N limit of QCD is a classical limit

$$\text{diag}\ (H_{\text{quantum}}) \longrightarrow \min_{\mathcal{C}} \left[\mathcal{H}(\mathcal{C})\right]$$

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- General paradigm realized in the coherent state approach for QCD **Yaffe**

unitary color-singlet operator

$$|\mathcal{C}\rangle = U(\mathcal{C})|0\rangle$$

arbitrary color-singlet "reference" state

$$\mathcal{H}(\mathcal{C}) = \langle\mathcal{C}|H_{\text{quantum}}|\mathcal{C}\rangle$$

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$$U(\mathcal{C}) = \exp\left[\mathcal{C}_F \times \psi_x^\dagger U_{x\to y}\psi_y\right] \times \exp\left[\mathcal{C}_g^{(1)} \times N\, tr(\hat{W}) + \mathcal{C}_g^{(2)} \times N\, tr(\hat{E}^i \lambda^i \hat{W})\right]$$

Coherent states, cont'd

- Dominance of glue over fermions :

$$\mathcal{H}(\mathcal{C}) = \underbrace{\mathcal{H}_{\rm glue}(\mathcal{C}_g)}_{\sim\, O(N^2)} + \underbrace{\mathcal{H}_{\rm F}(\mathcal{C}_{\rm F},\mathcal{C}_g)}_{\sim\, O(N)}$$

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  - Step A : Minimize leading

$$\mathcal{H}_{\rm glue}(\mathcal{C}_g) = \langle \mathcal{C}_g | H_{\rm glue} | \mathcal{C}_g \rangle \longrightarrow \langle \sum_a e^{i\varphi_a} \rangle, \langle \sum_a e^{2i\varphi_a} \rangle, \ldots$$

Coherent states, cont'd

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- Step B : Minimize sub-leading

$$\mathcal{H}_F(\mathcal{C}_g = \mathcal{C}_{g,\min}, \mathcal{C}_F) = \langle \mathcal{C}_{g,\min} | \langle \mathcal{C}_F | H_{\text{F}} | \mathcal{C}_F \rangle | \mathcal{C}_{g,\min} \rangle$$

$$\longrightarrow \langle \sum_a \psi_x^{\dagger a} \psi_x^a \rangle, \langle \sum_a \psi_x^{\dagger a} e^{i\varphi_a (y-x)/L_s} \psi_y^a \rangle, \ldots$$

OK, so roll up your sleeves and start working ...

$$
\begin{aligned}
H &= H_G + H_K + H_C, \\
H_G &= g^2 \sum_a \left(E^{aa}_{p=0}\right)^2 \\
H_K &= -\frac{i}{2} \sum_x \psi^{\dagger a}_x \, e^{i\varphi_a/L_s} \, \psi^a_{x+1} + h.c. + m \sum_x (-1)^x \, \psi^{\dagger a}_x \psi^a_x,
\end{aligned}
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$$H_C = \frac{g^2}{L_s} \sum_{\substack{ab \\ xyp}}{}' \frac{\rho^{ab}_x \, \rho^{ba}_y \, e^{ip(x-y)}}{4 \sin^2\left(\frac{(\varphi_a - \varphi_b)/L_s + p}{2}\right)} \qquad \text{with} \qquad \rho^{ab}_x \sim \psi^{\dagger b}_x \psi^a_x$$

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$$

- Glue : can actually solve exactly (for finite-N) Z_N vacuum of

$$
\langle \sum_a e^{iq\varphi_a} \rangle = N\delta_{q,0}
$$

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$\mathcal{H}_{\mathrm{F}}$

$$\boxed{\begin{aligned} H_K &= -\frac{i}{2}\sum_x \psi^{\dagger a}_x \, e^{i\varphi_a/L_s}\, \psi^a_{x+1} + h.c. + m\sum_x (-1)^x\, \psi^{\dagger a}_x \psi^a_x, \\ H_C &= \frac{g^2}{L_s} \sum_{\substack{ab\\ xyp}}{}' \frac{\rho^{ab}_x\, \rho^{ba}_y\, e^{ip(x-y)}}{4\sin^2\left(\frac{(\varphi_a-\varphi_b)/L_s+p}{2}\right)} \quad \text{with} \quad \rho^{ab}_x \sim \psi^{\dagger b}_x \psi^a_x \end{aligned}}$$

- Coherent state

$$|\mathcal{C}_F\rangle = U(\mathcal{C}_F)|0\rangle_B = \exp\left(-i\sum_{x=1}^{L_s}\sum_{y=-\infty}^{\infty} \mathcal{C}^{xy}_F\ \psi^\dagger_x\ e^{i\varphi(y-x)/L_s}\ \psi_y\right)|0\rangle_B$$

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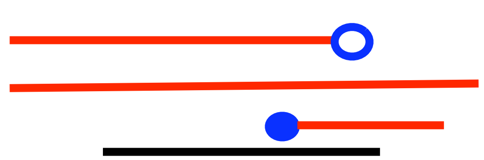

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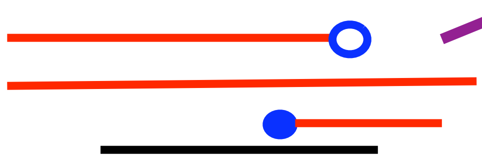
- Coherent state $\mathcal{H}_F(\rho^Q_{xy})$ with $\rho^Q_{xy} \sim \langle \, \psi^{\dagger a}_x \, e^{i\varphi_a(y-x+QL_s)/L_s} \, \psi^a_y \, \rangle$



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A Peek at $\mathcal{H}_F(\rho_{xy}^{Q})$

- More convenient : $\rho_{xy}(p) = \sum_{Q=-\infty}^{\infty} \rho_{xy}^{Q}\, e^{i\,Q\,p}$

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$$\mathcal{H}_F(\rho)/N \;=\; \int \frac{dp}{2\pi} \sum_{x \in Z_{L_s}} \left\{ \left(-\frac{i}{2}\rho_{x,x+1}(p) + c.c. \right) + m(-1)^x \rho_{xx}(p) \right\}$$

$$-\;\; \frac{g^2 N}{4} \int\!\!\int \frac{dp}{2\pi}\frac{dp'}{2\pi}\frac{1}{L_s} \sum_{xy \in Z_{L_s}} \sum_{l=1}^{L_s} \frac{\rho_{xy}(p)\,\rho_{yx}(p')\,e^{i2\pi l(x-y)/L_s}}{4\sin^2\left((p-p')/L_s + 2\pi l/L_s\right)/2}$$

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\end{aligned}
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- If assume ansatz : $\rho_{xy}^Q = f(x - y + Q L_s)$

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- If assume ansatz : $\rho^Q_{xy} = f(x - y + QL_s)$

$$\mathcal{H}_F/(NL_s) \;=\; \int_0^{2\pi} \frac{dp}{2\pi} \mathrm{tr}\left[\rho(p)\left(-\sigma_3 \sin(p) + m\,\sigma_1\right)\right] - \frac{g^2N}{4}\int\!\!\int \frac{dp}{2\pi}\frac{dq}{2\pi} \frac{\mathrm{tr}\,\left(\rho(p)\,\rho(q)\right)}{4\sin^2\left((p-q)/2\right)}$$

Large-N volume independence !!!

Remarks (I)

- Meaning $\rho^{Q}_{xy} \sim \langle\, \psi^{\dagger a}_{x}\, e^{i\varphi_a (y-x+QL_s)/L_s}\, \psi^{a}_{y} \,\rangle \;=\; f(x-y+QL_s)$

As if L is infinite

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As if L is infinite

- Papers "ignoring" 0-modes : have "set" $\varphi_a \to 0$ & break Z_N

$$\forall q : \langle \sum_a e^{iq\varphi_a} \rangle = N \quad \Longrightarrow \quad \rho^{Q}_{xy} \longrightarrow \rho_{xy}$$

No L independence : wrong

Minimizing $\mathcal{H}_F(\rho_{xy}^Q)$

- To minimize
  - solve, numerically, $\dfrac{\delta \mathcal{H}}{\delta \rho_{xy}^Q} = 0$: becomes a constraint Hartree-Fock

$$\sum_x \rho_{xx}^Q = B\delta_{Q,0}$$

  - regularize :

$$Q = 0, \pm 1, \pm 2, \pm 3, \cdots \pm \infty \longrightarrow Q = 0, \pm 1, \pm 2, \pm 3, \pm M$$

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- In practice

$M = 1, 5, 15, 25$

$L_s = 130 - 500$

$a\sqrt{\lambda} \simeq 0.62, 0.31, 0.15, 0.07$

$2 \lesssim L\sqrt{\lambda} \lesssim 80$

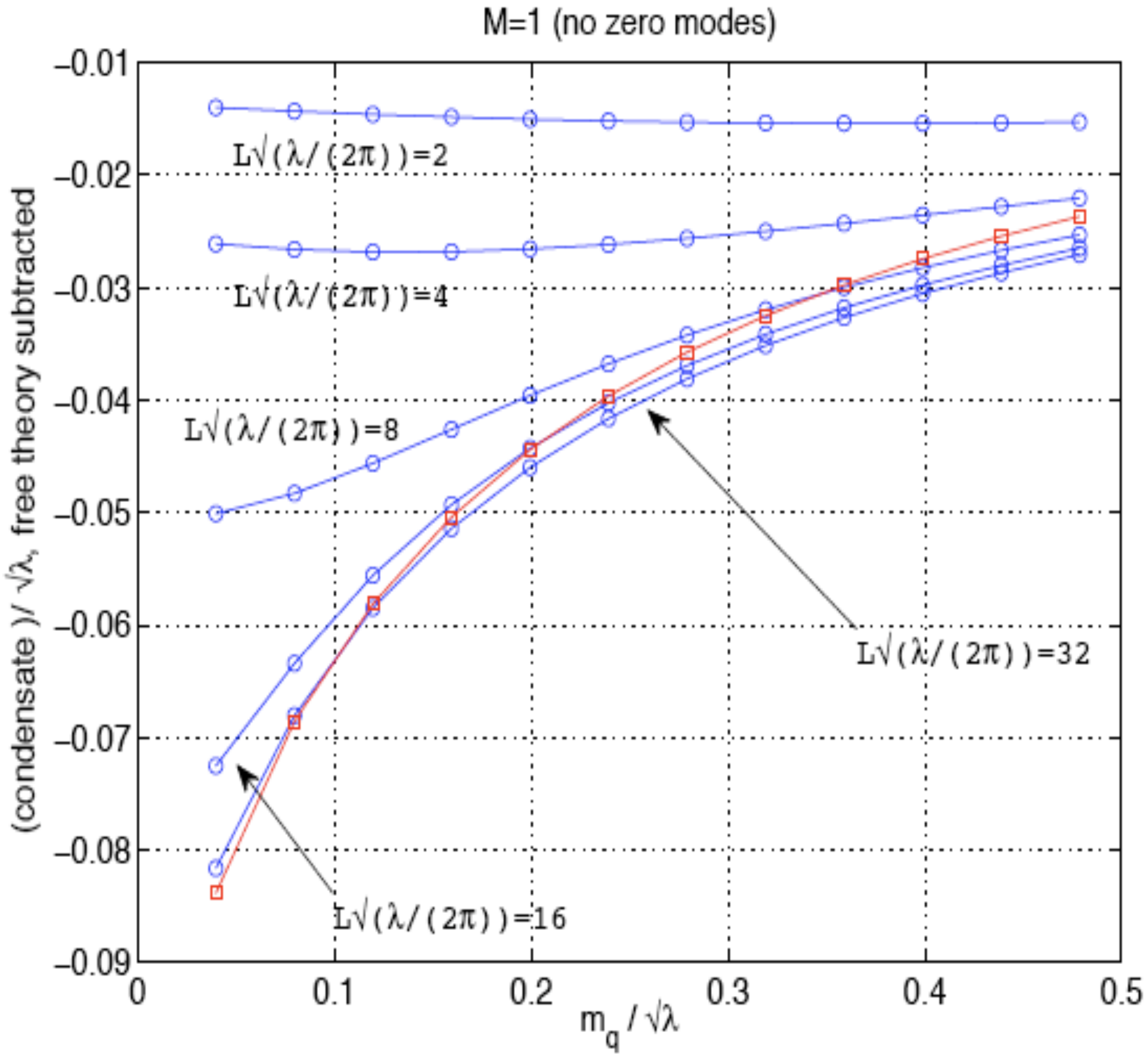
$m/\sqrt{\lambda} \simeq 0.02, 0.2, 0.4, 1.2, 2.4$

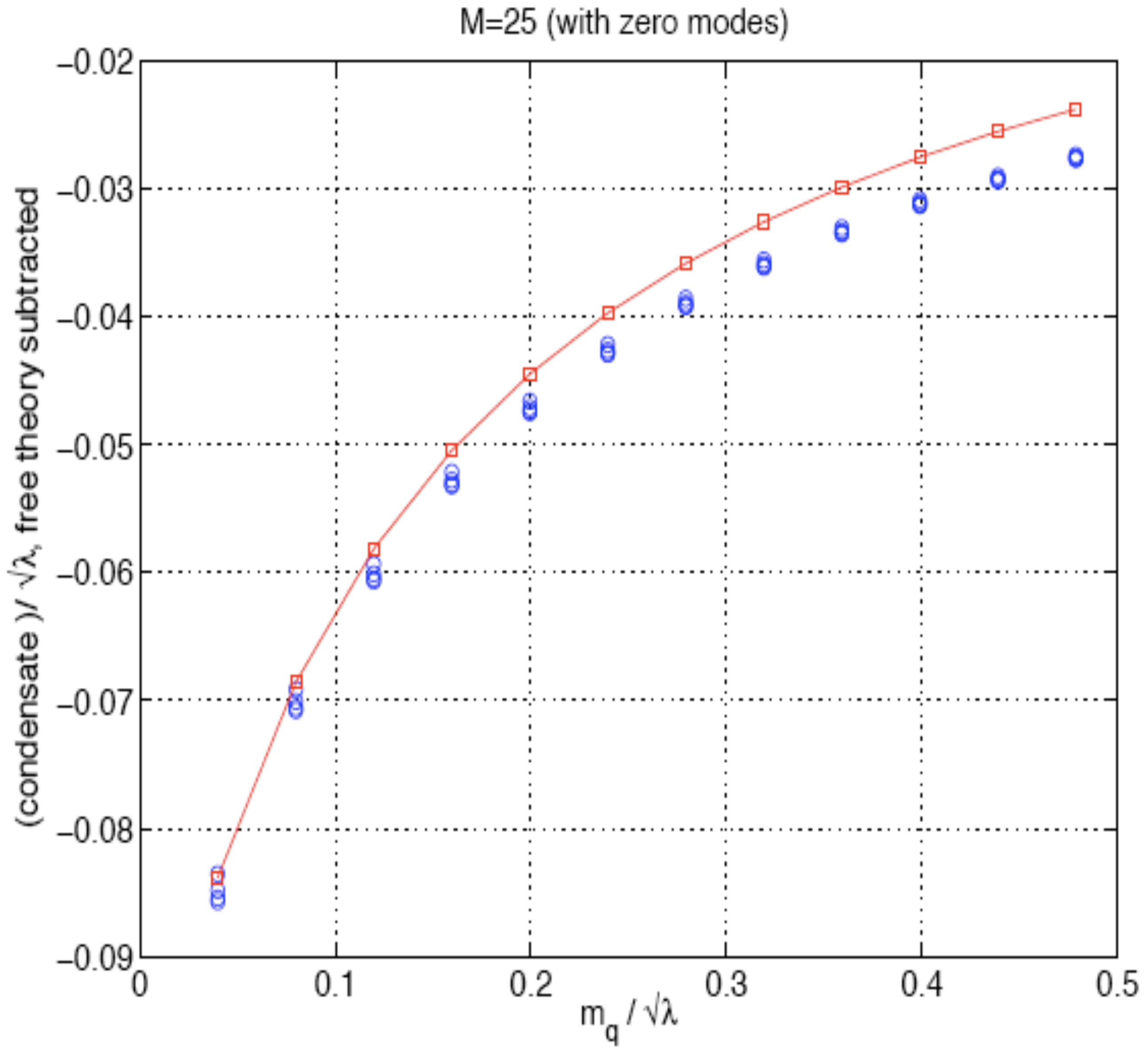
$B = 0, 1, 2, \ldots, 30$

$0 \leq n_B/\sqrt{\lambda} \lesssim 1.5$

$$\boxed{0 \leq \frac{\mu_B}{m_B/N} \lesssim 10}$$

Results : volume independence





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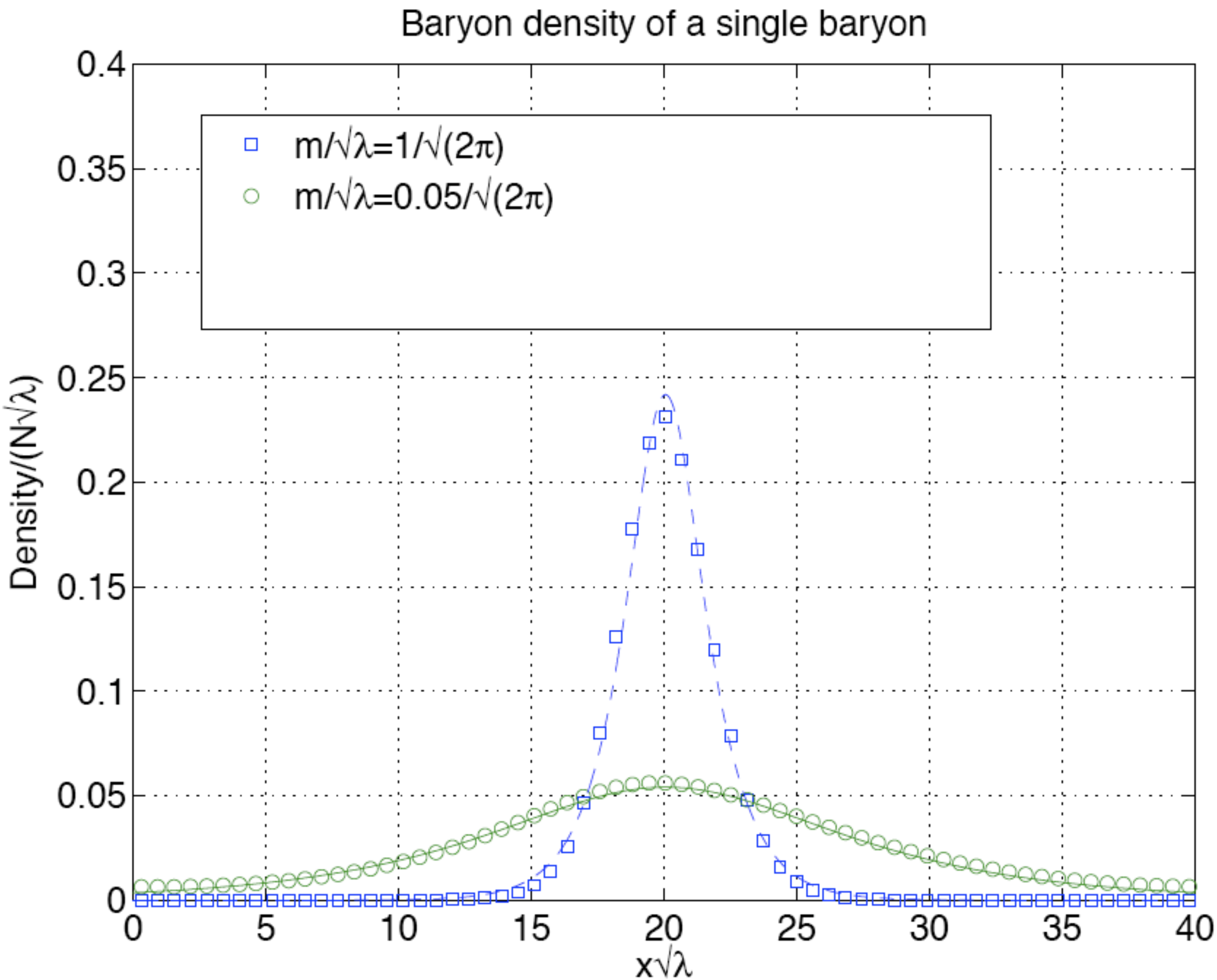
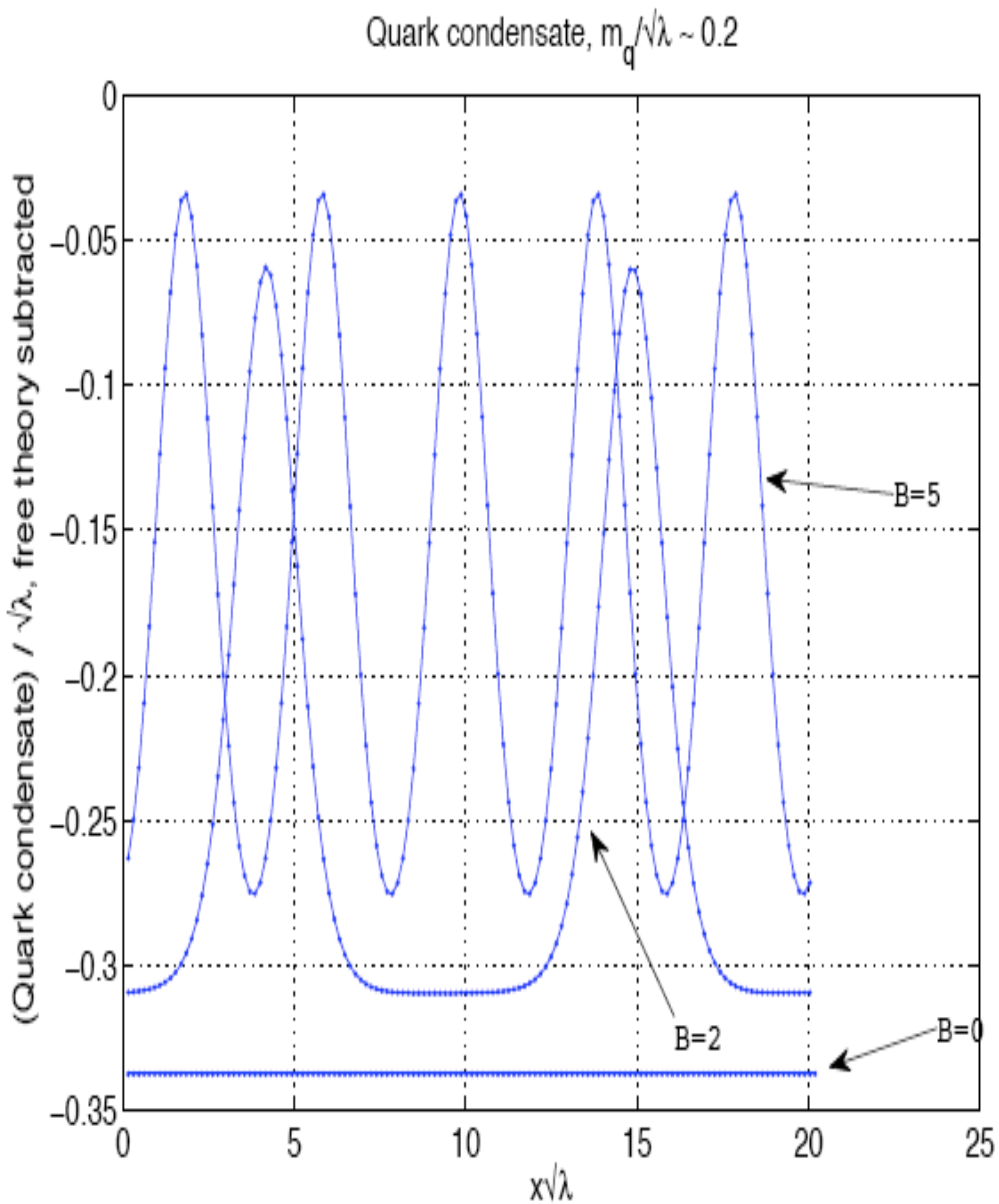
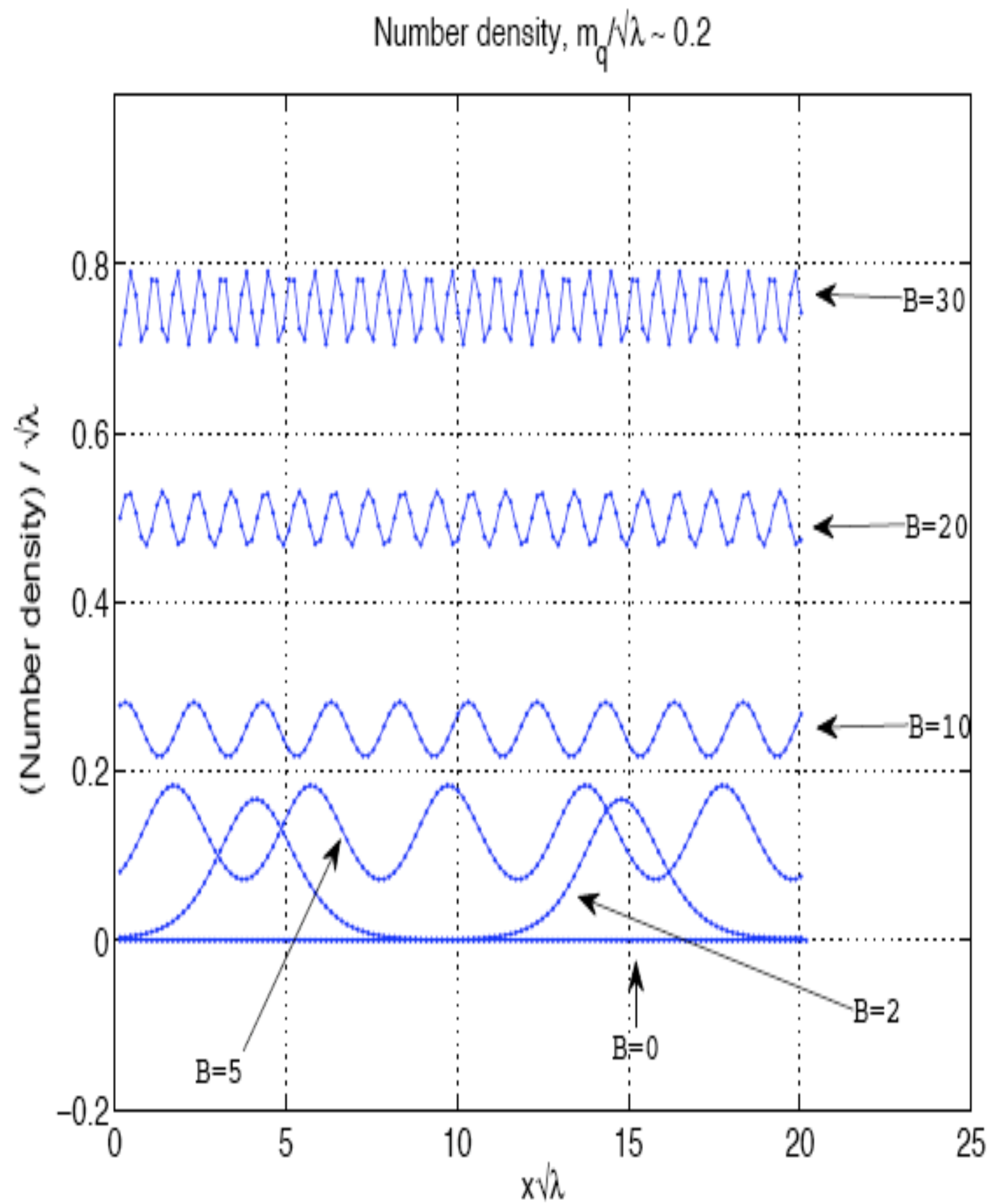
Results : **B=1**



$a\sqrt{\lambda} \simeq 0.15$

Results : **B>1**





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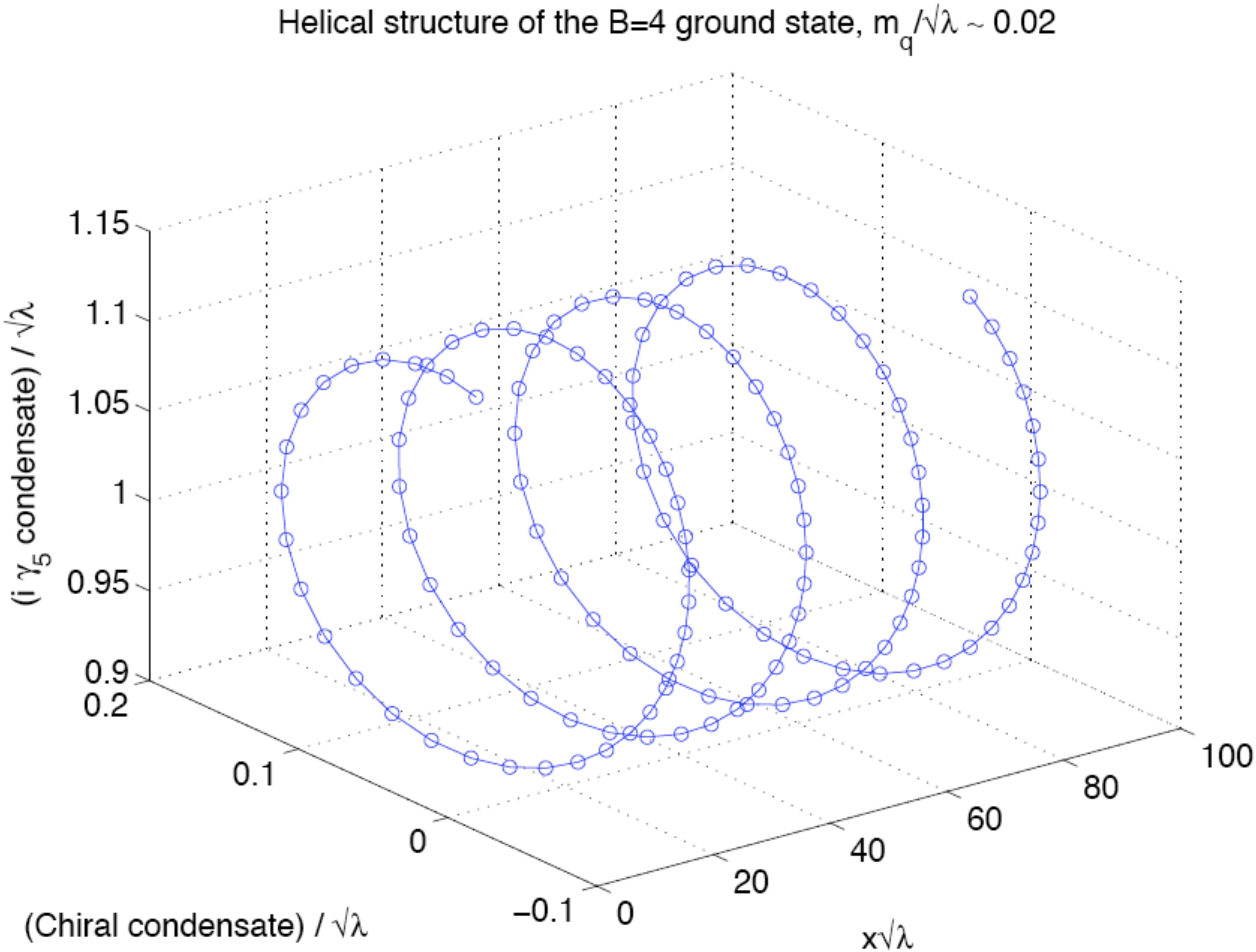
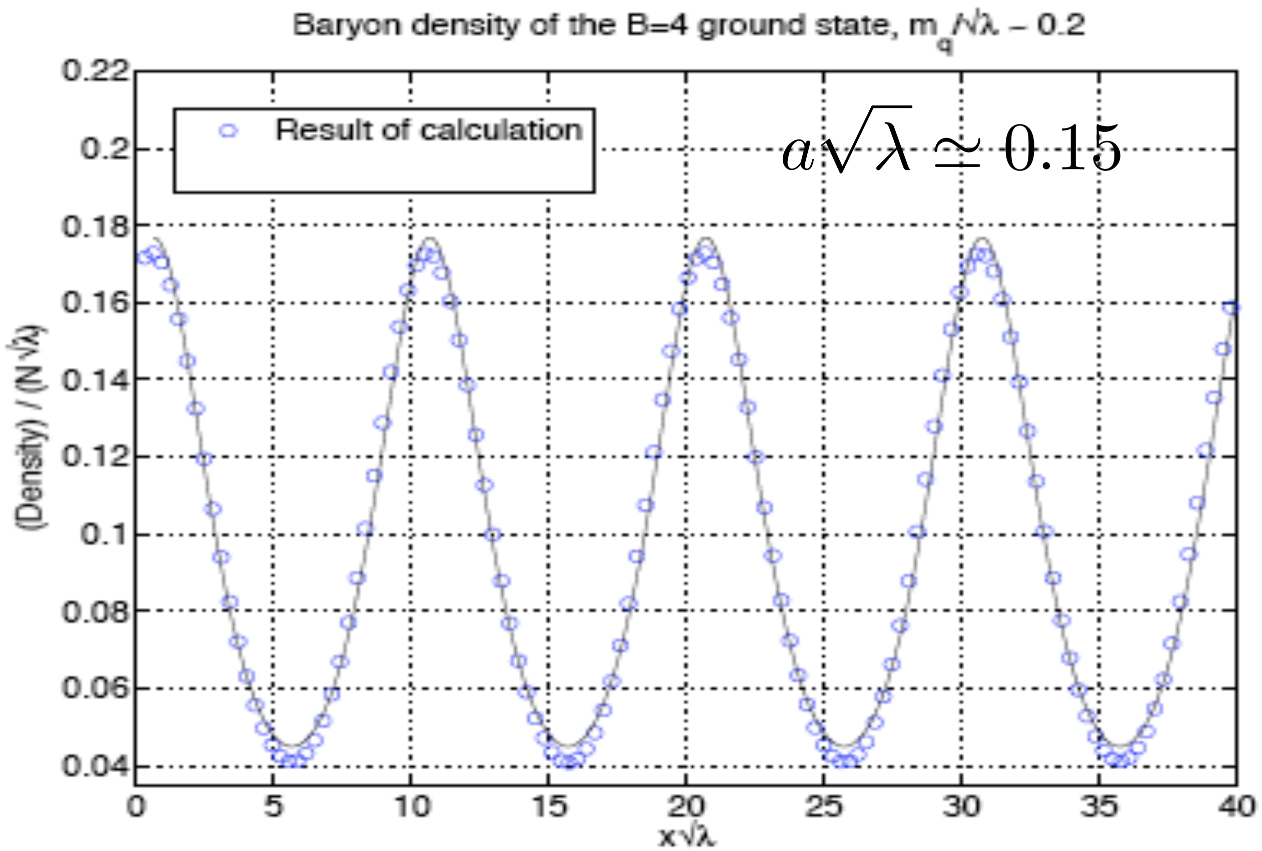
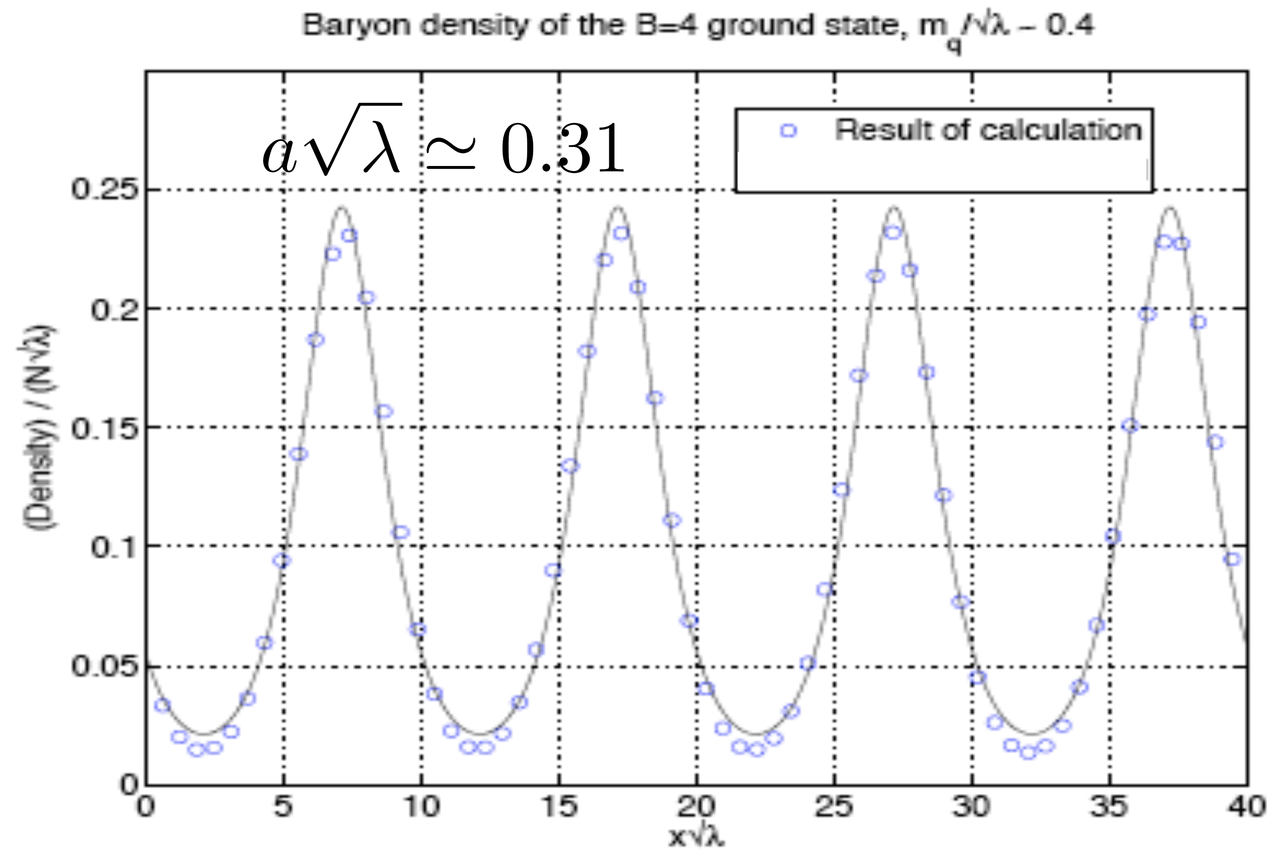
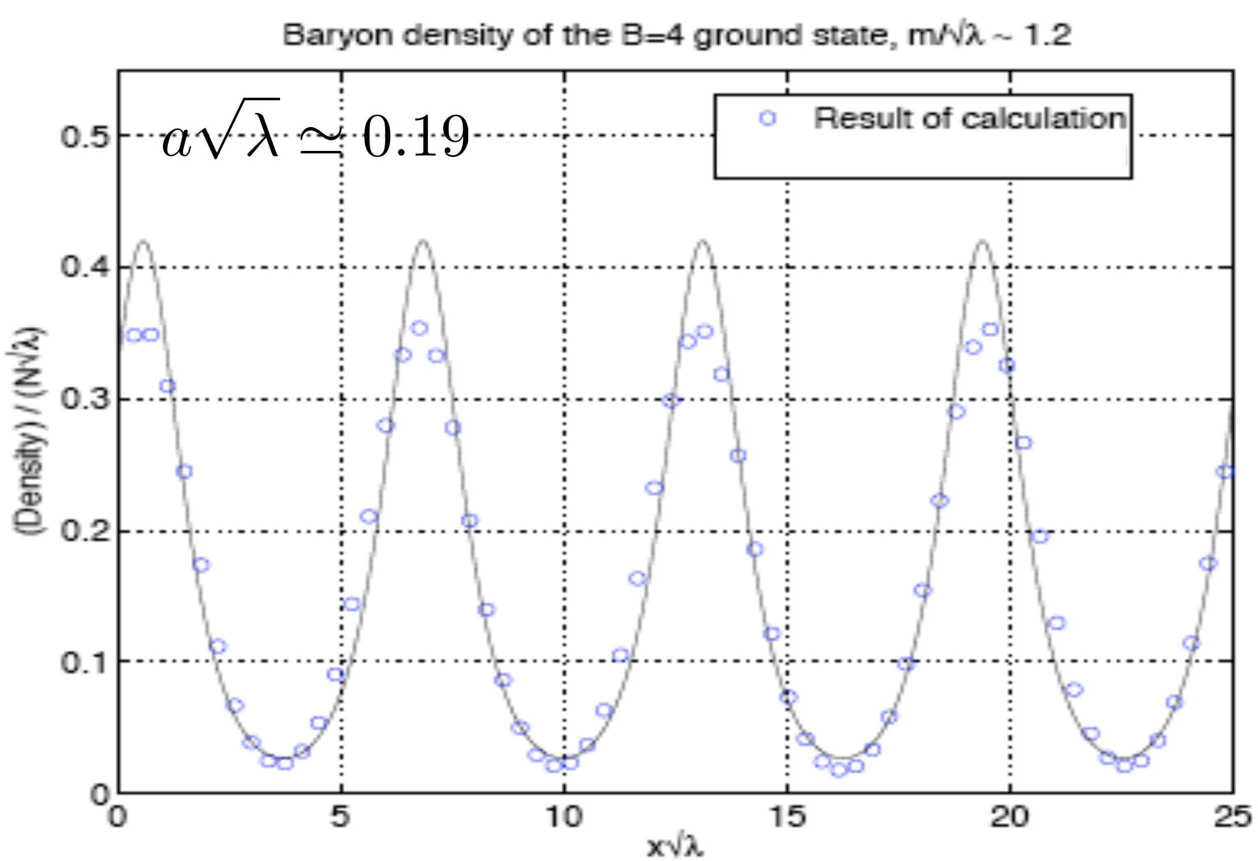
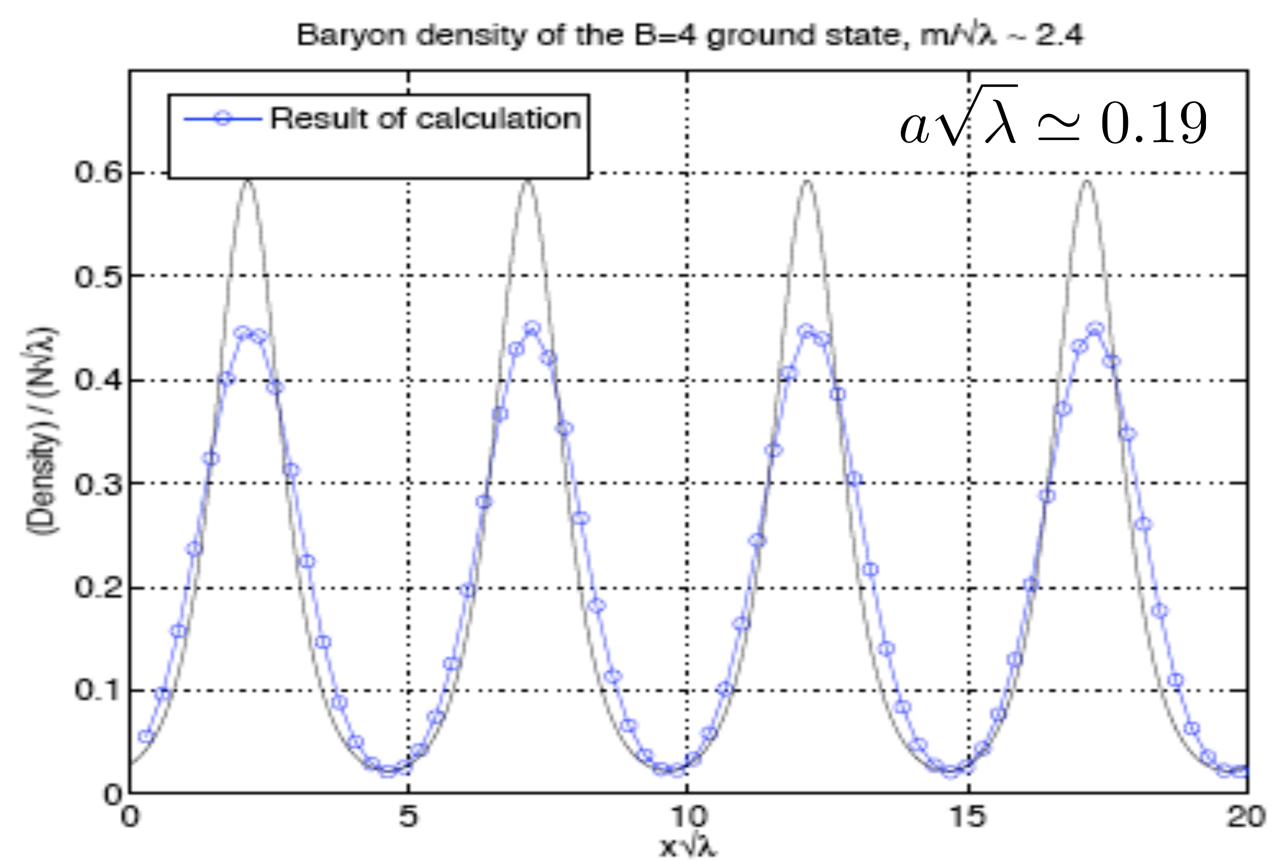


Results : **B>1**

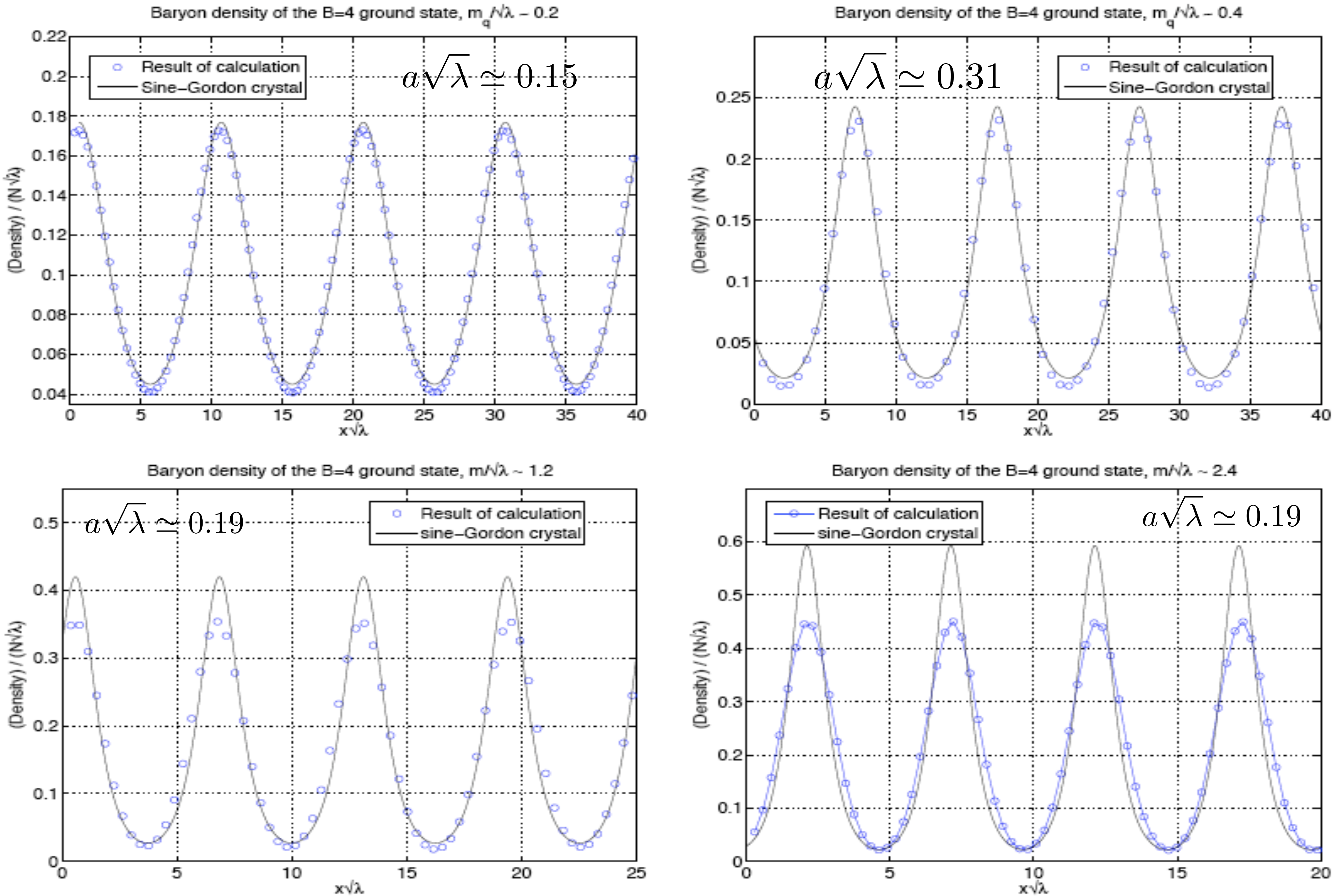

Baryon density of the B=4 ground state, m_q/√λ ~ 0.2
Result of calculation
$a\sqrt{\lambda} \simeq 0.15$
(Density) / (N√λ)
x√λ


Baryon density of the B=4 ground state, m_q/√λ ~ 0.4
Result of calculation
$a\sqrt{\lambda} \simeq 0.31$
(Density) / (N√λ)
x√λ


Baryon density of the B=4 ground state, m/√λ ~ 1.2
Result of calculation
$a\sqrt{\lambda} \simeq 0.19$
(Density) / (N√λ)
x√λ


Baryon density of the B=4 ground state, m/√λ ~ 2.4
Result of calculation
$a\sqrt{\lambda} \simeq 0.19$
(Density) / (N√λ)
x√λ

Results : **B>1**



Baryon density of the B=4 ground state, mq/√λ ~ 0.2
Result of calculation
Sine–Gordon crystal
$a\sqrt{\lambda} \simeq 0.15$
(Density) / (N√λ)
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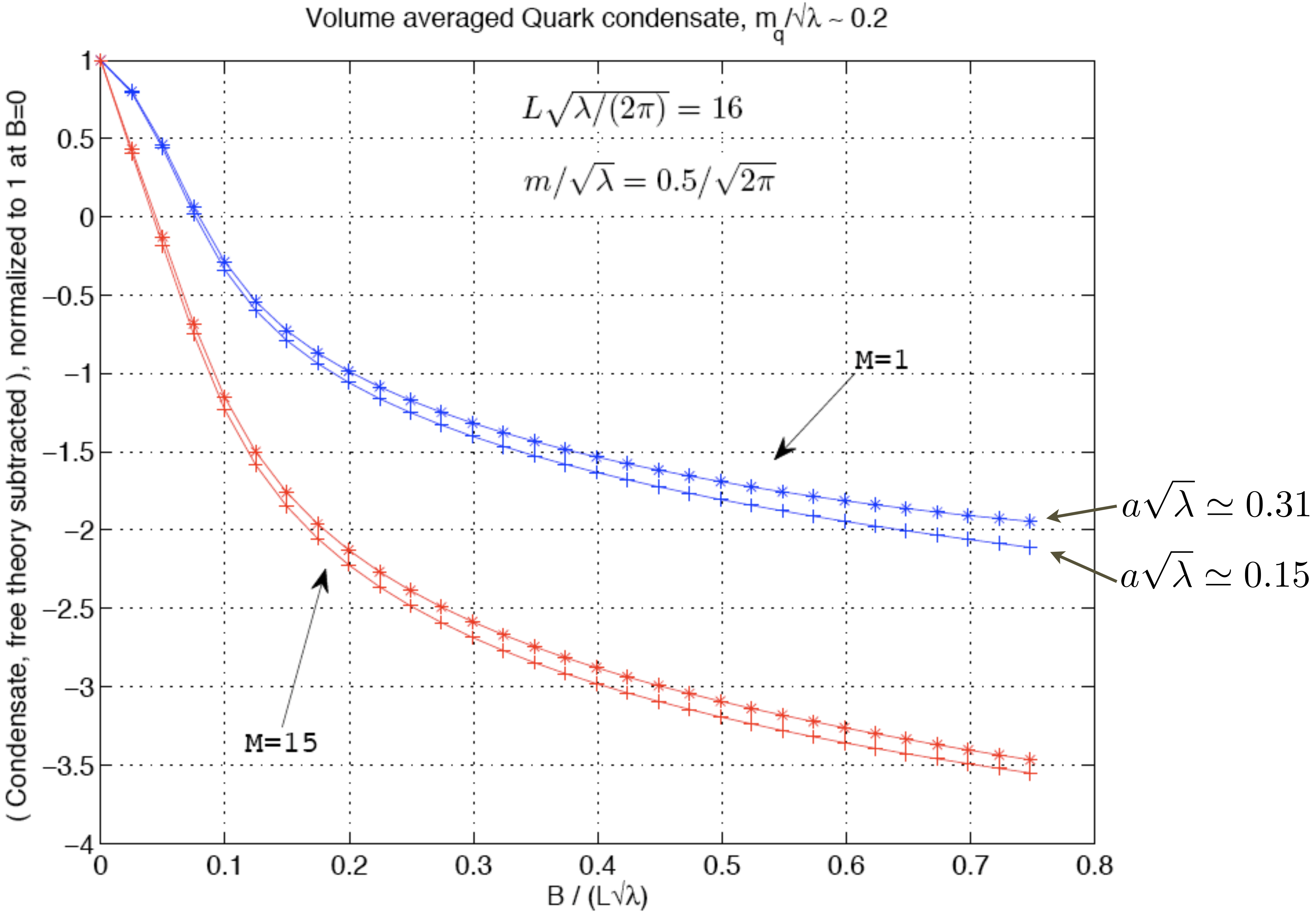
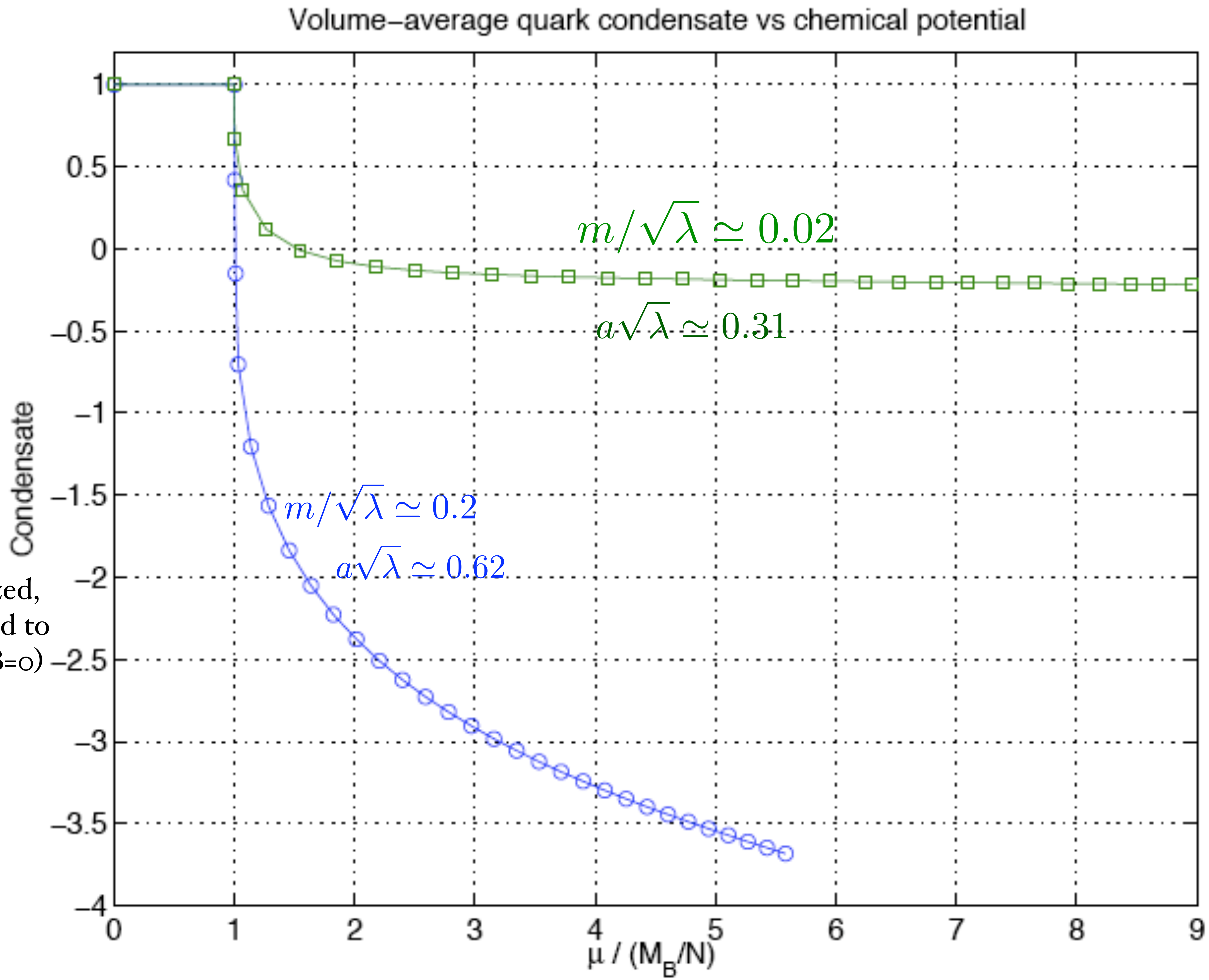


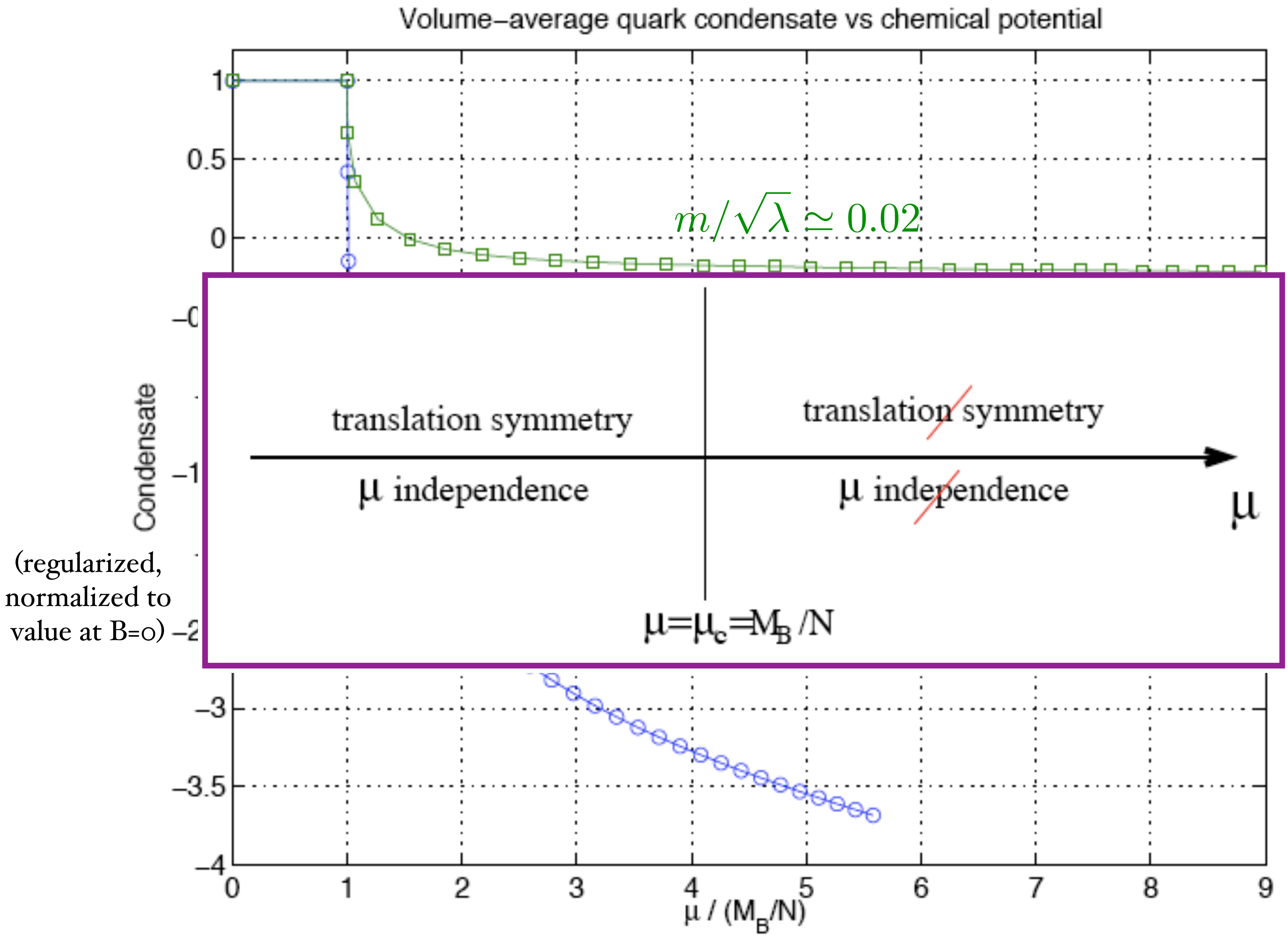
Volume averaged Quark condensate, $m_q/\sqrt{\lambda} \sim 0.2$
$L\sqrt{\lambda/(2\pi)} = 16$
$m/\sqrt{\lambda} = 0.5/\sqrt{2\pi}$
M=1
M=15
$a\sqrt{\lambda} \simeq 0.31$
$a\sqrt{\lambda} \simeq 0.15$
(Condensate, free theory subtracted), normalized to 1 at B=0
B / (L√λ)

Results : B>1



(regularized, normalized to value at B=0)

Results : B>1



Volume–average quark condensate vs chemical potential
$m/\sqrt{\lambda} \simeq 0.02$
translation symmetry
μ independence
translation symmetry
μ independence
μ
$\mu=\mu_c=M_B/N$
Condensate
(regularized, normalized to value at B=0)
μ / (M_B/N)

Go back to L independence :

- We saw that Hamiltonian is volume independence so long as

$$\rho_{xy}^{Q} = f(x - y + QL_s)$$

Z_N symmetry intact — guaranteed by gluons

→

Translation symmetry intact — Only for $\mu < m_B/N$

- Can be useful ?

Study sign problem in 1+1, large-N QCD, at V~0, and $\mu < m_B/N$

$$\langle \det \left(D + \mu\gamma_0 + m \right) \rangle \sim e^{-\# V}$$ *if V~ 0, a "soft" sign problem*

Use volume independence, cont'd

- \`t Hooft model on a single site

$$\mathcal{Z} = \int DU\, e^{\beta\,\mathrm{Re}\,\mathrm{Tr}\,[\,U_0\,U_1\,U_0^\dagger\,U_1^\dagger\,]/N}\ \det\left[m + \frac{1}{2}\gamma_0\left(U_0\,e^{\mu} + U_0^\dagger\,e^{-\mu}\right) + \gamma_1\left(U_1 + U_1^\dagger\right)\right]$$

- Naive (four-fold doubling, not an issue).

1. Generate configs with heat-bath (Fabricius-Haan)
2. Calculate explicitly determinant.
3. Re-weight.

6 x Intel(R)Core(TM) 2.66GHz
a couple of months
N = 10,20,40
$\beta/2N^2$ = 0.6,6,10

$m/\sqrt{\lambda} \simeq 0.56$

$m_\pi/(2\sqrt{\lambda}) \simeq 0.6 - 0.65$

$m_B/\sqrt{N\lambda} \simeq 0.758$

Use volume independence, cont'd

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$$\mathcal{Z} = \int DU\, e^{\beta\,\mathrm{Re\,Tr}\,[\,U_0\,U_1\,U_0^\dagger\,U_1^\dagger\,]/N}\ \det\left[m + \frac{1}{2}\,\gamma_0\left(U_0\,e^{\mu} + U_0^\dagger\,e^{-\mu}\right) + \gamma_1\left(U_1 + U_1^\dagger\right)\right]$$

$$U(N)\ :\ \mu\mathrm{-independence}$$

$$SU(N)\ :\ \mu\mathrm{-dependence\ through\ }\mu N$$

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- `t Hooft model on a single site

$$\mathcal{Z} = \underbrace{\int DU\, e^{\beta\, \mathrm{Re\,Tr}\left[U_0\, U_1\, U_0^\dagger\, U_1^\dagger\right]/N}}_{\text{invariant to center}} \det\left[m + \frac{1}{2}\gamma_0 \left(U_0\, e^{\mu} + U_0^\dagger\, e^{-\mu}\right) + \gamma_1 \left(U_1 + U_1^\dagger\right)\right]$$

$U(N)$: μ−independence $\qquad U \to U e^{i\alpha}$

$SU(N)$: μ−dependence through μN $\qquad U \to U \times \left[e^{2\pi i/N}, e^{4\pi i/N}, \ldots, 1\right]$

Use volume independence, cont'd

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Try to "re-sum" path integral over all Z_N vacua ...

Use volume independence, cont'd

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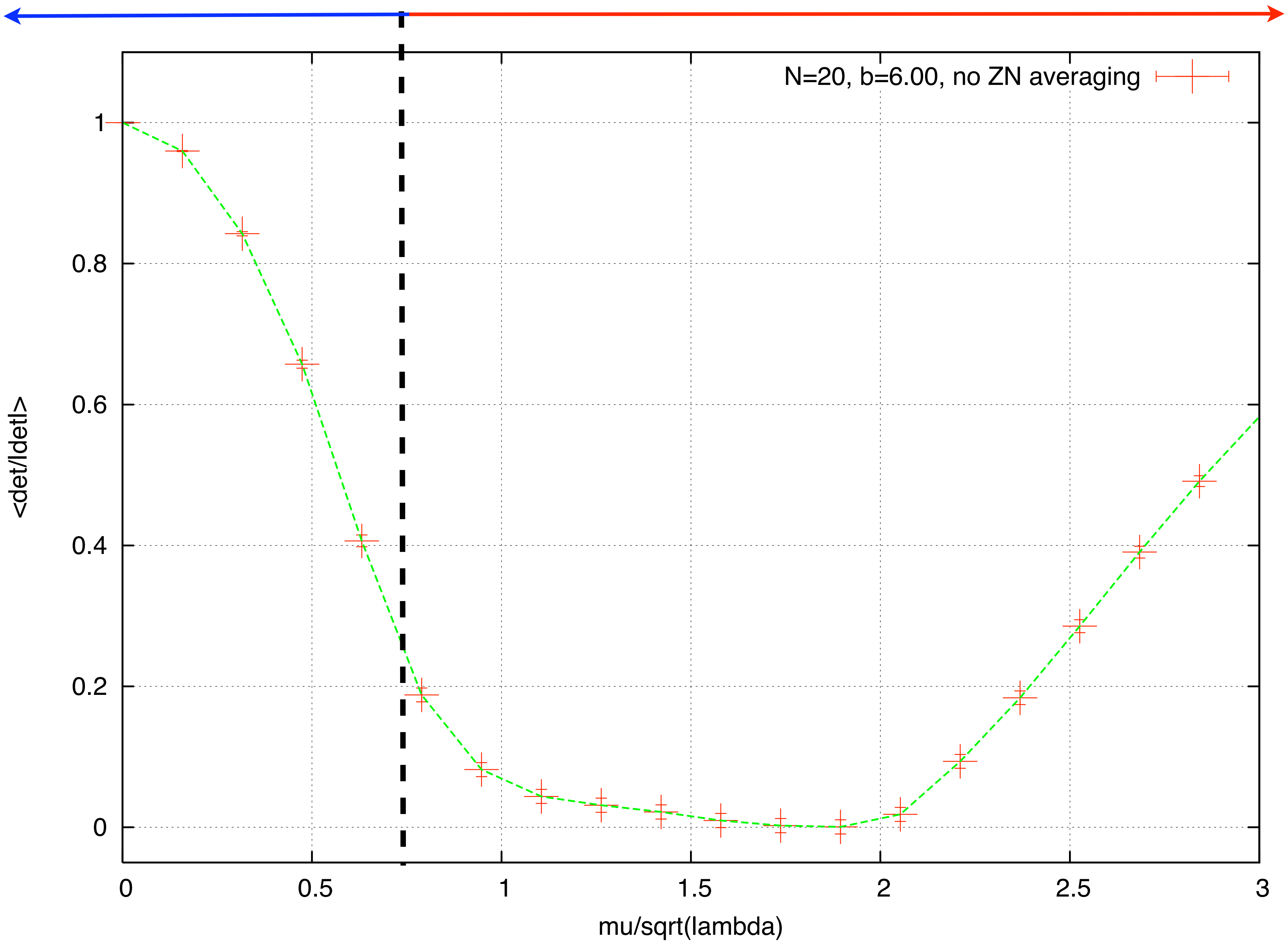
Try to "re-sum" path integral over all Z_N vacua ...

Results : average sign

$$\left\langle \frac{\det\ (D+\mu\gamma_0)}{|\det\,(D+\mu\gamma_0)|} \right\rangle$$

equivalent to large-N QCD2D

Model for sign problem



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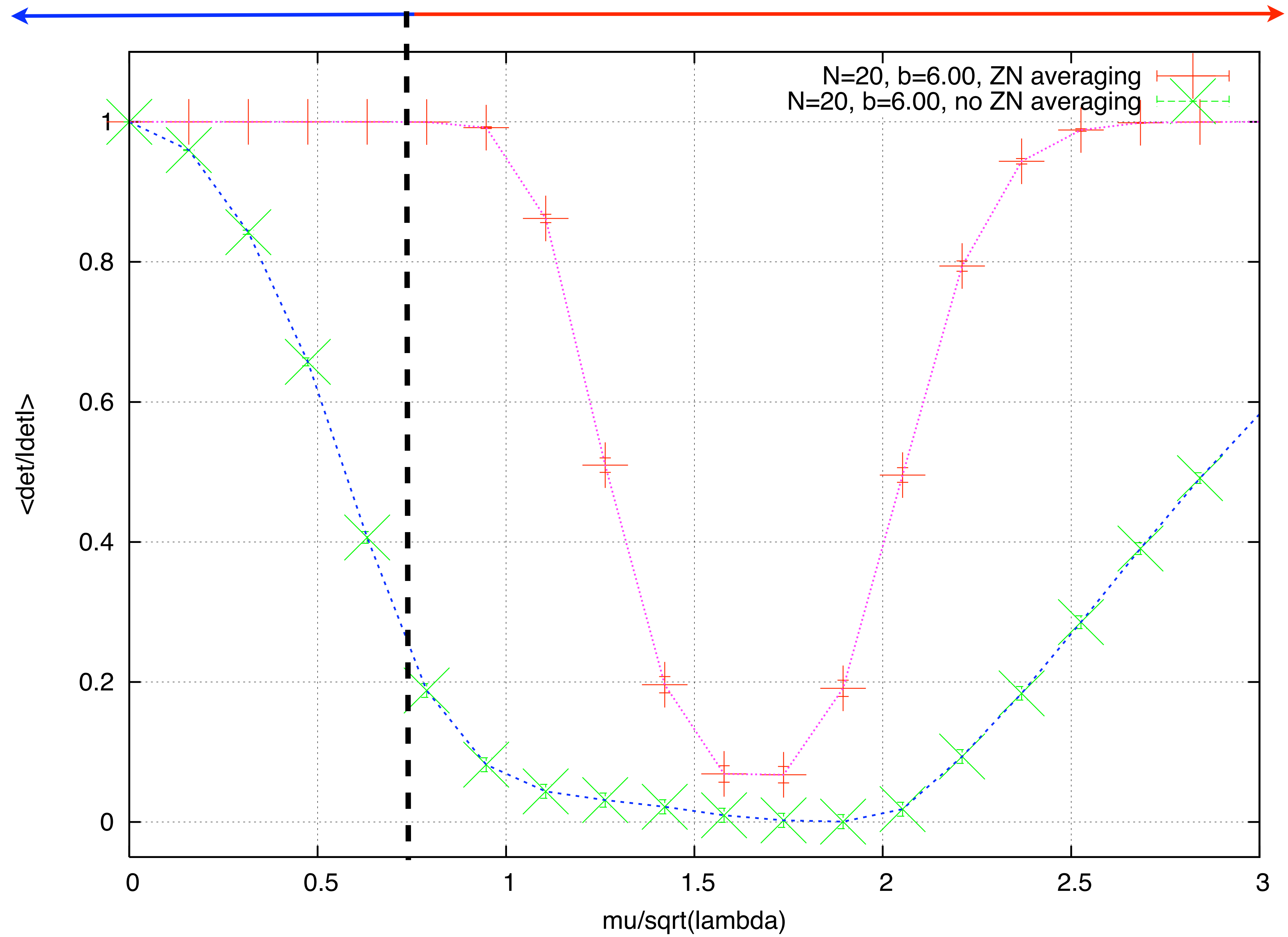
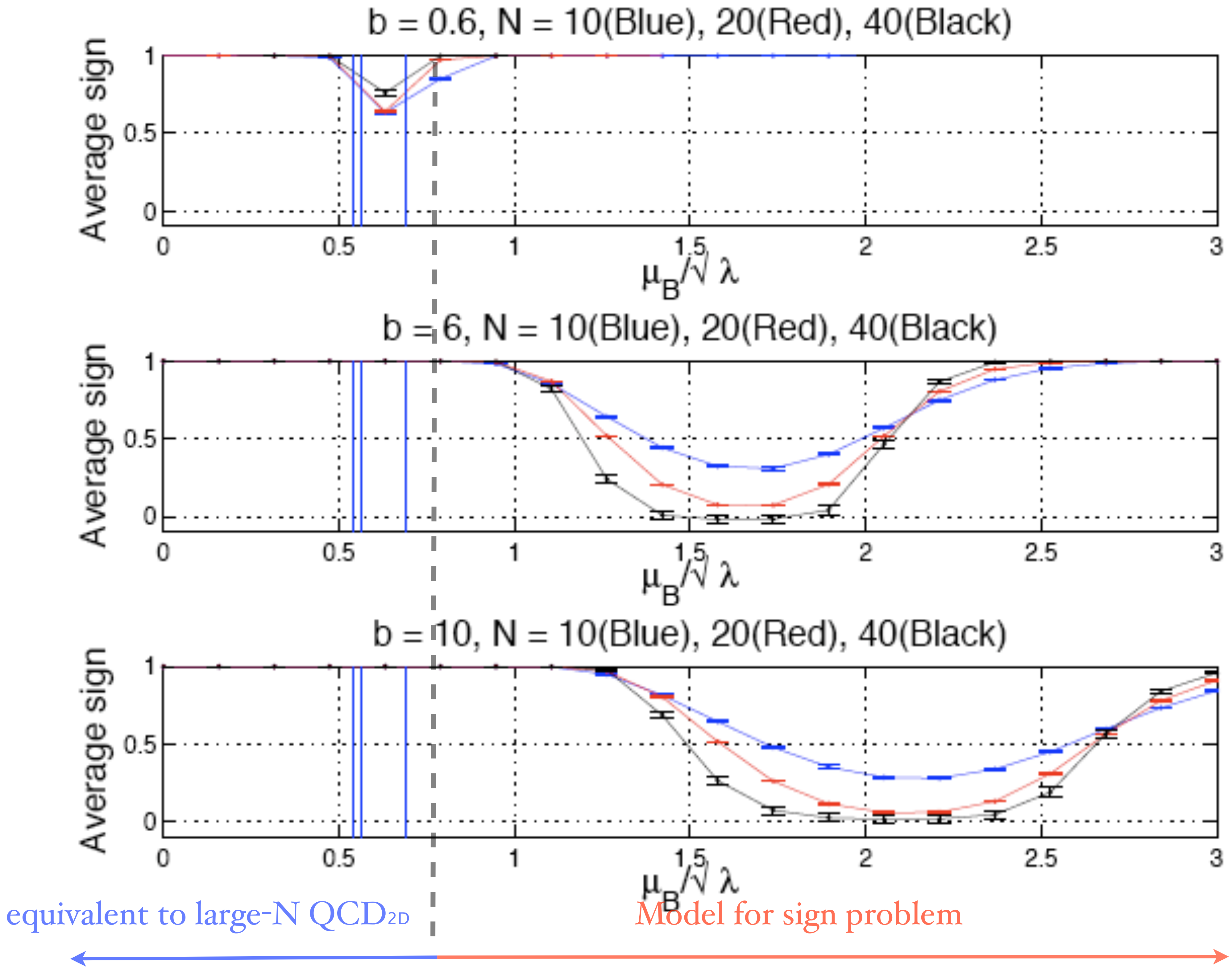
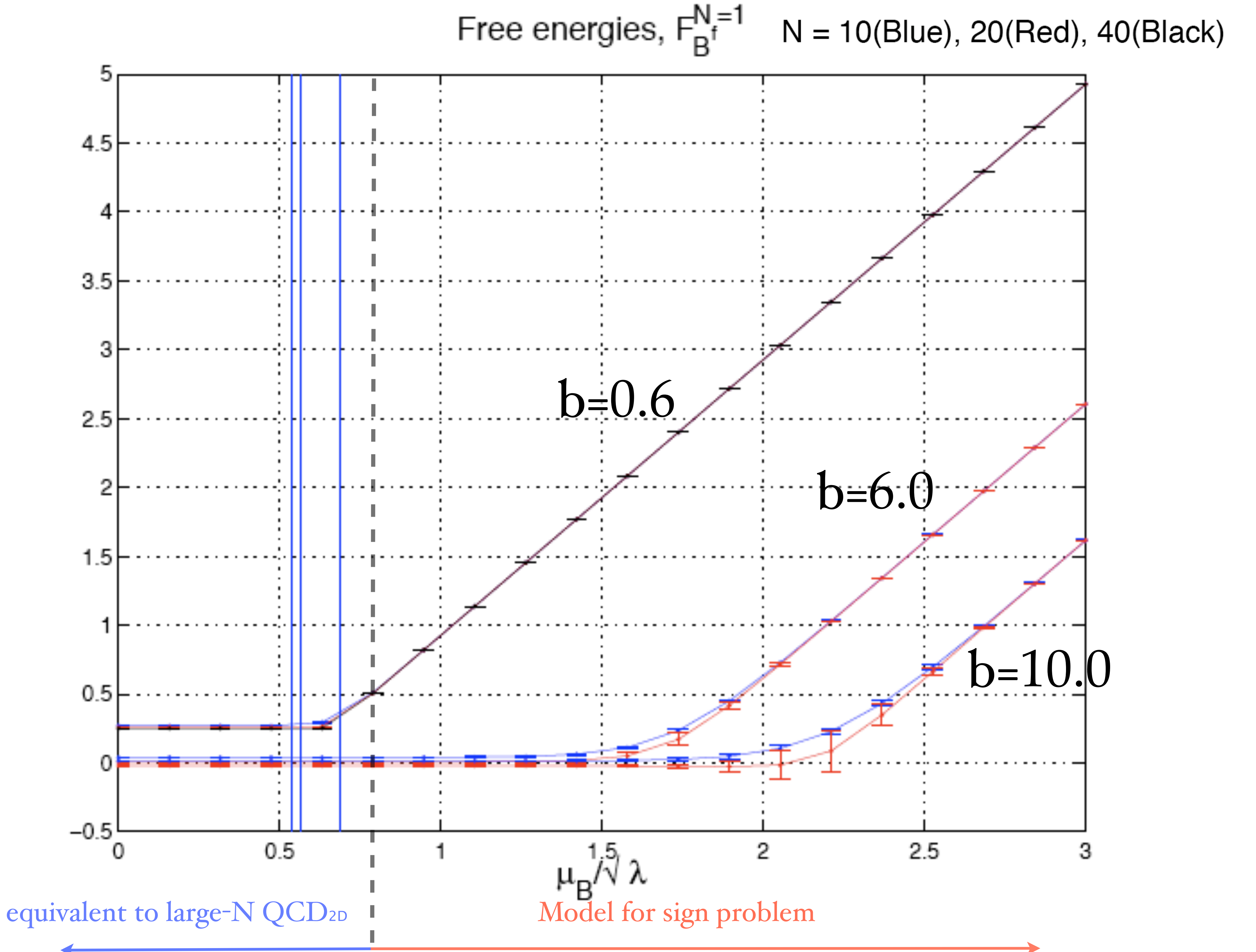


Results : average sign (Z_N averaging)

$$\left\langle \frac{\det\,(D+\mu\gamma_0)}{|\det\,(D+\mu\gamma_0)|} \right\rangle$$



Results : unquenched free energies



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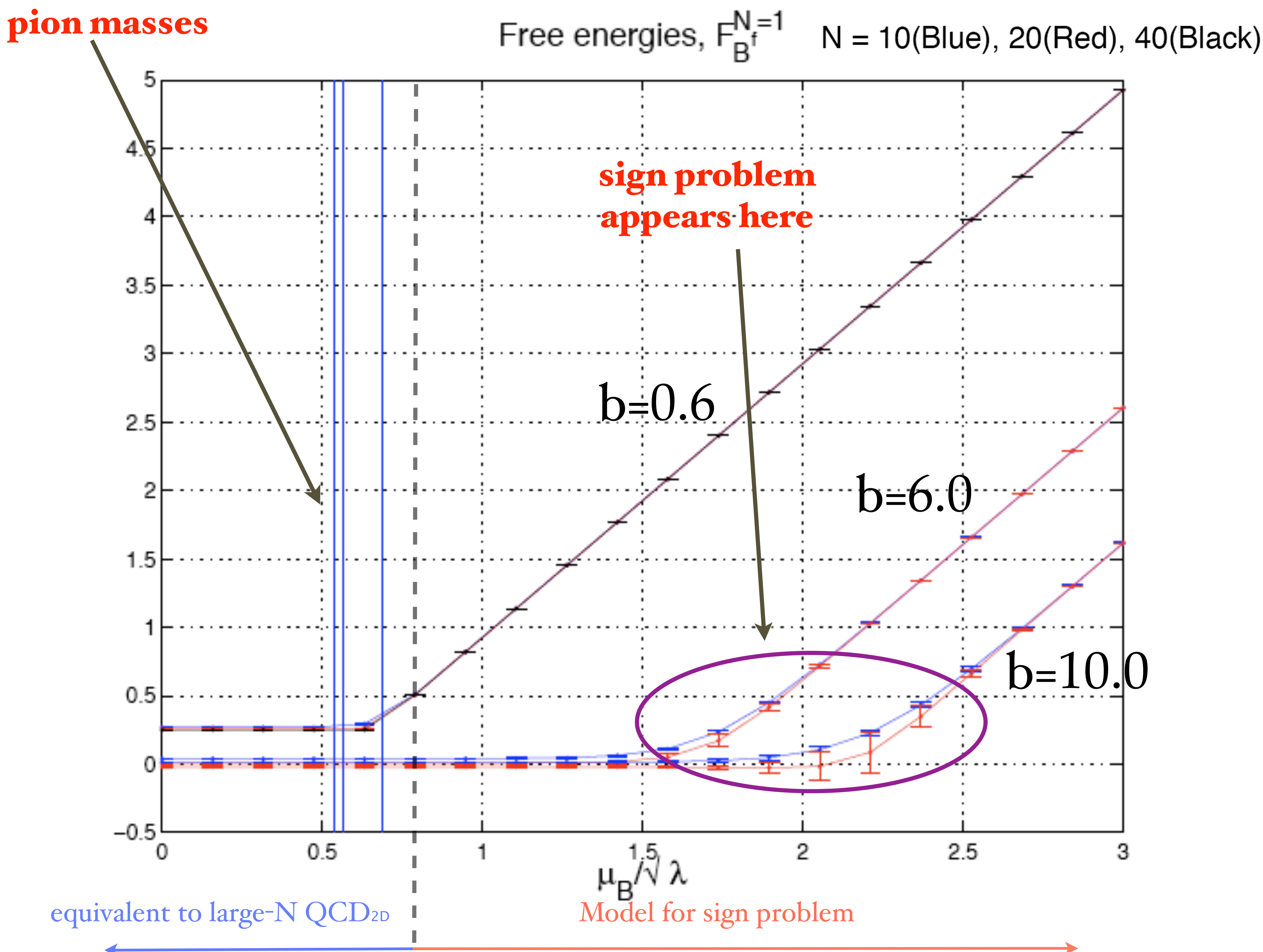

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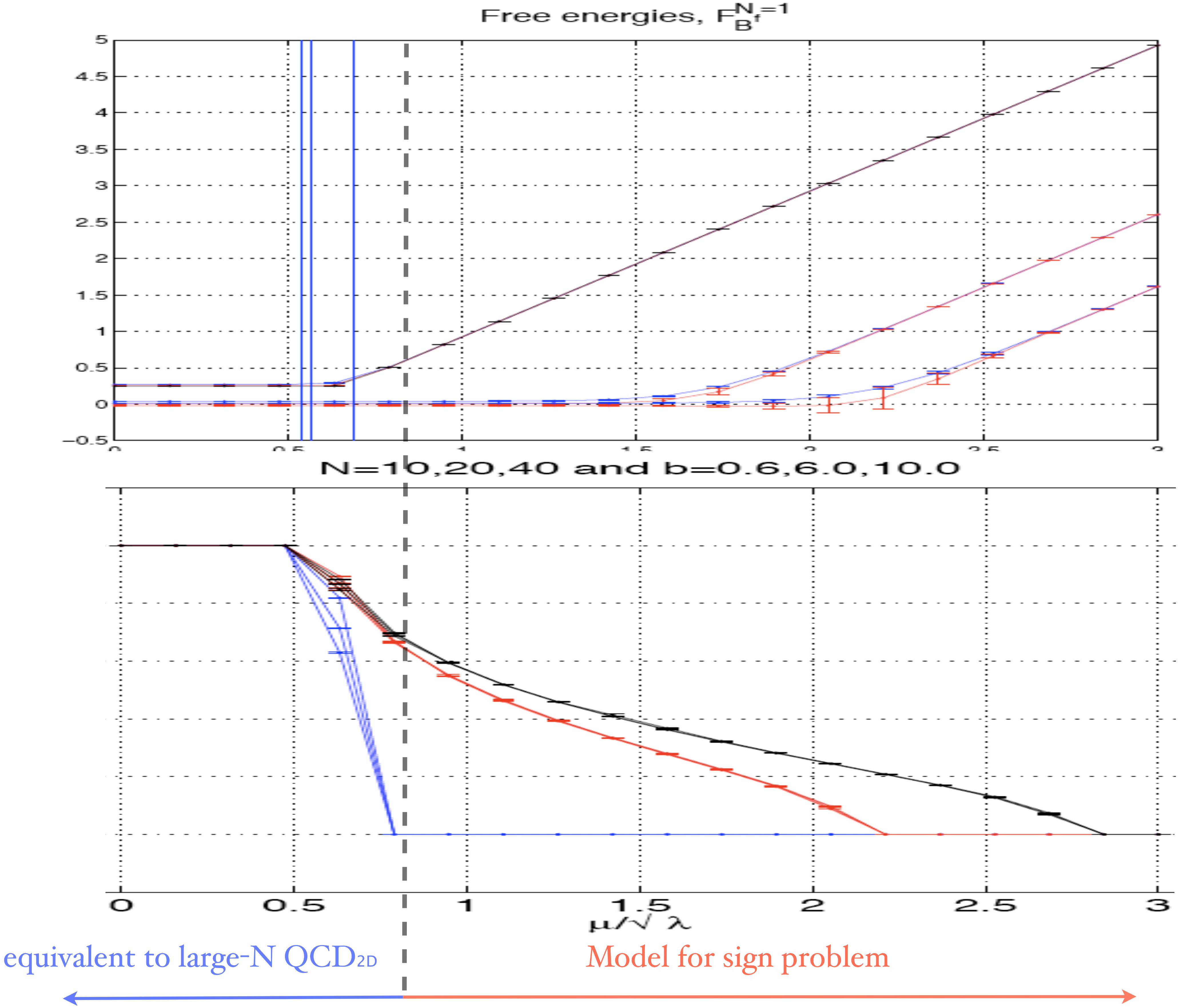
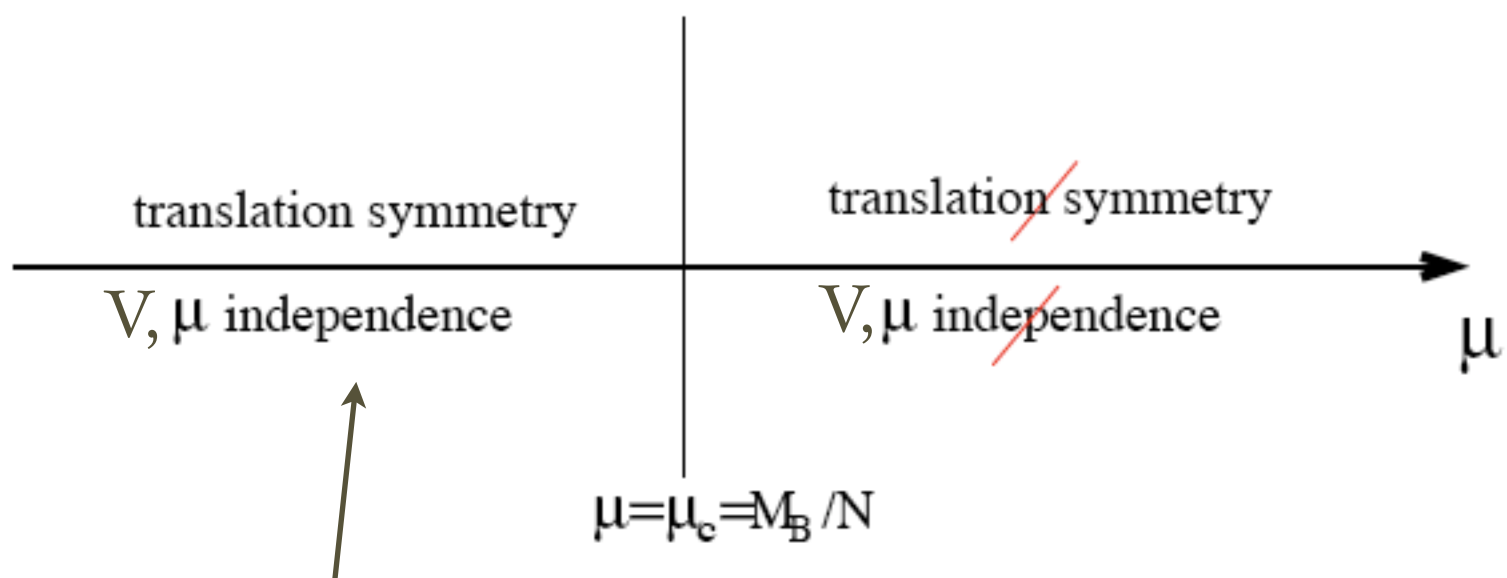


Conclusions

- Solved two-dimensional QCD with nonzero density :
  - Arbitrary mass, B, and L.
  - No ansatz.
  - Treat 0-modes correctly (crucial at large densities)
- Large-N `t Hooft model contains a helical crystal at nonzero B.



- Can study sign problem here with (very) modest resources
  (orthogonal approach)

Conclusions

- Numerical results in "Eguchi-Kawai" model = QCD_{2D} for $\mu < m_B/N$.

    - Sign problem solved for $\mu < m_B/N$ by using Z_N averaging.

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    - Perhaps not outrageous to simulate & re-weight **N=3** on large volumes ?

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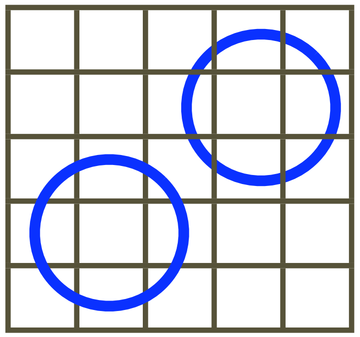
- Can show (numerically and analytically), **at large-N :**
  (as long as confined)

$B = 1, L/B$



B, L

Zero modes

Differential realization of $E^I_{p=0}$: with some work can realize

$$2\sqrt{L_s}\sum_I \lambda^I_a E^I_{p=0} = -i\left(\frac{\delta}{\delta\varphi_a} - \frac{1}{N}\sum_c \frac{\delta}{\delta\varphi_c}\right) - \frac{i}{2}\left[\frac{\delta\,\log\Delta^2(\varphi)}{\delta\varphi_a} - \frac{1}{N}\sum_c \frac{\delta\log\,\Delta^2}{\delta\varphi_c}\right],$$

$$\Delta^2(\{\varphi\}) = \prod_{a<b}\sin^2\left(L_s\frac{\varphi_a-\varphi_b}{2}\right).$$

Redefining wave function of zero modes

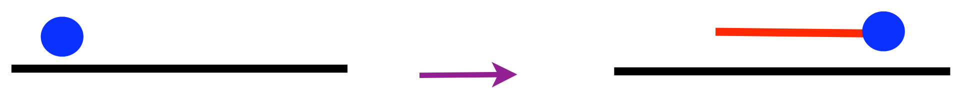
$$\Psi(\varphi) \equiv \frac{\Psi_{\mathrm{new}}(\varphi)}{\prod_{a<b}\sin\left(L_s\frac{\varphi_a-\varphi_b}{2}\right)}.$$

$$\frac{g^2}{2}\sum_I\left(E^I_{p=0}\right)^2 = -\frac{g^2L_s}{4}\sum_{d=1}^N\left(\frac{\delta}{\delta\varphi_d} - \frac{1}{N}\sum_c\frac{\delta}{\delta\varphi_c}\right)^2$$

Coherent states, cont'd

- Fermionic sector : choose reference state $|0\rangle = |B \text{ static baryons}\rangle$.

$$|\theta\rangle \equiv \mathcal{U}(\theta)|0\rangle = \exp\left(-i \sum_{x\in Z_{L_s}} \sum_{y\in Z} \theta_{xy}\, \psi_x^{\dagger\, a}\, e^{i\varphi_a(y-x)/L_s}\, \psi_y^a\right) |0\rangle,$$



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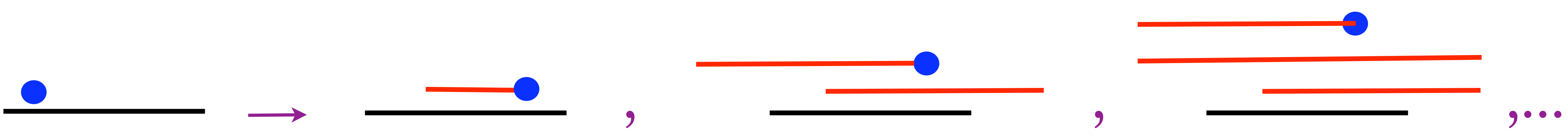

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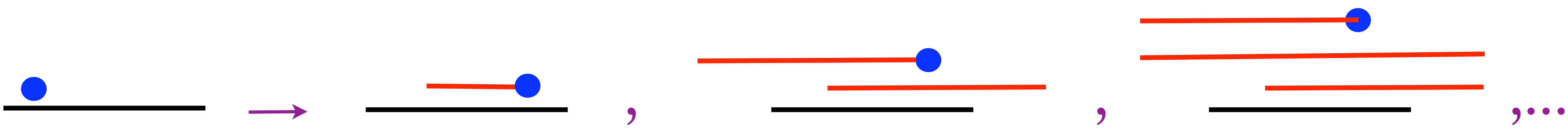

- Next, calculate $\mathcal{H}_F(\theta) = \langle\theta|H_F|\theta\rangle$

with

$$H_F = -\frac{i}{2}\sum_x \psi_x^{\dagger a}\, e^{i\varphi_a/L_s}\, \psi_{x+1}^a + h.c. + m\sum_x (-1)^x\, \psi_x^{\dagger a}\psi_x^a \quad + \quad \frac{g^2}{L_s}\sum_{abp}{}' \frac{\rho_{F,x}^{ab}\, \rho_{F,y}^{ba}\, e^{ip(y-x)}}{4\sin^2\left(\frac{(\varphi_a-\varphi_b)/L_s+p}{2}\right)}$$

takes some time, only show what $\mathcal{H}_F$ depends on ...

Coherent states, cont'd

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Coherent states, cont'd

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- Calculate

$$\mathcal{H}_F = \langle\theta|H_F(\psi_x)|\theta\rangle = \langle 0|H_F(U_\theta\,\psi_x\,U_\theta^\dagger)|0\rangle$$

,...

Coherent states, cont'd

$$H_F = -\frac{i}{2}\sum_x \psi_x^{\dagger a}\, e^{i\varphi_a/L_s}\, \psi^a_{x+1} + h.c. + m\sum_x (-1)^x\, \psi_x^{\dagger a}\psi_x^a \quad + \quad \frac{g^2}{L_s}\sum_{abp}{}' \frac{\rho^{ab}_{F,x}\,\rho^{ba}_{F,y}\, e^{ip(y-x)}}{4\sin^2\left(\frac{(\varphi_a-\varphi_b)/L_s+p}{2}\right)}$$

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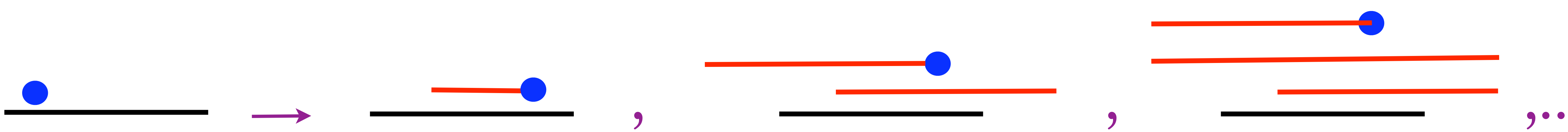
$$\mathcal{H}_F = \langle\theta|H_F(\psi_x)|\theta\rangle = \langle 0|H_F(U_\theta\,\psi_x\,U_\theta^\dagger)|0\rangle$$

$$U_\theta\,\psi_x^a\,U_\theta^\dagger = \sum_{y\in Z}[\mathcal{A}(\theta)]_{xy}\; e^{i\varphi_a(y-x)/L_s}\;\psi_y^a$$



,...

Coherent states, cont'd

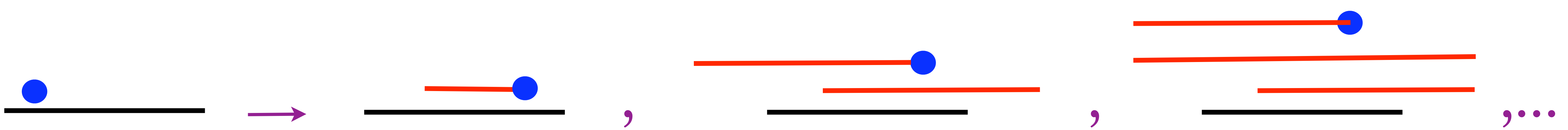
$$H_F = -\frac{i}{2}\sum_x \psi_x^{\dagger a}\, e^{i\varphi_a/L_s}\, \psi_{x+1}^a + h.c. + m\sum_x (-1)^x\, \psi_x^{\dagger a}\psi_x^a \quad + \quad \frac{g^2}{L_s}{\sum_{abp}}' \frac{\rho_{F,x}^{ab}\,\rho_{F,y}^{ba}\, e^{ip(y-x)}}{4\sin^2\left(\frac{(\varphi_a-\varphi_b)/L_s+p}{2}\right)}$$

$$|\theta\rangle \equiv \mathcal{U}(\theta)|0\rangle = \exp\left(-i\sum_{x\in Z_{L_s}}\sum_{y\in Z}\theta_{xy}\,\psi_x^{\dagger\, a}\, e^{i\varphi_a(y-x)/L_s}\,\psi_y^a\right)|0\rangle,$$

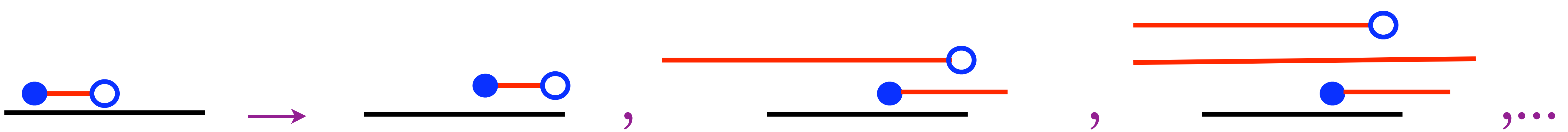
- Calculate

$$\mathcal{H}_F = \langle\theta|H_F(\psi_x)|\theta\rangle = \langle 0|H_F(U_\theta\,\psi_x\,U_\theta^\dagger)|0\rangle$$

$$U_\theta\,\psi_x^a\,U_\theta^\dagger = \sum_{y\in Z}[\mathcal{A}(\theta)]_{xy}\; e^{i\varphi_a(y-x)/L_s}\;\psi_y^a$$



$$U_\theta\left(\sum_a \psi_x^{\dagger a}\, e^{i\varphi_a(y-x)/L_s}\;\psi_y^a\right)U_\theta^\dagger = \sum_{wv\in Z}[\mathcal{A}(\theta)]_{yv}\;[\mathcal{A}^\star(\theta)]_{xw}\left(\sum_a \psi_w^{\dagger a}\, e^{i\varphi_a(v-w)/L_s}\;\psi_v^a\right)$$



Coherent states, cont'd

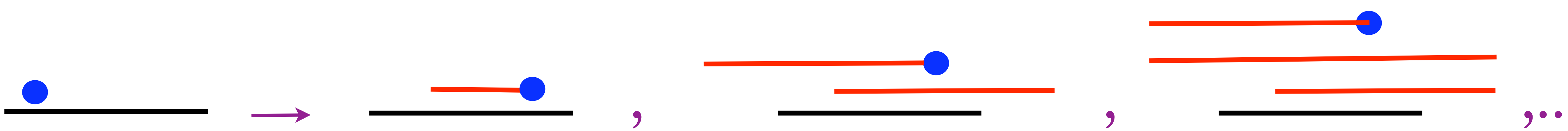
$$H_F = -\frac{i}{2}\sum_x \psi_x^{\dagger a}\, e^{i\varphi_a/L_s}\, \psi_{x+1}^a + h.c. + m\sum_x (-1)^x\, \psi_x^{\dagger a}\psi_x^a \quad + \quad \frac{g^2}{L_s}\sum_{abp}{}' \frac{\rho_{F,x}^{ab}\,\rho_{F,y}^{ba}\, e^{ip(y-x)}}{4\sin^2\left(\frac{(\varphi_a-\varphi_b)/L_s+p}{2}\right)}$$

$$|\theta\rangle \equiv \mathcal{U}(\theta)|0\rangle = \exp\left(-i\sum_{x\in Z_{L_s}}\sum_{y\in Z}\theta_{xy}\,\psi_x^{\dagger a}\, e^{i\varphi_a(y-x)/L_s}\,\psi_y^a\right)|0\rangle,$$

- Calculate

$$\mathcal{H}_F = \langle\theta|H_F(\psi_x)|\theta\rangle = \langle 0|H_F(U_\theta\,\psi_x\,U_\theta^\dagger)|0\rangle$$

$$U_\theta\,\psi_x^a\,U_\theta^\dagger = \sum_{y\in Z}[\mathcal{A}(\theta)]_{xy}\; e^{i\varphi_a(y-x)/L_s}\;\psi_y^a$$



$$U_\theta\left(\sum_a \psi_x^{\dagger a}\, e^{i\varphi_a(y-x)/L_s}\,\psi_y^a\right)U_\theta^\dagger = \sum_{wv\in Z}[\mathcal{A}(\theta)]_{yv}\;[\mathcal{A}^\star(\theta)]_{xw}\left(\sum_a \psi_w^{\dagger a}\, e^{i\varphi_a(v-w)/L_s}\,\psi_v^a\right)$$

Coherent states, cont'd

$$H_F = -\frac{i}{2}\sum_x \psi_x^{\dagger a}\, e^{i\varphi_a/L_s}\, \psi_{x+1}^a + h.c. + m\sum_x (-1)^x\, \psi_x^{\dagger a}\psi_x^a \quad + \quad \frac{g^2}{L_s}\sum_{abp}{}' \frac{\rho_{F,x}^{ab}\,\rho_{F,y}^{ba}\, e^{ip(y-x)}}{4\sin^2\left(\frac{(\varphi_a-\varphi_b)/L_s+p}{2}\right)}$$

$$|\theta\rangle \equiv \mathcal{U}(\theta)|0\rangle = \exp\left(-i \sum_{x\in Z_{L_s}} \sum_{y\in Z} \theta_{xy}\, \psi_x^{\dagger\, a}\, e^{i\varphi_a(y-x)/L_s}\, \psi_y^a\right) |0\rangle,$$

- Calculate

$$\mathcal{H}_F = \langle\theta|H_F(\psi_x)|\theta\rangle = \langle 0|H_F(U_\theta\, \psi_x\, U_\theta^\dagger)|0\rangle$$

$$\boxed{U_\theta \left(\sum_a \psi_x^{\dagger a}\, e^{i\varphi_a(y-x)/L_s}\, \psi_y^a\right) U_\theta^\dagger = \sum_{wv\in Z} [\mathcal{A}(\theta)]_{yv}\, [\mathcal{A}^\star(\theta)]_{xw} \left(\sum_a \psi_w^{\dagger a}\, e^{i\varphi_a(v-w)/L_s}\, \psi_v^a\right)}$$

Coherent states, cont'd

$$H_F = -\frac{i}{2}\sum_x \psi_x^{\dagger a}\, e^{i\varphi_a/L_s}\, \psi_{x+1}^a + h.c. + m\sum_x (-1)^x\, \psi_x^{\dagger a}\psi_x^a \quad + \quad \frac{g^2}{L_s}\sum_{abp}{}' \frac{\rho_{F,x}^{ab}\, \rho_{F,y}^{ba}\, e^{ip(y-x)}}{4\sin^2\left(\frac{(\varphi_a-\varphi_b)/L_s+p}{2}\right)}$$

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$$\sum_a\sum_{q\in Z} e^{i\varphi_a(v-w)/L_s}\delta_{v-w,qL_s} = \sum_{q\in Z} P_q\delta_{v-w,qL_s}$$

Coherent states, cont'd

$$H_F = -\frac{i}{2}\sum_x \psi_x^{\dagger a}\, e^{i\varphi_a/L_s}\,\psi_{x+1}^a + h.c. + m\sum_x (-1)^x\,\psi_x^{\dagger a}\psi_x^a \quad + \quad \frac{g^2}{L_s}\sum_{abp}{}' \frac{\rho_{F,x}^{ab}\,\rho_{F,y}^{ba}\,e^{ip(y-x)}}{4\sin^2\left(\frac{(\varphi_a-\varphi_b)/L_s+p}{2}\right)}$$

$$|\theta\rangle \equiv \mathcal{U}(\theta)|0\rangle = \exp\left(-i\sum_{x\in Z_{L_s}}\sum_{y\in Z}\theta_{xy}\,\psi_x^{\dagger a}\,e^{i\varphi_a(y-x)/L_s}\,\psi_y^a\right)|0\rangle,$$

- Calculate

$$\mathcal{H}_F = \langle\theta|H_F(\psi_x)|\theta\rangle = \langle 0|H_F(U_\theta\,\psi_x\,U_\theta^\dagger)|0\rangle$$

$$\boxed{\langle 0|U_\theta\left(\sum_a \psi_x^{\dagger a}\,e^{i\varphi_a(y-x)/L_s}\,\psi_y^a\right)U_\theta^\dagger|0\rangle = \sum_{wv\in Z}[\mathcal{A}(\theta)]_{yv}\,[\mathcal{A}^\star(\theta)]_{xw}\,\langle 0|\left(\sum_a \psi_w^{\dagger a}\,e^{i\varphi_a(v-w)/L_s}\,\psi_v^a\right)|0\rangle}$$

$$\sum_a\sum_{q\in Z} e^{i\varphi_a(v-w)/L_s}\delta_{v-w,qL_s} = \sum_{q\in Z}P_q\delta_{v-w,qL_s}$$

$$\langle 0|U_\theta\left(\sum_a \psi_x^{\dagger a}\,e^{i\varphi_a(y-x)/L_s}\,\psi_y^a\right)U_\theta^\dagger|0\rangle = \sum_{q\in Z}\sum_{w\in Z}[\mathcal{A}(\theta)]_{y,w+qL_s}\,[\mathcal{A}^\star(\theta)]_{xw}\,\times P_q$$

Coherent states, cont'd

$$H_F = -\frac{i}{2}\sum_x \psi_x^{\dagger a}\, e^{i\varphi_a/L_s}\, \psi_{x+1}^a + h.c. + m\sum_x (-1)^x\, \psi_x^{\dagger a}\psi_x^a \quad + \quad \frac{g^2}{L_s}{\sum_{abp}}' \frac{\rho_{F,x}^{ab}\,\rho_{F,y}^{ba}\, e^{ip(y-x)}}{4\sin^2\left(\frac{(\varphi_a-\varphi_b)/L_s+p}{2}\right)}$$

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$$\boxed{\langle 0|U_\theta\left(\sum_a \psi_x^{\dagger a}\, e^{i\varphi_a(y-x)/L_s}\,\psi_y^a\right)U_\theta^\dagger|0\rangle = \sum_{wv\in Z}[\mathcal{A}(\theta)]_{yv}\,[\mathcal{A}^\star(\theta)]_{xw}\,\langle 0|\left(\sum_a \psi_w^{\dagger a}\, e^{i\varphi_a(v-w)/L_s}\,\psi_v^a\right)|0\rangle}$$

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$$\langle 0|U_\theta\left(\sum_a \psi_x^{\dagger a}\, e^{i\varphi_a(y-x)/L_s}\,\psi_y^a\right)U_\theta^\dagger|0\rangle \equiv \sum_{q\in Z}\rho_{xy}^q\times P_q$$

Coherent states, cont'd

$$H_F = -\frac{i}{2}\sum_x \psi_x^{\dagger a}\, e^{i\varphi_a/L_s}\, \psi^a_{x+1} + h.c. + m\sum_x (-1)^x\, \psi_x^{\dagger a}\psi_x^a \quad + \quad \frac{g^2}{L_s}\sum_{abp}{}' \frac{\rho^{ab}_{F,x}\,\rho^{ba}_{F,y}\, e^{ip(y-x)}}{4\sin^2\left(\frac{(\varphi_a-\varphi_b)/L_s+p}{2}\right)}$$

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$$\mathcal{H}_F = \langle\theta|H_F(\psi_x)|\theta\rangle = \langle 0|H_F(U_\theta\,\psi_x\,U_\theta^\dagger)|0\rangle$$

$$\boxed{\langle 0|U_\theta\left(\sum_a \psi_x^{\dagger a}\, e^{i\varphi_a(y-x)/L_s}\,\psi_y^a\right)U_\theta^\dagger|0\rangle = \sum_{wv\in Z}[\mathcal{A}(\theta)]_{yv}\,[\mathcal{A}^\star(\theta)]_{xw}\,\langle 0|\left(\sum_a \psi_w^{\dagger a}\, e^{i\varphi_a(v-w)/L_s}\,\psi_v^a\right)|0\rangle}$$

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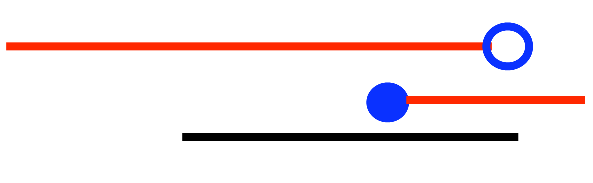
$$\langle 0|U_\theta\left(\sum_a \psi_x^{\dagger a}\, e^{i\varphi_a(y-x)/L_s}\,\psi_y^a\right)U_\theta^\dagger|0\rangle = \sum_{q\in Z}\sum_{w\in Z}[\mathcal{A}(\theta)]_{y,w+qL_s}\,[\mathcal{A}^\star(\theta)]_{xw}\times P_q$$

$$\langle 0|U_\theta\left(\sum_a \psi_x^{\dagger a}\, e^{i\varphi_a(y-x-QL_s)/L_s}\,\psi_y^a\right)U_\theta^\dagger|0\rangle \equiv \sum_{q\in Z}\rho^q_{xy}\times P_{q-Q}$$



Coherent states, cont'd

- So find $\mathcal{H}_F$ depends on

$$\sum_{q\in Z}\rho^q_{xy}\times P_{q-Q}=\sum_{q\in Z}\left\{\int_{-\pi}^{\pi}\frac{dk}{2\pi}\,e^{iqk}\rho_{xy}(k)\right\}\times P_{q-Q}$$

- Glue dynamics : Z_N vacuum of

$$\langle P_{q-Q}\rangle = N\,\delta_{q-Q}$$

$$\begin{aligned}\mathcal{H}_F(\rho)/N \;&=\; \int\frac{dp}{2\pi}\sum_{x\in Z_{L_s}}\left\{\left(-\frac{i}{2}\rho_{x,x+1}(p)+c.c.\right)+m(-1)^x\rho_{xx}(p)\right\}\\ &+\;\frac{g^2N}{4}\int\!\!\int\frac{dp}{2\pi}\frac{dp'}{2\pi}\,\frac{1}{L_s}\sum_{xy\in Z_{L_s}}\sum_{l=1}^{L_s}\frac{\rho_{xy}(p)\,\bar{\rho}_{yx}(p')\,e^{i2\pi l(x-y)/L_s}}{4\sin^2\left((p-p')/L_s+2\pi l/L_s\right)/2}\end{aligned}$$

Remarks (I)

- If assume translation invariance : $\theta_{x,y} = \theta_{x-y} \longrightarrow \rho^q_{xy} = \rho_{x-y+qL_s}$

$$\begin{aligned}\mathcal{H}_F(\rho)/N \;=\; & \int \frac{dp}{2\pi} \sum_{x\in Z_{L_s}} \left\{ \left(-\frac{i}{2}\rho_{x,x+1}(p) + c.c. \right) + m(-1)^x \rho_{xx}(p) \right\} \\ & + \frac{g^2 N}{4} \int\!\!\int \frac{dp}{2\pi}\frac{dp'}{2\pi}\frac{1}{L_s} \sum_{xy\in Z_{L_s}} \sum_{l=1}^{L_s} \frac{\rho_{xy}(p)\,\bar{\rho}_{yx}(p')\,e^{i2\pi l(x-y)/L_s}}{4\sin^2\left((p-p')/L_s + 2\pi l/L_s\right)/2}\end{aligned}$$

$$\mathcal{H}_F/(NL_s) \;=\; \int_0^{2\pi} \frac{dp}{2\pi}\,\mathrm{tr}\,[\rho(p)\,(-\sigma_3\sin(p) + m\,\sigma_1)] - \frac{g^2N}{4}\int\!\!\int \frac{dp}{2\pi}\frac{dq}{2\pi}\frac{\mathrm{tr}\,(\rho(p)\,\rho(q))}{4\sin^2((p-q)/2)}$$

L independence !

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$$+ \frac{g^2 N}{4} \int\int \frac{dp}{2\pi}\frac{dp'}{2\pi}\frac{1}{L_s} \sum_{xy \in Z_{L_s}} \sum_{l=1}^{L_s} \frac{\rho_{xy}(p)\,\bar{\rho}_{yx}(p')\,e^{i2\pi l(x-y)/L_s}}{4\sin^2\left((p-p')/L_s + 2\pi l/L_s\right)/2}$$

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L independence !

- Papers “ignoring” 0-modes : have “set” $\varphi_a \rightarrow 0$ & break Z_N

$$\forall q : P_q = N \qquad \longrightarrow \sum_{q \in Z} \rho^q_{xy} \times \cancelto{N}{P_{q-Q}} = \sum_{q \in Z} \left\{ \int_{-\pi}^{\pi} \frac{dk}{2\pi} e^{iqk} \rho_{xy}(k) \right\} \times \cancelto{N}{P_{q-Q}} = \rho_{xy}(k=0)$$

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$\rho^q_{xy} \rightarrow \rho_{xy}$ No L independence !

Remarks (II)

$$\mathcal{H}_F(\rho)/N = \int \frac{dp}{2\pi} \sum_{x \in Z_{L_s}} \left\{ \left(-\frac{i}{2} \rho_{x,x+1}(p) + c.c. \right) + m(-1)^x \rho_{xx}(p) \right\}$$
$$- \frac{g^2 N}{4} \int\int \frac{dp}{2\pi} \frac{dp'}{2\pi} \frac{1}{L_s} \sum_{xy \in Z_{L_s}} \sum_{l=1}^{L_s} \frac{\rho_{xy}(p)\, \rho_{yx}(p')\, e^{i2\pi l(x-y)/L_s}}{4 \sin^2\left((p-p')/L_s + 2\pi l/L_s \right)/2}$$

- B enters through :
    - extra B-dependent, yet constants, terms.
    - constraints on $\rho_{xy}(p)$ obeys : $\sum_x \rho_{xx}(p) = B$.

- To minimize $\mathcal{H}_{\mathcal{F}}(\rho)$
    - solve, numerically, $\dfrac{\delta \mathcal{H}}{\delta \rho_{xy}(p)} = 0$.
    - descretize $\rho_{xy}(p \in (0, 2\pi]) \longrightarrow \rho_{xy}(p = \frac{2\pi}{M}, \frac{4\pi}{M}, \ldots, 2\pi)$

Remarks (II)

$$\mathcal{H}_F(\rho)/N = \int \frac{dp}{2\pi} \sum_{x\in Z_{L_s}} \left\{ \left(-\frac{i}{2}\rho_{x,x+1}(p) + c.c. \right) + m(-1)^x \rho_{xx}(p) \right\}$$

$$- \frac{g^2 N}{4} \int\int \frac{dp}{2\pi}\frac{dp'}{2\pi}\frac{1}{L_s} \sum_{xy\in Z_{L_s}} \sum_{l=1}^{L_s} \frac{\rho_{xy}(p)\,\rho_{yx}(p')\,e^{i2\pi l(x-y)/L_s}}{4\sin^2\left((p-p')/L_s + 2\pi l/L_s\right)/2}$$

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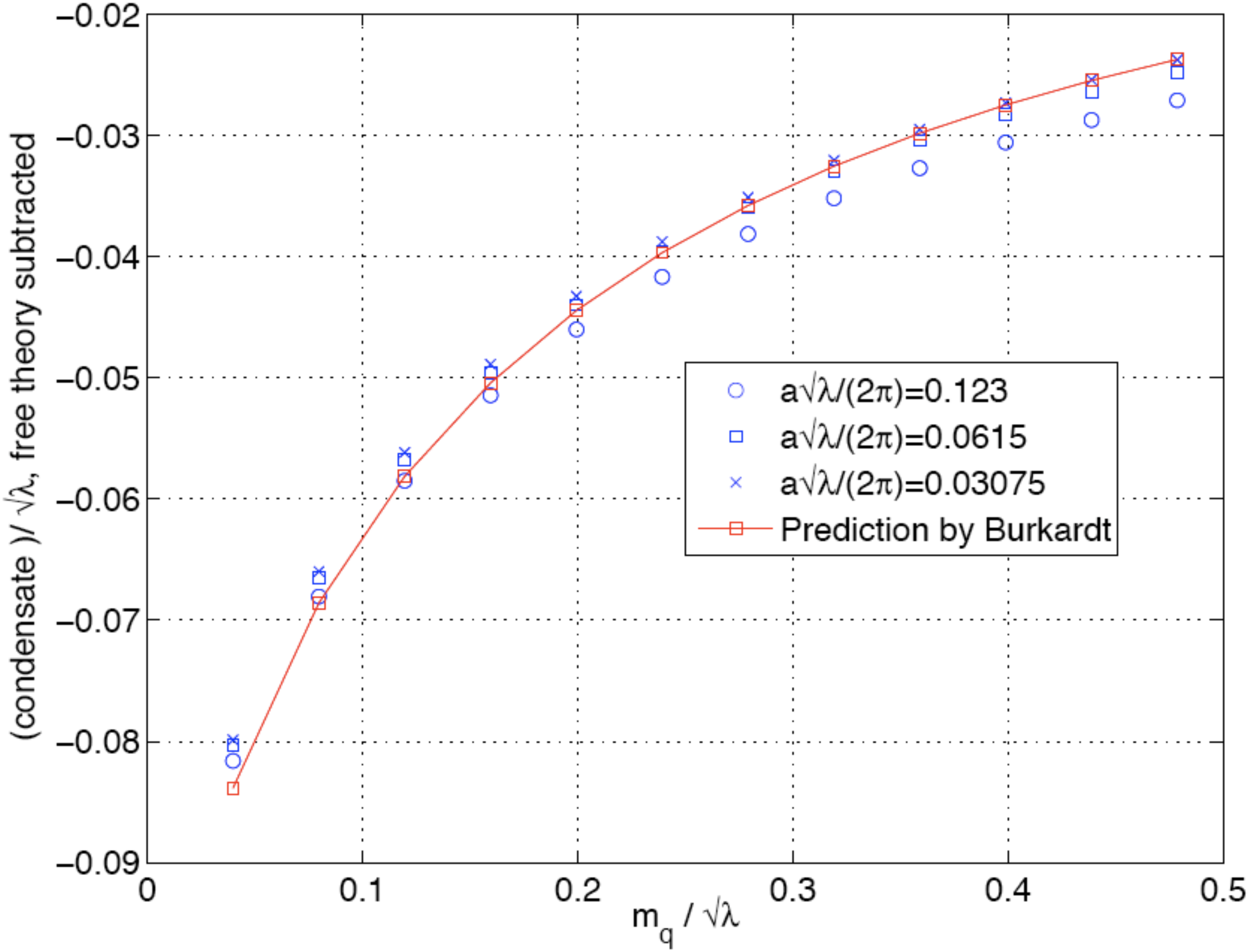
$M = \infty$ **incorporate 0-modes**

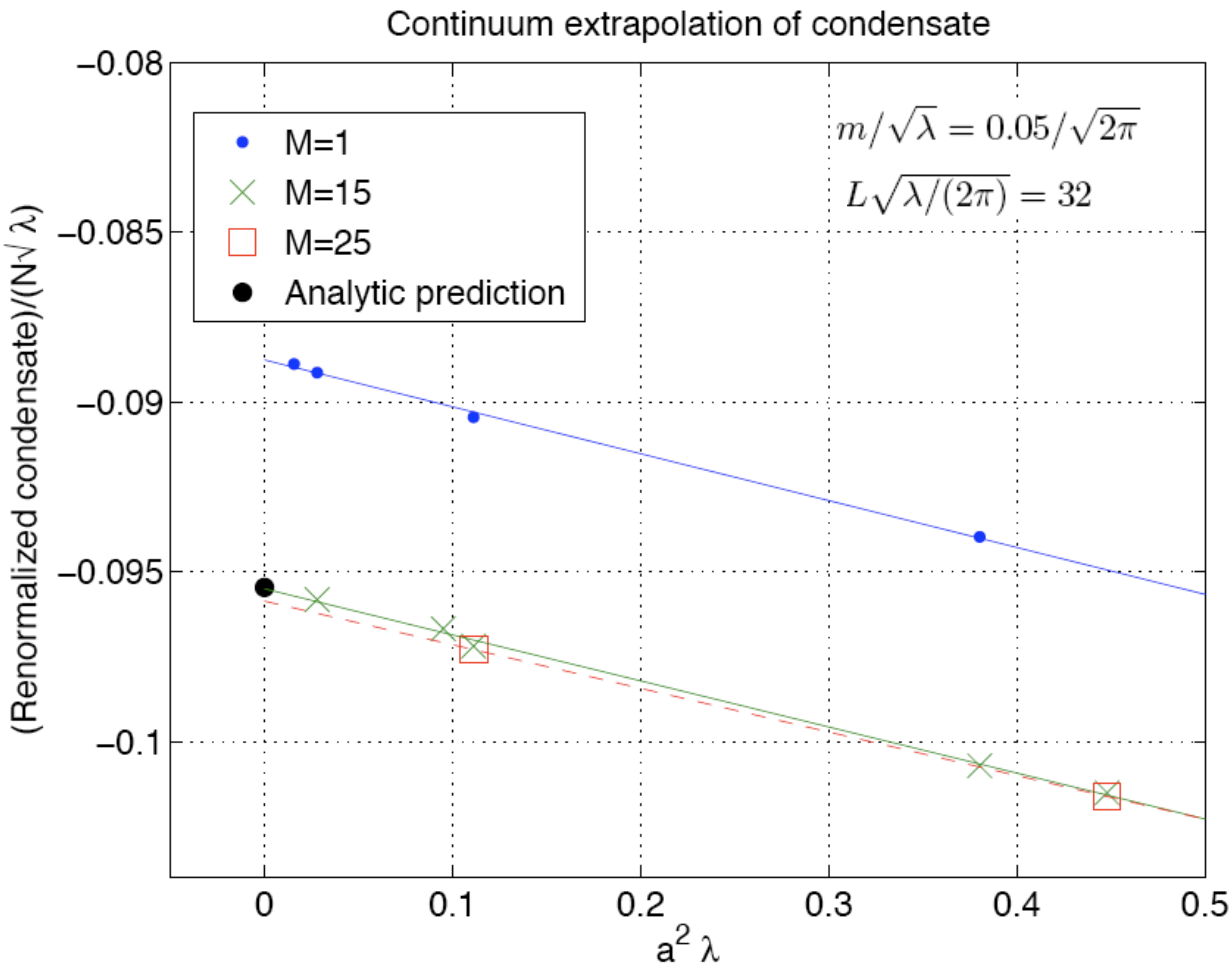
$M = 1$ **neglect 0-modes**

Results : continuum extrapolation

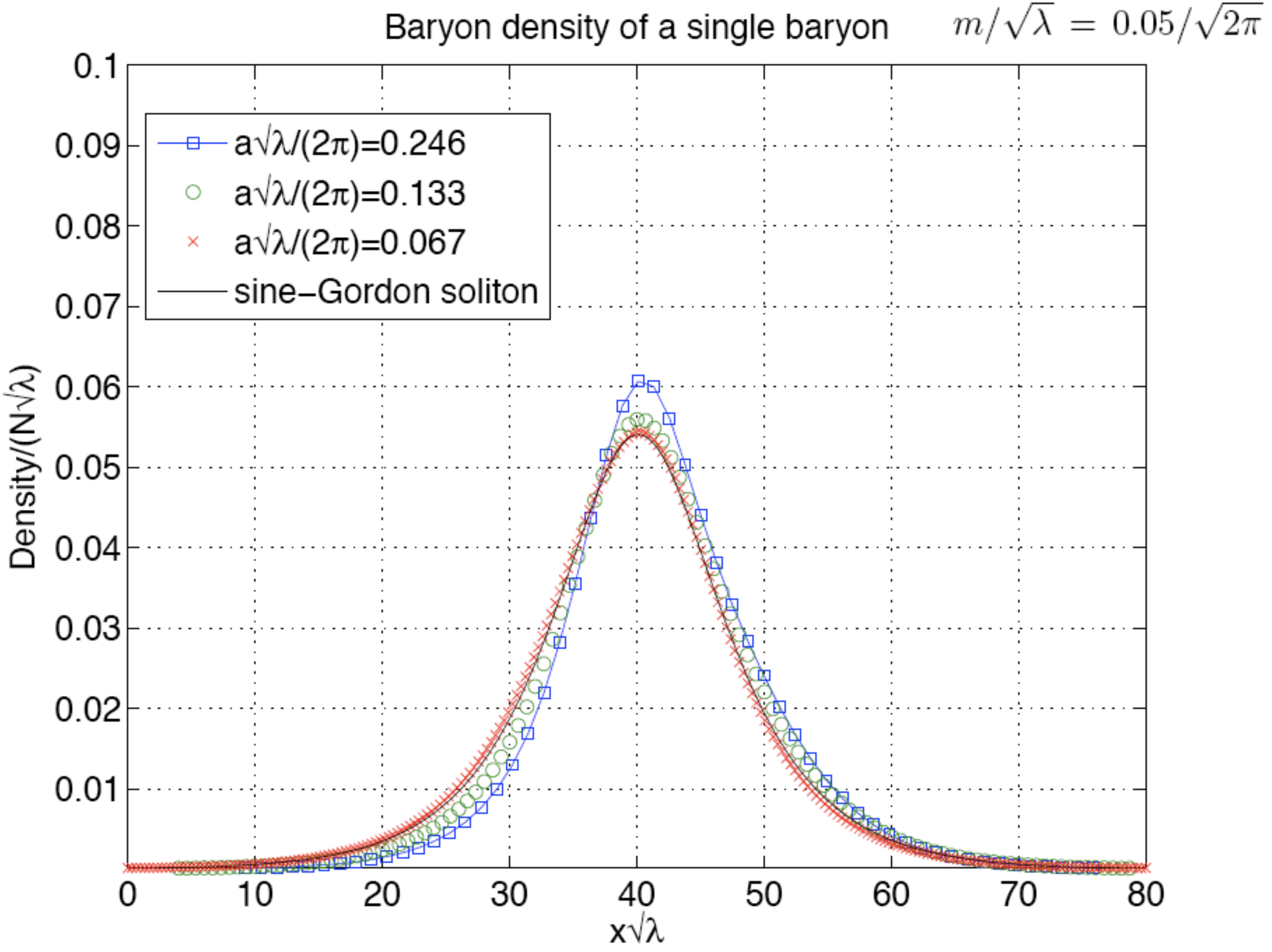
$M = 1$, but a relatively large volume of $L\sqrt{\lambda/(2\pi)} = 32$.



Results : continuum extrapolation



Results : B=1

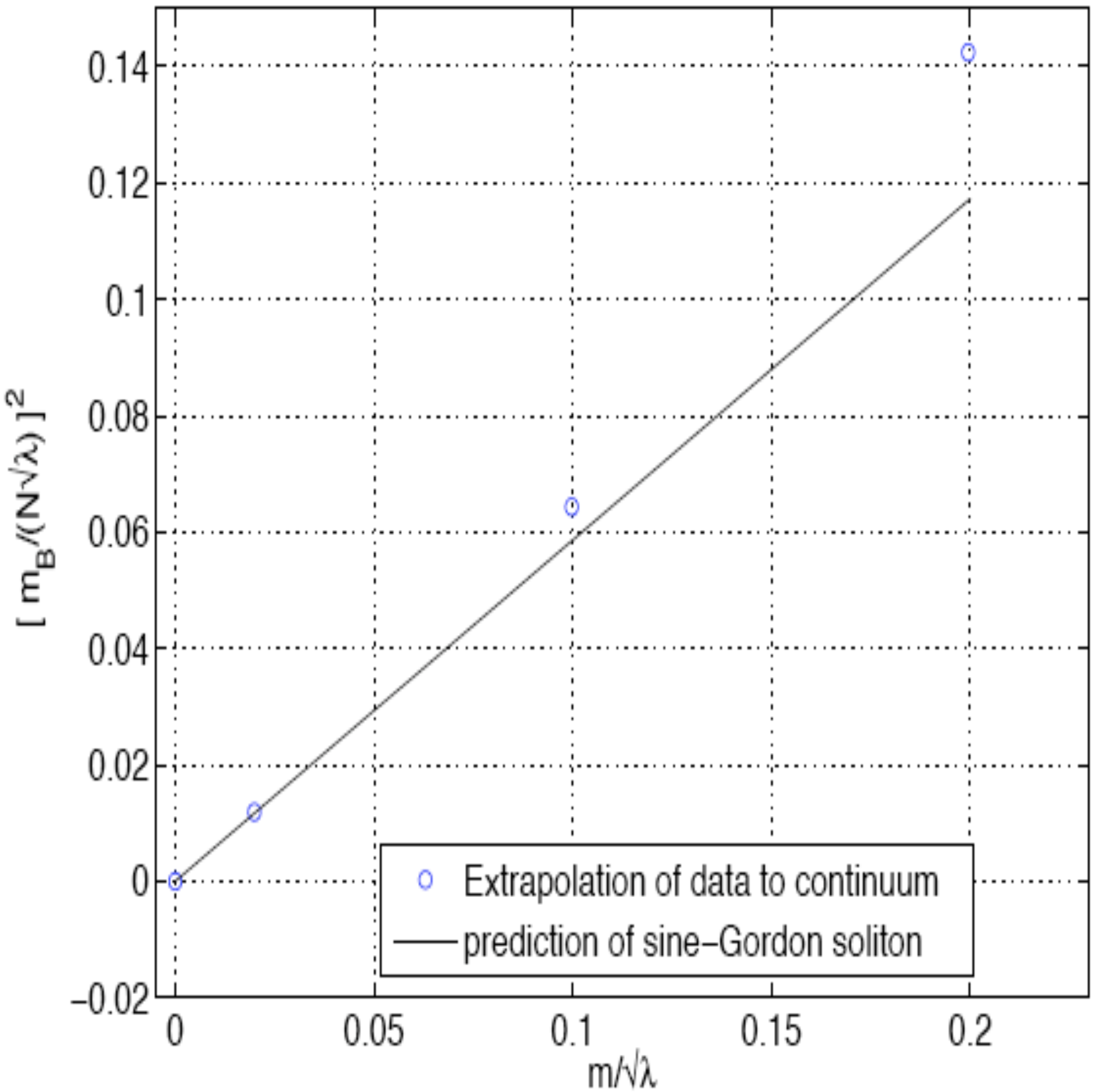


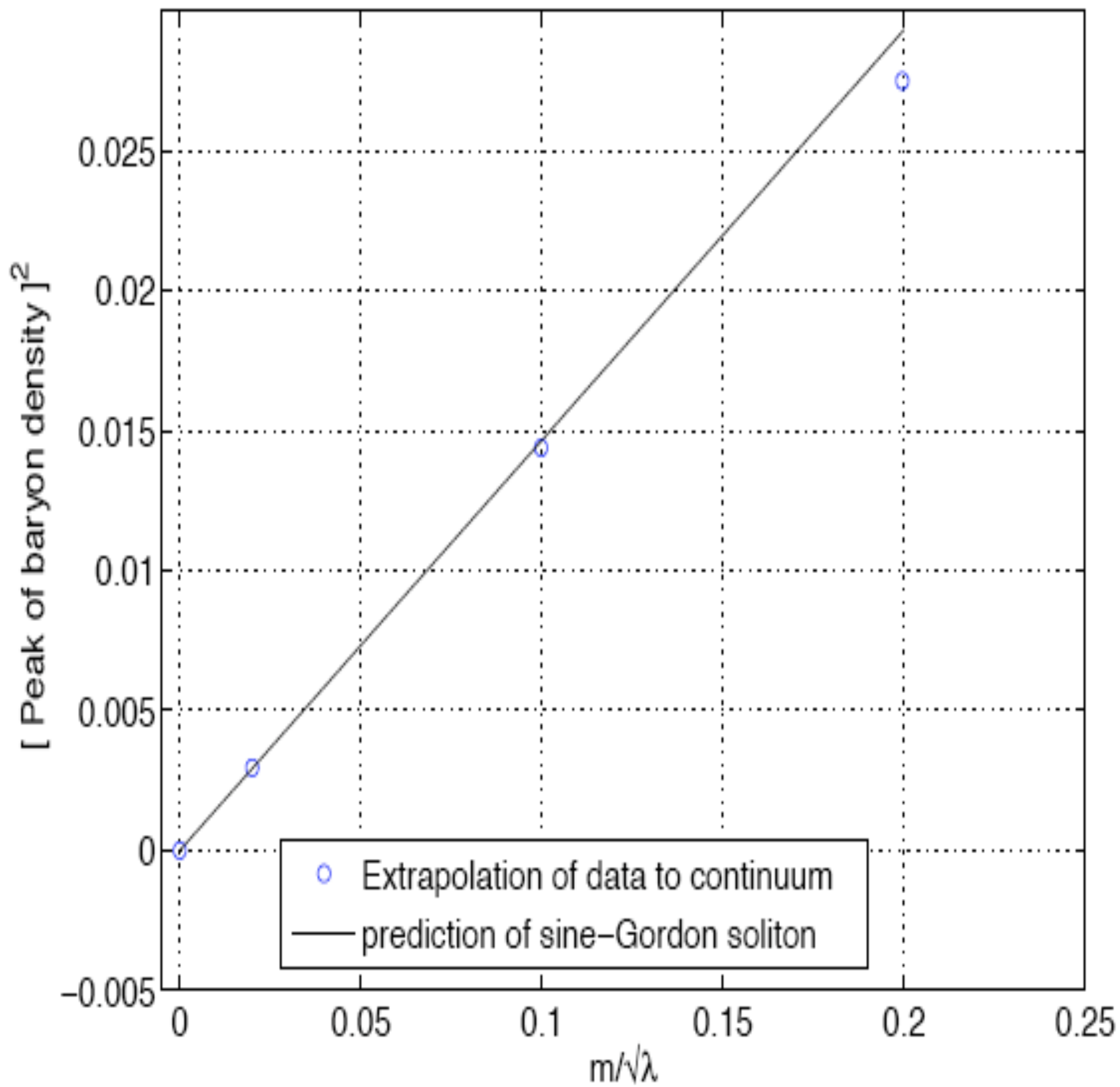
Baryon density of a single baryon
$m/\sqrt{\lambda} = 0.05/\sqrt{2\pi}$
a√λ/(2π)=0.246
a√λ/(2π)=0.133
a√λ/(2π)=0.067
sine−Gordon soliton
Density/(N√λ)
x√λ
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
10
20
30
40
50
60
70
80

Results : **B=1**

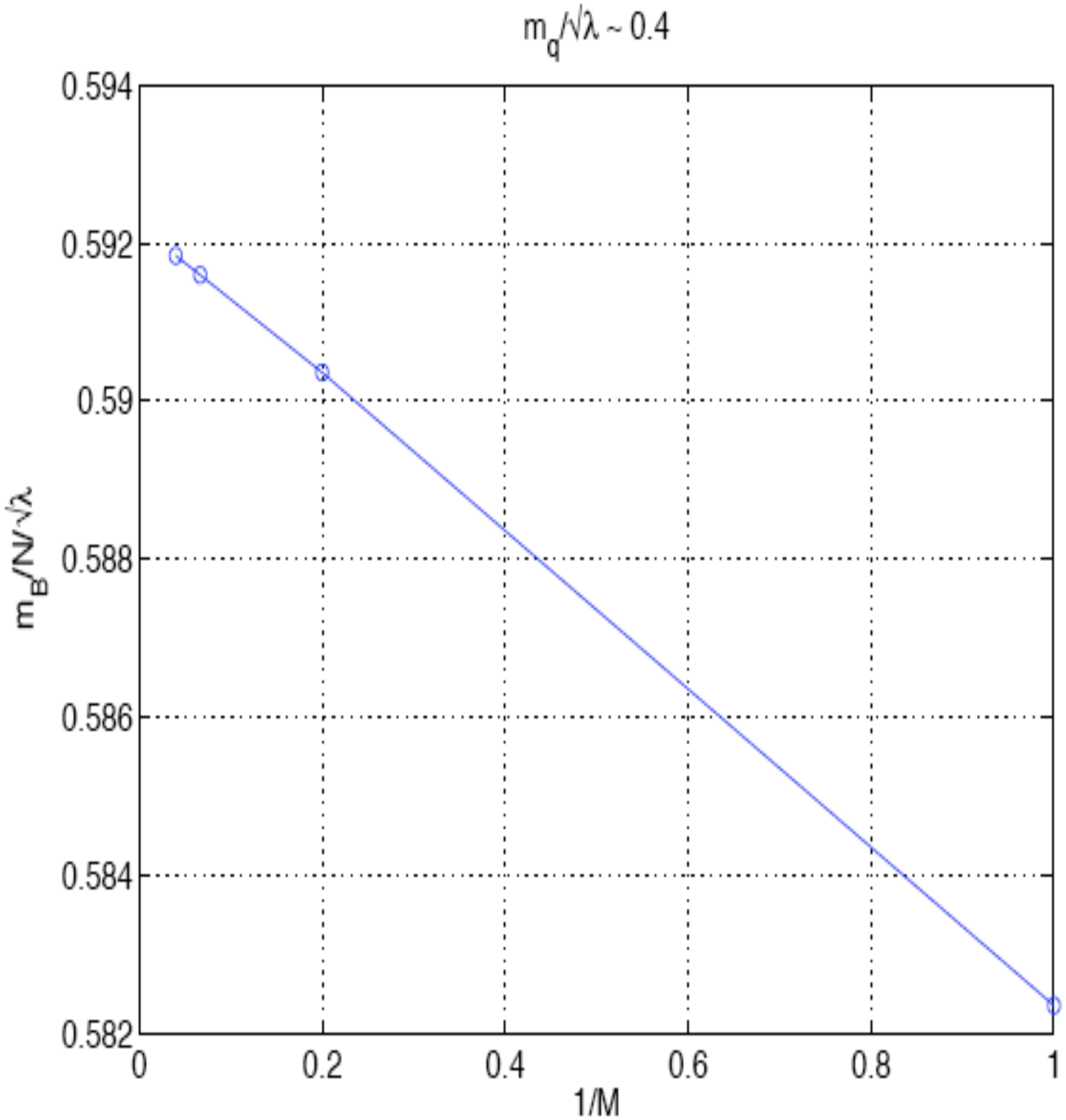
$$L\sqrt{\lambda/(2\pi)} = 16, 24, 48$$

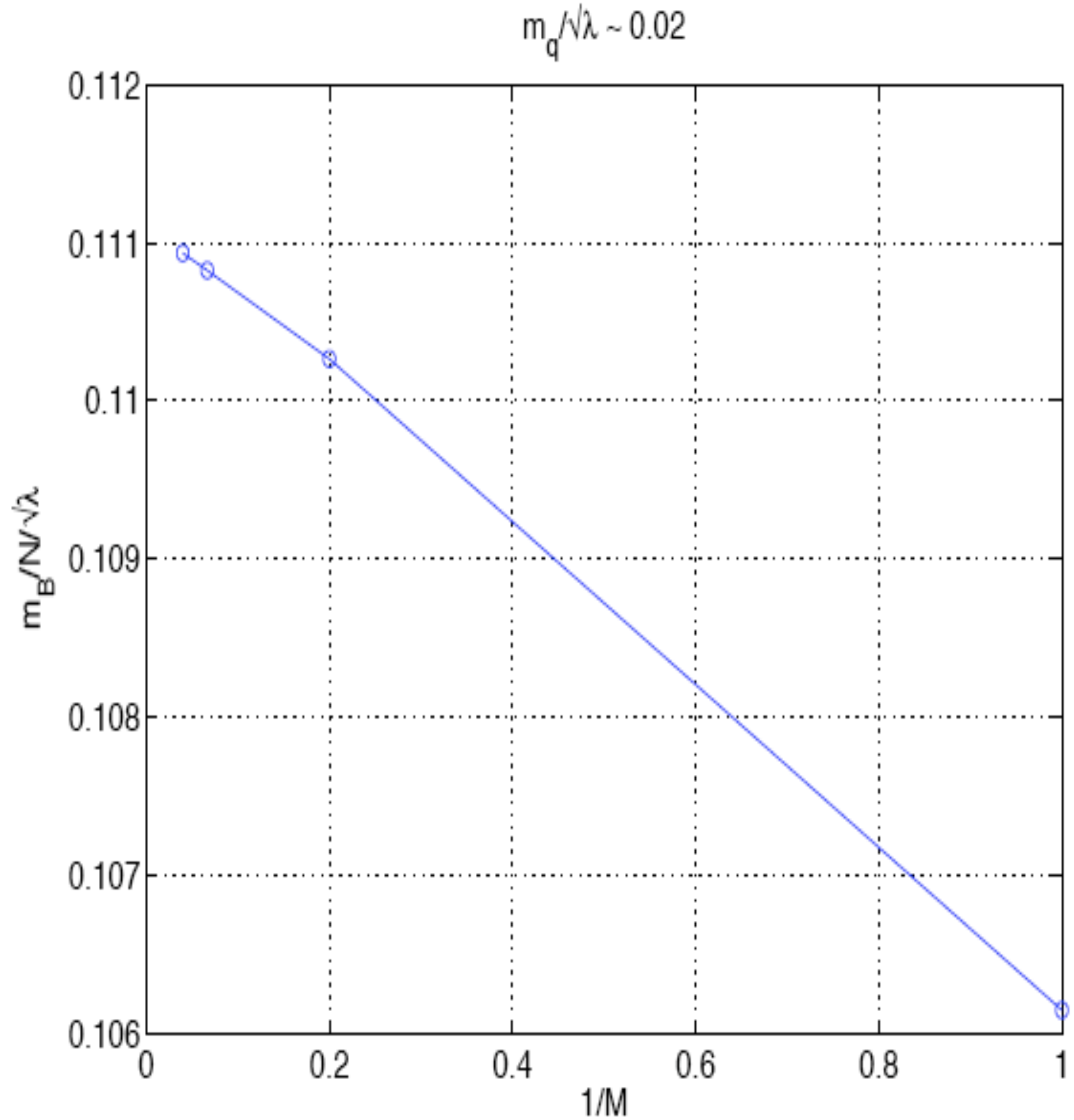
$$m/\sqrt{\lambda} = (0.5, 0.25, 0.05)/\sqrt{2\pi}$$





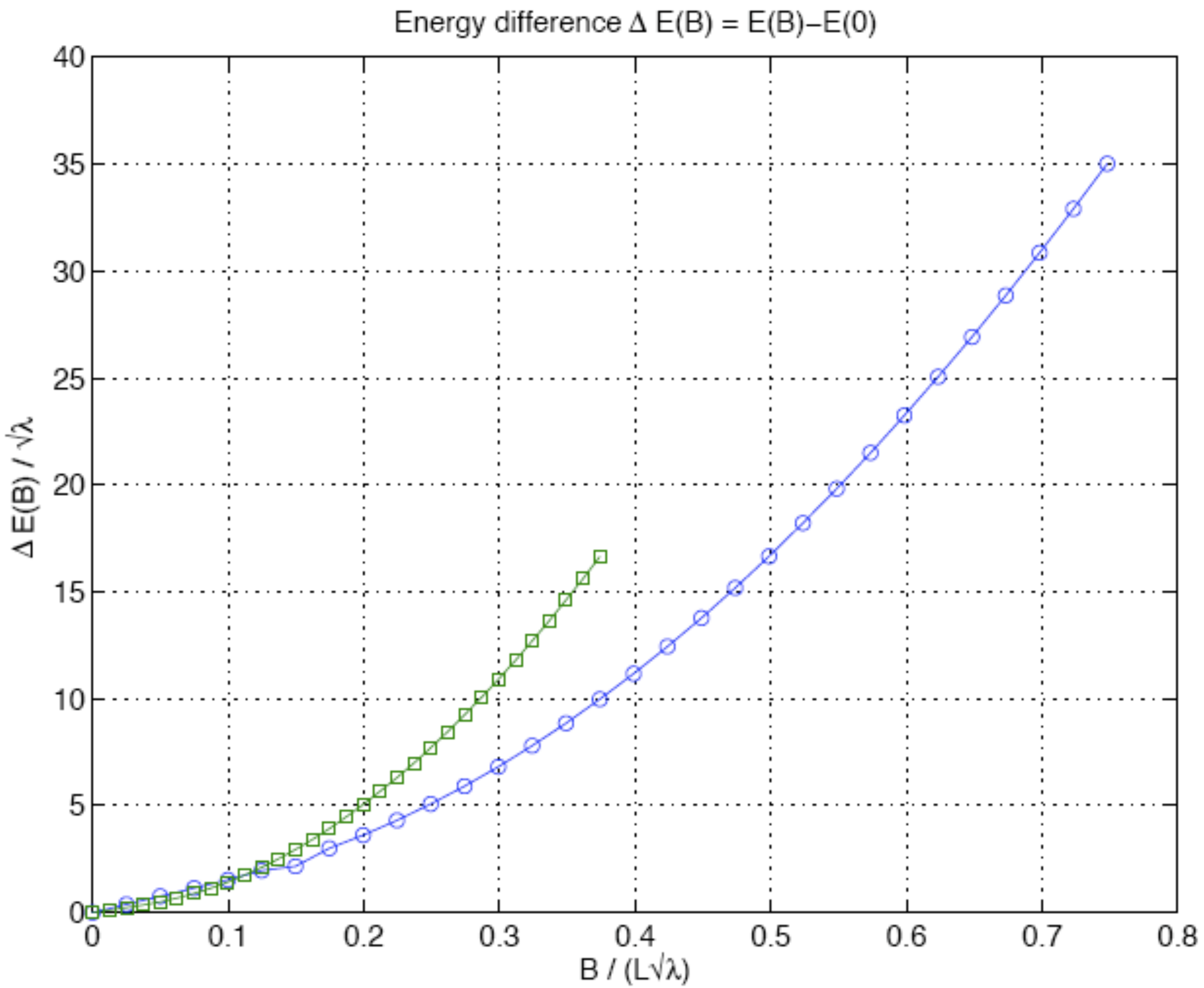
Results : **B=1**



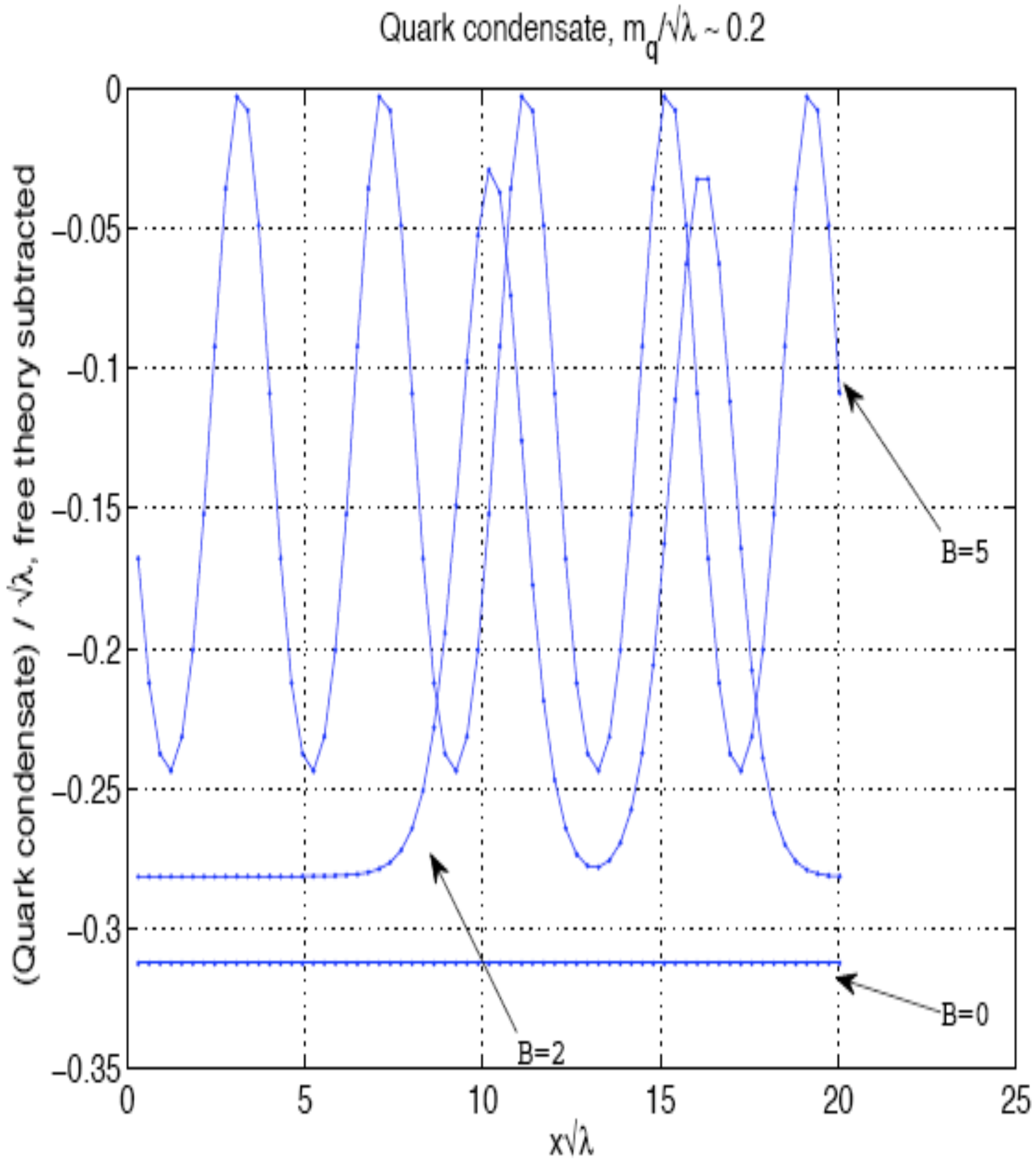


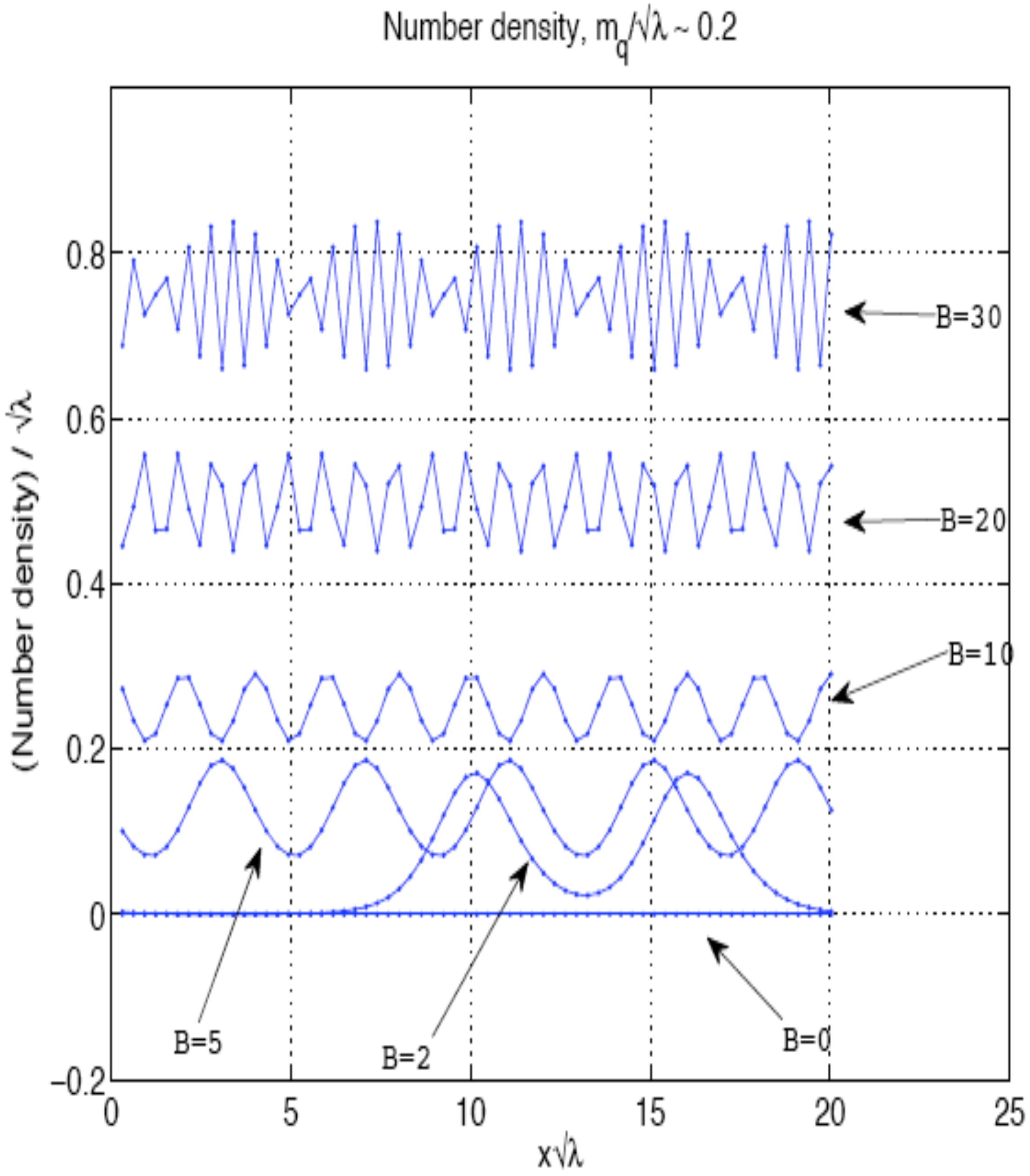
Baryon mass at $a\sqrt{\lambda/(2\pi)} = 0.123$ and $L\sqrt{\lambda/(2\pi)} = 16$ as a function of $1/M$

Results : B>1


Energy difference Δ E(B) = E(B)−E(0)
Δ E(B) / √λ
B / (L√λ)
0
5
10
15
20
25
30
35
40
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8

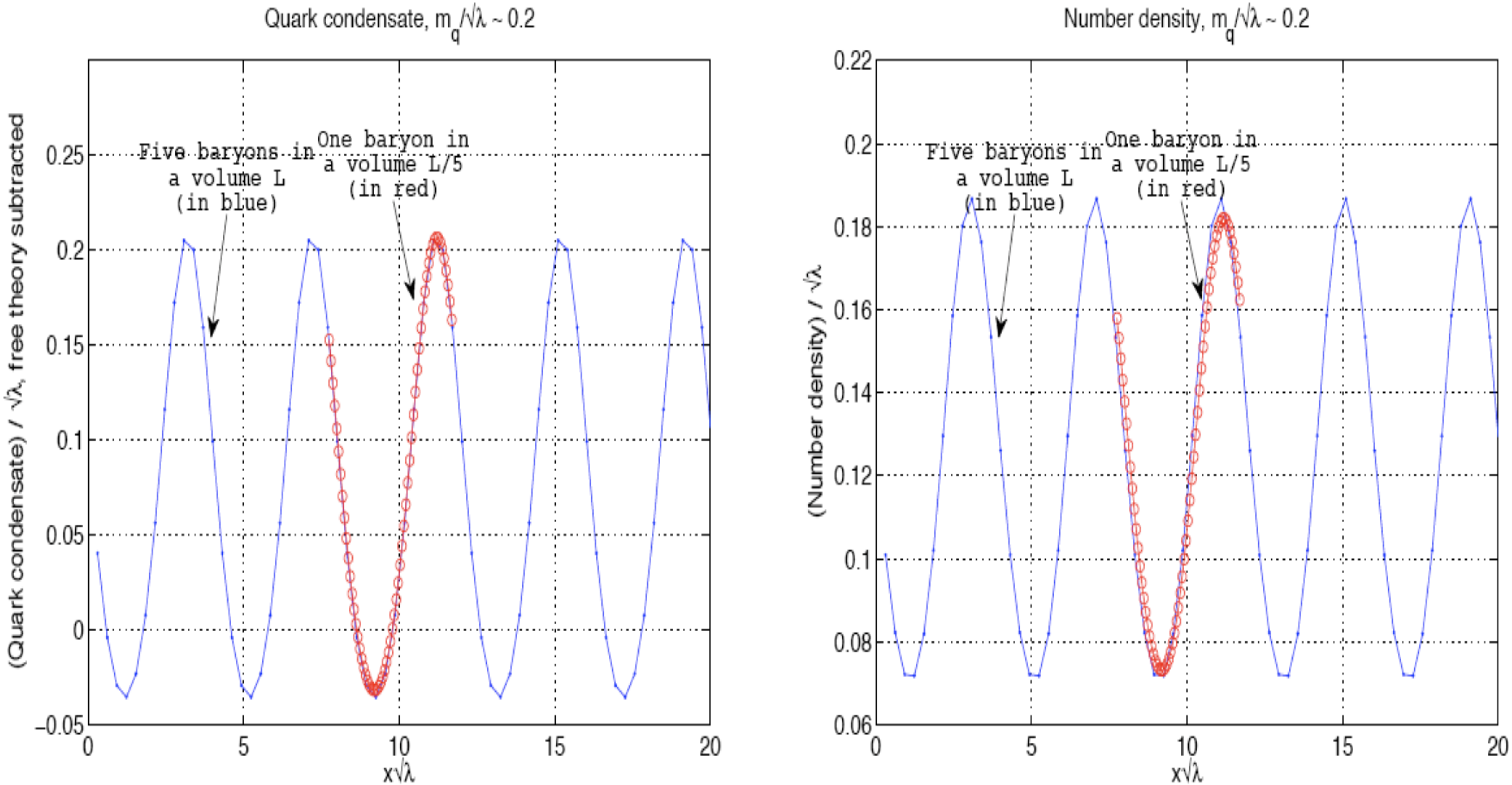
Results : B=1





Here $a\sqrt{\lambda/(2\pi)} = 0.123,\ m/\sqrt{\lambda} = 0.5/\sqrt{2\pi},\ L\sqrt{\lambda/(2\pi)} = 16.$

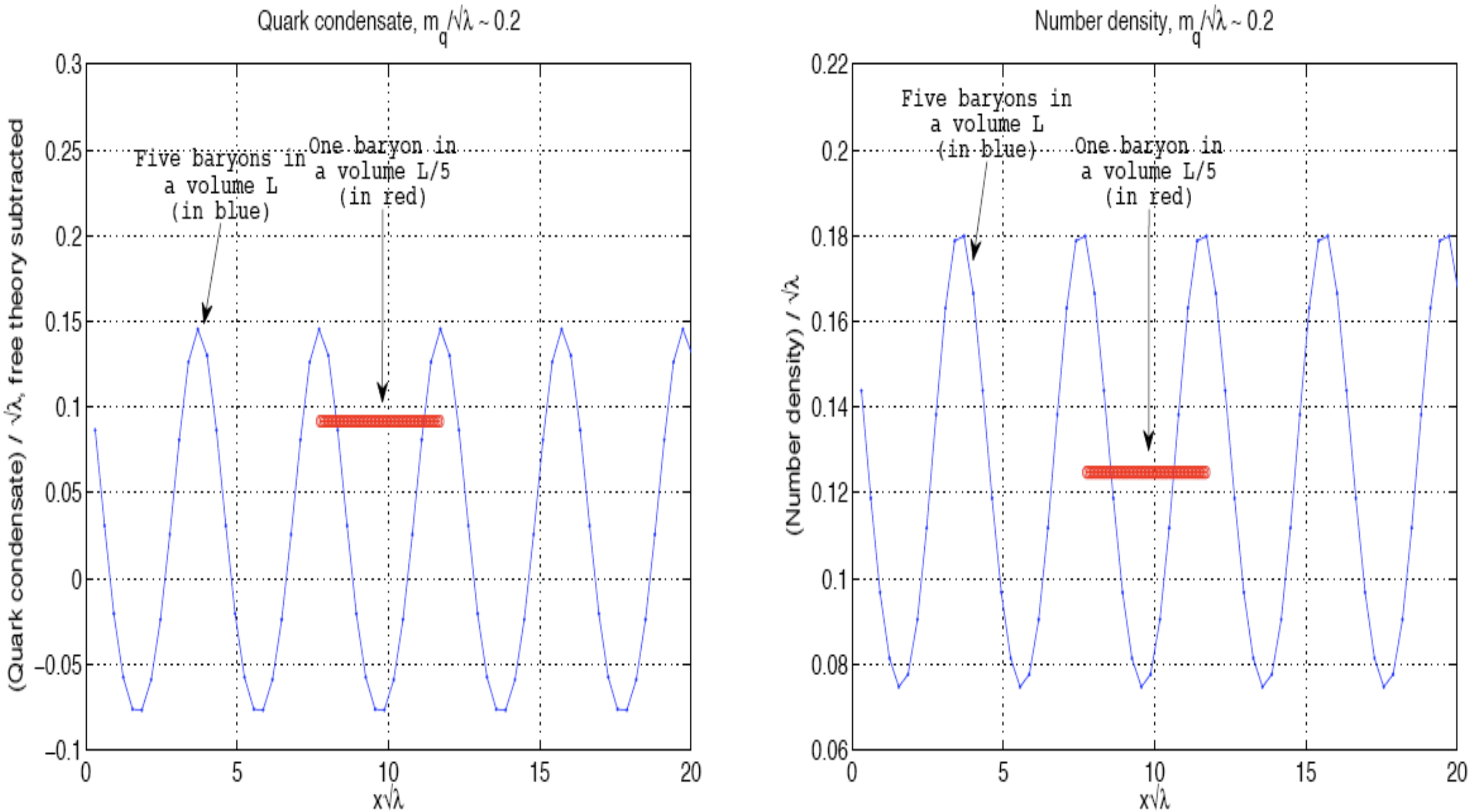
Results : **B>1**



$$M = 25$$

$$L\sqrt{\lambda/(2\pi)} = 16$$

Results : B>1



$$M = 25$$

$$L\sqrt{\lambda/(2\pi)} = 16$$

Coherent states

- QCD admits an overcomplete "coherent state" basis $|\mathcal{C}\rangle = U(\mathcal{C})|0\rangle$

$|0\rangle =$ some state in Hilbert space

$$U(\mathcal{C}) = \exp\left[\mathcal{C}_F \times \psi_x^\dagger U_{x\to y}\psi_y\right] \times \exp\left[\mathcal{C}_g^{(1)} \times N\, tr(\hat{W}) + \mathcal{C}_g^{(2)} \times N\, tr(\hat{E}^i \lambda^i \hat{W})\right]$$

which becomes classical at large-N

$$\left\{\frac{\partial \hat{A}}{\partial t} = -i[\hat{A}, H]\right\} \xrightarrow{N\to\infty} \left\{\frac{\partial a(\mathcal{C})}{\partial t} = -\{a(\mathcal{C}), \mathcal{H}(\mathcal{C})\}_{\mathrm{PB}}\right\}$$ with $$a(\mathcal{C}) = \langle \mathcal{C}|\hat{A}|\mathcal{C}\rangle \qquad \mathcal{H}(\mathcal{C}) = \langle \mathcal{C}|H|\mathcal{C}\rangle$$

- Effectively makes whole dynamics classical and :

$$\mathrm{diag}\ (H_{\mathrm{quantum}}) \longrightarrow \min_{\mathcal{C}}\left[\mathcal{H}(\mathcal{C})\right]$$ and $$\langle \sum_a e^{i\varphi_a}\rangle,\ \langle \sum_a \bar{\psi}^a\, \psi^a\rangle = f(\mathcal{C})_{|\mathcal{C}=\mathcal{C}_{\min}}$$

Coherent states, cont'd

- Dominance of glue over fermions :

$$\mathcal{H}(\mathcal{C}) = \underbrace{\mathcal{H}_{\text{glue}}(\mathcal{C}_g)}_{O(N^2)} + \underbrace{\mathcal{H}_{\text{F}}(\mathcal{C}_{\text{F}}, \mathcal{C}_g)}_{O(N)}$$

- Step A : Minimize leading

$$\mathcal{H}_{\text{glue}}(\mathcal{C}_g) = \langle \mathcal{C}_g | H_{\text{glue}} | \mathcal{C}_g \rangle \longrightarrow \text{fix } \mathcal{C}_g = \mathcal{C}_{g,\text{min}}$$

$$\Downarrow$$

$$\langle \sum_a e^{i\varphi_a} \rangle, \langle \sum_a e^{2i\varphi_a} \rangle = f(\mathcal{C}_{g,\text{min}})$$

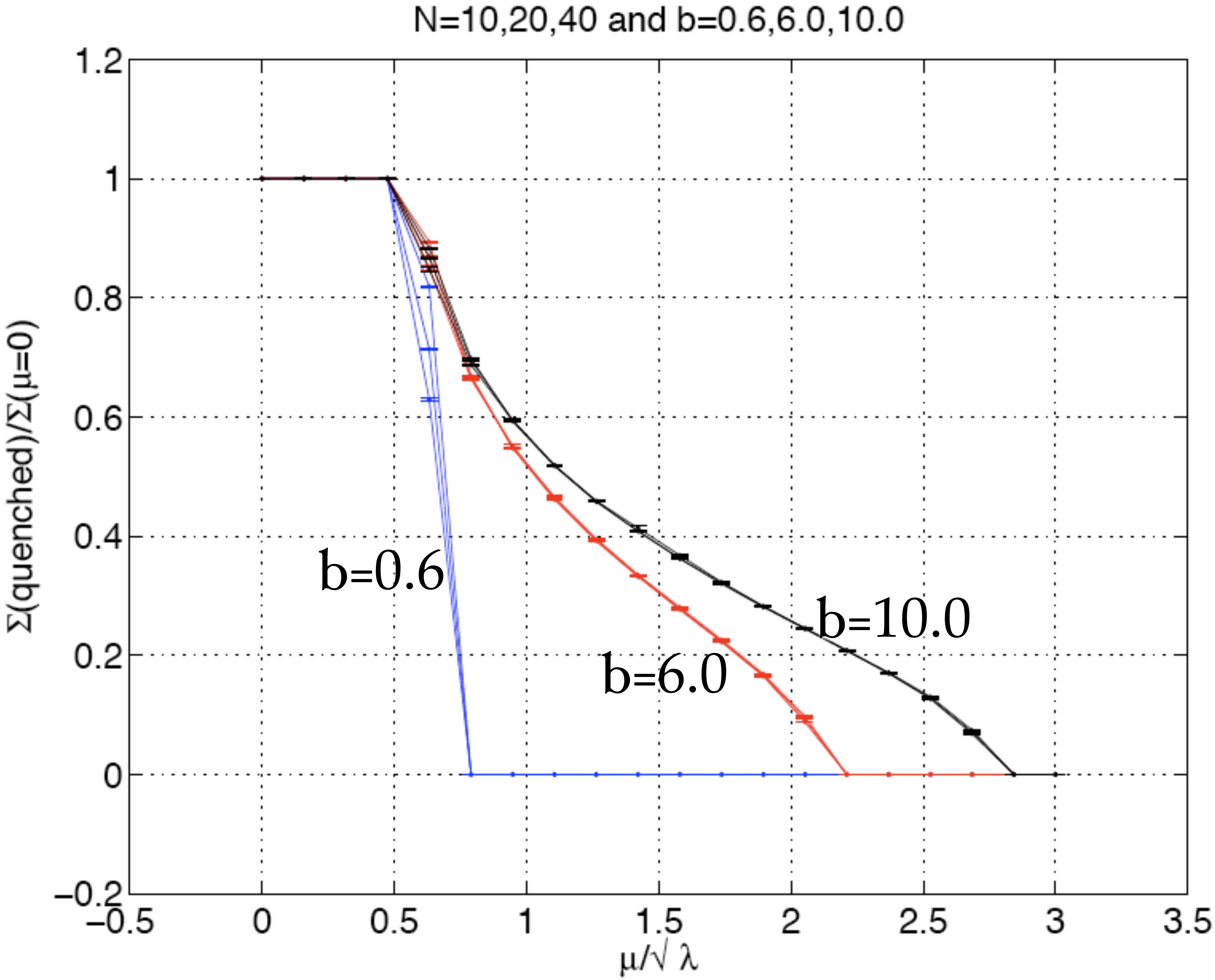
- Step B : Minimize sub-leading

$$\mathcal{H}_F(\mathcal{C}_g = \mathcal{C}_{g,\text{min}}, \mathcal{C}_F) = \langle \mathcal{C}_{g,\text{min}} | \langle \mathcal{C}_F | H_{\text{F}} | \mathcal{C}_F \rangle | \mathcal{C}_{g,\text{min}} \rangle \longrightarrow \text{fix } \mathcal{C}_F = \mathcal{C}_{F,\text{min}}$$

$$\Downarrow$$

$$\langle \bar{\psi} \psi \rangle = f(\mathcal{C}_{g,\text{min}}, \mathcal{C}_{F,\text{min}})$$

Results : quenched condensates



Results : try Z_N average

Re-sum path integral

$$\langle \mathcal{O} \rangle = \frac{1}{N_{\rm cnfg}} \sum_{i=1}^{N_{\rm cnfg}} \mathcal{O}_i \longrightarrow \frac{1}{N_{\rm cnfg}} \sum_{i=1}^{N_{\rm cnfg}} \left(\frac{1}{N^d} \sum_{k_1}^{N} \sum_{k_2=1}^{N} \cdots \sum_{k_d=1}^{N} \mathcal{O}_i^k \right)$$