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Finiteness of hot classical scalar field theory and the plasmon damping rate

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Abstract

We investigate the renormalizability of the classical ϕ^4 theory at finite temperature. We calculate the time-dependent two-point function to two loop order and show that it can be rendered finite by the counterterms of the classical static theory. As an application the classical plasmon damping rate is found to be $\gamma = \lambda^2T^2/1536\pi m$. When we use the high temperature expression for m given by dimensional reduction, the rate is found to agree with the quantum mechanical result.

1. Introduction

In recent years there has been an increasing interest in real-time phenomena in finite temperature quantum field theory. The complexity of the phenomena often calls for numerical simulations. Because it is extremely difficult to simulate quantum mechanical real-time quantities, approximations have to be made. In particular the classical approximation has been used to simulate diffusion of the Chern-Simons number in gauge theories [1–4] and the dynamical properties of the electroweak phase transition [5]. It has been argued that the classical description can be accurate for physical quantities which are mainly determined by low momentum modes of the theory, because these modes will be highly occupied at high temperatures. However, a clear demonstration is complicated by the fact that there may be modes with momentum higher that the temperature. Excluding these by a cutoff in an effective field theory allows for a controlled derivation, but it leads to very complicated dynamics with nonlocal noise terms [6]. It is much simpler to use cutoffs much higher than the temperature and deal with dynamics which is local at this cutoff scale. In this letter we shall study the limit of infinite cutoff of the (noiseless) classical theory in perturbation theory, taking the ϕ^4 theory as an example.

Classical time dependent correlation functions are determined by solving classical equations of motion for arbitrary initial conditions, and performing a Boltzmann weighted sum over these initial conditions. For time

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independent correlation functions, this prescription reduces to calculations in a three-dimensional, euclidean field theory. In fact, this static theory is of the same form as the dimensional reduction approximation in finite temperature field theory [2,4]. In one method of dimensional reduction, the values of the coupling constants are precisely specified by a matching procedure between the four- and the three-dimensional theory [7]. In the classical theory the coupling constants are a priori arbitrary. The dimensionally reduced theory has been used extensively for the study of the electroweak phase transition [8].

The euclidean theory coming from dimensional reduction is superrenormalizable and therefore, time independent correlation functions can be made finite by merely a mass renormalization. It is not a priori clear that time dependent correlation functions can be made finite by the same renormalization. In [9] it is shown for a ϕ^4-theory that the time dependent two-point function can be made one loop finite by supplying the mass counterterm of the static theory. However, at two loops the situation is different because then the correlation functions of the static theory mix with time dependent Green functions. We shall study this question by calculating the classical time dependent two-point function to two loop order.

We will solve the equations of motion by applying naive perturbation theory, which leads to resonant or secular terms in each step of the expansion. These can be avoided by adding counterterms to eliminate the secular terms. We will show that the counterterms needed to make the theory finite can be chosen such that they also partially take care of the resonances.

We also calculate the classical plasmon rate and show that it is equal to the quantum mechanical expression at high T, when the connection is made with dimensional reduction.

2. Definition of the correlation functions

We consider a scalar field theory at temperature $T = 1/\beta$, with the following hamiltonian

$$H = \int d^3x \left(\tfrac{1}{2}\pi^2 + \tfrac{1}{2}(\nabla\phi)^2 + \tfrac{1}{2}\mu^2\phi^2 + \frac{1}{4!}\lambda\phi^4 + \epsilon \right).$$

Classical real time n point functions are defined in the following way

$$\langle \phi(\boldsymbol{x}_1, t_1) \ldots \phi(\boldsymbol{x}_n, t_n) \rangle = \frac{1}{Z} \int D\pi D\phi e^{-\beta H(\pi,\phi)} \phi(\boldsymbol{x}_1, t_1) \ldots \phi(\boldsymbol{x}_n, t_n),$$

where

$$Z = \int D\pi D\phi e^{-\beta H(\pi,\phi)},$$

and $\phi(\boldsymbol{x}, t)$ is the solution of the equations of motion

$$\dot\phi(\boldsymbol{x}, t) = \pi(\boldsymbol{x}, t), \quad \dot\pi(\boldsymbol{x}, t) = \nabla^2\phi(\boldsymbol{x}, t) - \mu^2\phi(\boldsymbol{x}, t) - \frac{\lambda}{3!}\phi^3(\boldsymbol{x}, t),$$

with initial conditions

$$\phi(\boldsymbol{x}, t_0) = \phi(\boldsymbol{x}), \quad \pi(\boldsymbol{x}, t_0) = \pi(\boldsymbol{x}).$$

The integrals over π and ϕ are performed at $t = t_0$ (summing over the initial conditions). Since we consider equilibrium correlation functions, t_0 is arbitrary. In particular the two-point function will only depend on $t_1 - t_2$. Choosing infinite volume it depends also only on $\boldsymbol{x}_1 - \boldsymbol{x}_2$. In the following we choose parameters such that the theory is in its symmetric phase.

For equal times $t_1 = \ldots = t_n = t_0$ the π-field decouples and we are left with a three-dimensional theory for ϕ. This three-dimensional theory is superrenormalizable and it can be made finite with merely a mass renormalization. The counterterms are determined by the leaf diagram and the momentum independent part of the setting sun diagram. Writing

$$\mu^2 = m^2 - \delta m^2, \quad \delta m^2 = \lambda m_1^2 + \lambda^2 m_2^2,$$

we will use

$$m_1^2 = \tfrac{1}{2}T \int \frac{d^3k}{(2\pi)^3} \frac{1}{\omega_k^2} = \frac{1}{4\pi^2}\Lambda T - \frac{1}{8\pi} mT, \tag{1}$$

$$m_2^2 = -\tfrac{1}{6}T^2 \int \frac{d^3k_1}{(2\pi)^3} \frac{d^3k_2}{(2\pi)^3} \frac{d^3k_3}{(2\pi)^3} \frac{(2\pi)^3\delta(k_1+k_2+k_3)}{\omega_{k_1}^2\omega_{k_2}^2\omega_{k_3}^2} = -\frac{T^2}{32\pi^2}\log(\frac{\Lambda^2}{m^2}) + \text{finite}, \tag{2}$$

with Λ the momentum cutoff and $\omega_k^2 = k^2 + m^2$. The choice of the finite parts of the counterterms implied by the above expressions will turn out to be convenient for the moment. With this mass renormalization the equal time correlation functions are finite.

The first few terms of a perturbative solution $\phi = \phi_0 + \lambda\phi_1 + \lambda^2\phi_2 + \cdots$ to the equations of motion can easily be constructed. The zeroth order solution is given by

$$\phi_0(x,t) = \int \frac{d^3k}{(2\pi)^3} e^{ik\cdot x}\left[\phi(k)\cos(\omega_k(t-t_0)) + \frac{\pi(k)}{\omega_k}\sin(\omega_k(t-t_0))\right],$$

where $\phi(k)$ and $\pi(k)$ are the Fourier components of the initial conditions. Integration over the initial conditions gives the free two-point function

$$\langle\phi_0(x,t)\phi_0(x',t')\rangle_0 \equiv S_0(x-x',t-t') = T\int \frac{d^3k}{(2\pi)^3} e^{ik\cdot(x-x')}\frac{\cos(\omega_k(t-t'))}{\omega_k^2}.$$

Imposing the initial conditions $\phi_1 = \dot\phi_1 = \phi_2 = \dot\phi_2 = \cdots = 0$ the higher order terms can be found with the help of the retarded Green function

$$G_0^R(x-x',t-t') = \int \frac{d^3k}{(2\pi)^3} e^{ik\cdot(x-x')}\theta(t-t')\frac{\sin(\omega_k(t-t'))}{\omega_k}.$$

The solution to second order is given by

$$\begin{aligned}
\phi(x,t) = {} & \phi_0(x,t) - \lambda\int dt'\int d^3x'\, G_0^R(x-x',t-t')\left[\frac{1}{3!}\phi_0^3(x',t') - m_1^2\phi_0(x',t')\right] \\
& + \lambda^2\int dt'\int d^3x'\, G_0^R(x-x',t-t')m_2^2\phi_0(x',t') \\
& + \lambda^2\int dt'\int d^3x'\int dt''\int d^3x''\, G_0^R(x-x',t-t')G_0^R(x'-x'',t'-t'') \\
& \times\left[\frac{1}{2!}\phi_0^2(x',t') - m_1^2\right]\left[\frac{1}{3!}\phi_0^3(x'',t'') - m_1^2\phi_0(x'',t'')\right],
\end{aligned}$$

where the integrals over t', t'' start at t_0. We expressed μ^2 in the equation of motion in terms of m^2 and δm^2 and expanded in λ, using the same coefficients m_1^2 and m_2^2 as found earlier for the equal time case. Expanding also the Boltzmann weight, all correlation functions can be expressed in terms of the free n-point functions of $\phi_0(x,t)$, which are just products of the two-point function S_0, by Wick's theorem. The various propagators are given in Fig. 1. It is not clear at this point that the static counterterms suffice also for making the time dependent correlation functions finite, but we shall see in the following that this is indeed the case.

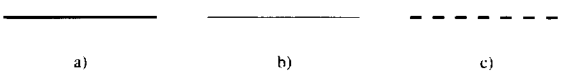

Fig. 1. Propagators, a) $S_0(\boldsymbol{x}-\boldsymbol{x}',t-t')$, b) $S_0(\boldsymbol{x}-\boldsymbol{x}',0)$, c) $G_0^R(\boldsymbol{x}-\boldsymbol{x}',t-t')$.

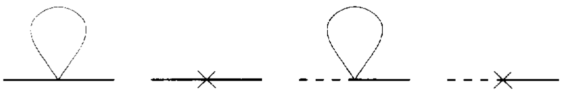

Fig. 2. First order contribution to the two-point function, the first order counterterm is denoted with a cross.

3. First and second order calculation of the two-point function

Using the framework set up in the previous paragraph, we calculate the time dependent two-point function to first order in λ. The result is

$$\begin{aligned}\langle\phi(\boldsymbol{x}_1,t_1)\phi(\boldsymbol{x}_2,t_2)\rangle &= S_0(\boldsymbol{x}_1-\boldsymbol{x}_2,t_1-t_2)\\ &-\tfrac{1}{2}\beta\lambda\int d^3x S_0(\boldsymbol{x}_1-\boldsymbol{x},t_1-t_0)(S_0(\boldsymbol{0},0)-2m_1^2)S_0(\boldsymbol{x}-\boldsymbol{x}_2,t_0-t_2)\\ &-\tfrac{1}{2}\lambda\int dt\int d^3x G_0^R(\boldsymbol{x}_1-\boldsymbol{x},t_1-t)(S_0(\boldsymbol{0},0)-2m_1^2)S_0(\boldsymbol{x}-\boldsymbol{x}_2,t-t_2)+(\boldsymbol{x}_1,t_1)\leftrightarrow(\boldsymbol{x}_2,t_2).\end{aligned}$$

The corresponding diagrams are given in Fig. 2. Performing the time integral, the correlation function becomes in momentum space

$$\begin{aligned}S(\boldsymbol{p},t) &\equiv \int d^3x\, e^{-i\boldsymbol{p}\cdot\boldsymbol{x}}\,\langle\phi(\boldsymbol{x},t_1)\phi(\boldsymbol{0},t_2)\rangle\\ &= \frac{T\cos(\omega_{\boldsymbol{p}}t)}{\omega_{\boldsymbol{p}}^2}-\frac{\lambda T}{4}\left(T\int\frac{d^3k}{(2\pi)^3}\frac{1}{\omega_{\boldsymbol{k}}^2}-2m_1^2\right)\frac{1}{\omega_{\boldsymbol{p}}^4}\left[\omega_{\boldsymbol{p}}t\sin(\omega_{\boldsymbol{p}}t)+2\cos(\omega_{\boldsymbol{p}}t)\right],\end{aligned}$$

where $t=t_1-t_2$. We see that the one loop divergence in this time dependent correlation function is cancelled by the one loop counterterm (1) of the static theory. The particular choice (1) for the finite terms also cancels the resonance due to the secular term.

We now turn to the second order calculation of the correlation function. Using the result for the first order counterterm, which cancels completely all the one loop contributions, the contribution to second order is

$$\begin{aligned}\langle\phi(\boldsymbol{x}_1,t_1)\phi(\boldsymbol{x}_2,t_2)\rangle &= S_0(\boldsymbol{x}_1-\boldsymbol{x}_2,t_1-t_2)\\ \text{(a)}\quad &+\tfrac{1}{6}\beta^2\lambda^2\int d^3x\,d^3x'\,S_0(\boldsymbol{x}_1-\boldsymbol{x},t_1-t_0)S_0^3(\boldsymbol{x}-\boldsymbol{x}',0)S_0(\boldsymbol{x}'-\boldsymbol{x}_2,t_0-t_2)\\ \text{(b)}\quad &+\beta m_2^2\lambda^2\int d^3x\,S_0(\boldsymbol{x}_1-\boldsymbol{x},t_1-t_0)S_0(\boldsymbol{x}-\boldsymbol{x}_2,t_0-t_2)\\ \text{(c)}\quad &+\tfrac{1}{6}\beta\lambda^2\int dt\int d^3x\,d^3x'\,G_0^R(\boldsymbol{x}_1-\boldsymbol{x},t_1-t)S_0^3(\boldsymbol{x}-\boldsymbol{x}',t-t_0)\\ &\quad\times S_0(\boldsymbol{x}'-\boldsymbol{x}_2,t_0-t_2)+(\boldsymbol{x}_1,t_1)\leftrightarrow(\boldsymbol{x}_2,t_2)\\ \text{(d)}\quad &+\tfrac{1}{2}\lambda^2\int dt\,dt'\int d^3x\,d^3x'\,G_0^R(\boldsymbol{x}_1-\boldsymbol{x},t_1-t)G_0^R(\boldsymbol{x}-\boldsymbol{x}',t-t')\\ &\quad\times S_0^2(\boldsymbol{x}-\boldsymbol{x}',t-t')S_0(\boldsymbol{x}'-\boldsymbol{x}_2,t'-t_2)+(\boldsymbol{x}_1,t_1)\leftrightarrow(\boldsymbol{x}_2,t_2)\\ \text{(e)}\quad &+Tm_2^2\lambda^2\int dt\int d^3x\,G_0^R(\boldsymbol{x}_1-\boldsymbol{x},t_1-t)S_0(\boldsymbol{x}-\boldsymbol{x}_2,t-t_2)+(\boldsymbol{x}_1,t_1)\leftrightarrow(\boldsymbol{x}_2,t_2)\\ \text{(f)}\quad &+\tfrac{1}{6}\lambda^2\int dt\,dt'\int d^3x\,d^3x'\,G_0^R(\boldsymbol{x}_1-\boldsymbol{x},t_1-t)S_0^3(\boldsymbol{x}-\boldsymbol{x}',t-t')G_0^R(\boldsymbol{x}_2-\boldsymbol{x}',t_2-t').\end{aligned}$$

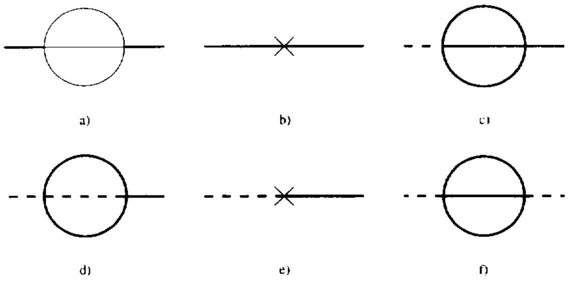

Fig. 3. Second order contribution to the two-point function; here the cross denotes the second order counterterm.

The diagrams are given in Fig. 3. Contributions (a) and (b) come from the second order expansion of the Boltzmann factor, (c) from a combination of the first order expansion of the Boltzmann factor and the first order solution of the equation of motion, (d) and (e) from the second order solution and (f) from the combination of two first order solutions to the equations of motion; (a) and (b) are the normal equal time contributions, (c), (d), (e) and (f) vanish when $t_1 = t_2 = t_0$.

4. Finiteness of the second order contribution

We will now see if the counterterm can be chosen in such a way that the second order contribution is finite. First we go over to three-dimensional momentum space. Approximating in expressions (a) to (f) the sines and the cosines by 1, we see that there are possible linear ((d)) and logarithmic ((a), (c), (f)) divergences (actually, the indication of a linear divergence disappears after doing the time integrals). Inspection then shows that the momentum dependent part is finite, such that only the momentum independent part needs to be studied. We will take the external momentum $\boldsymbol{p}$ to be zero, the internal momenta are labelled as $\boldsymbol{k}_1, \boldsymbol{k}_2$ and $\boldsymbol{k}_3 = -\boldsymbol{k}_1 - \boldsymbol{k}_2$. Using the fact that the correlation function only depends on $t_1 - t_2$, we take $t_2 = t_0 = 0$ and $t_1 = t$. This immediately sets to zero half of the contribution from (c), (d), (e) and the complete contribution from (f). Doing the time integrals, the expressions $\omega_{k_3} \pm \omega_{k_1} \pm \omega_{k_2}$ (with all four combinations of the $\pm$'s) often pop up, and we will denote these by

$$\Omega = \epsilon_1 \omega_{k_1} + \epsilon_2 \omega_{k_2} + \epsilon_3 \omega_{k_3},$$

with $\epsilon_i = \pm 1$.

Performing the time integrals, using the symmetry of the resulting expression to replace $3\Omega\epsilon_3\omega_{k_3} \to \Omega^2$ and combining several contributions, we find the following result:

$$S(\mathbf{0}, t) = \frac{T\cos(mt)}{m^2}$$

$$\text{(a+b)} \quad +\frac{T\lambda^2\cos(mt)}{m^4}\left[\frac{T^2}{6}\int\frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}\frac{d^3k_3}{(2\pi)^3}\frac{(2\pi)^3\delta(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)}{\omega^2_{\boldsymbol{k}_1}\omega^2_{\boldsymbol{k}_2}\omega^2_{\boldsymbol{k}_3}}+m_2^2\right]$$

$$\text{(d+e)} \quad +\frac{T\lambda^2 t\sin(mt)}{2m^3}\left[\frac{T^2}{6}\int\frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}\frac{d^3k_3}{(2\pi)^3}\frac{(2\pi)^3\delta(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)}{\omega^2_{\boldsymbol{k}_1}\omega^2_{\boldsymbol{k}_2}\omega^2_{\boldsymbol{k}_3}}+m_2^2\right]$$

$$\text{(c+d)} \quad +\frac{T^3\lambda^2}{48}\int\frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}\frac{d^3k_3}{(2\pi)^3}\frac{(2\pi)^3\delta(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)}{\omega^2_{\boldsymbol{k}_1}\omega^2_{\boldsymbol{k}_2}\omega^2_{\boldsymbol{k}_3}}$$

$$\times\sum_{\epsilon_1\epsilon_2\epsilon_3}\frac{1}{\Omega^2-m^2}\left[\frac{t\sin(mt)}{2m}+\frac{\cos(\Omega t)-\cos(mt)}{\Omega^2-m^2}\right].$$

In parentheses we have indicated which diagrams contributed. Both (a+b) and (d+e) give the same result for the second order counterterm, it is given by (2). This is an important result because it shows that the counterterms of the static theory are sufficient to make the time dependent two-point function two loop finite. With the choice (2) (again in particular the finite part), the above (a+b) and (d+e) are zero. The contribution indicated by (c+d) is ultraviolet finite. The possible divergences for $\Omega \to \pm m$ are actually not present. For example, for $\Omega \to m$,

$$\frac{1}{\Omega^2-m^2}\left[\frac{t\sin(mt)}{2m}+\frac{\cos(\Omega t)-\cos(mt)}{\Omega^2-m^2}\right]=\frac{t\sin mt}{8m^3}-\frac{t^2\cos mt}{8m^2}+O(\Omega-m).$$

Let us record here also the fact that the two loop correction is $O(t^4)$ for $t \to 0$.

5. Damping rate

As an application we derive an expression for the damping rate γ, assuming that the dominant large time behavior of the correlation function at zero momentum is given by $Z(T/m^2)\exp(-\gamma t)\cos((m+\delta)t)$. Since γ and $Z-1$ are $O(\lambda^2)$ and δ at least $O(\lambda)$ we can identify γ from the coefficient of $-t\cos(mt)$ in the large time behavior of (3). In terms of

$$w(\Omega)=\frac{\lambda^2}{6}\sum_{\epsilon_1\epsilon_2\epsilon_3}\int\prod_{j=1}^{3}\left[\frac{d^3k_j}{(2\pi)^3 2\omega_{\boldsymbol{k}_j}}\frac{T}{\omega_{\boldsymbol{k}_j}}\right](2\pi)^4\delta(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)\,\delta(\epsilon_1\omega_{\boldsymbol{k}_1}+\epsilon_2\omega_{\boldsymbol{k}_2}+\epsilon_3\omega_{\boldsymbol{k}_3}-\Omega),$$

we can rewrite the $O(\lambda^2)$ contribution to (3) in the form

$$S^{(2)}(\boldsymbol{0},t)=\int\frac{d\Omega}{2\pi}\,w(\Omega)\,\frac{1}{\Omega^2-m^2}\left[\frac{t\sin mt}{2m}+\frac{\cos\Omega t-\cos mt}{\Omega^2-m^2}\right].$$

The function $w(\Omega)$ is even in $\Omega \to -\Omega$, continuous and vanishes like Ω^{-1} at large Ω. A large time behavior $\propto t\cos mt$ can only come from the singularities at $\Omega = \pm m$, any other portion of the Ω integral gives subdominant behavior. Hence we may conveniently replace $w(\Omega) \to w(m)$ and evaluate the resulting integral,

$$S^{(2)}(\boldsymbol{0},t) \to w(m)\left[\frac{\sin mt}{4m^3}-\frac{t\cos mt}{4m^2}\right],$$

from which we identify

$$\gamma=\frac{w(m)}{4T}.$$

The ϵ's giving nonvanishing contributions to $w(\Omega)$ are $\epsilon_1 = \epsilon_2 = 1$, $\epsilon_3 = -1$ (cycl) for $\Omega > 0$ and similar for $\Omega < 0$. Hence $w(\Omega)$ has the form of a phase space integral for the scattering $(\mathbf{0}, \Omega) + (\mathbf{k}_3, \omega_{k_3}) \to (\mathbf{k}_1, \omega_{k_1}) + (\mathbf{k}_2, \omega_{k_2})$, etc. The particles are distributed according to T/ω, which is the high temperature form of the quantal distribution $[\exp(\omega/T) - 1]^{-1}$. Evaluation of $w(m)$ gives for the damping rate

$$\gamma = \frac{\lambda^2 T^2}{1536 \pi m}. \tag{3}$$

6. Matching to the quantum theory

The classical theory can be seen as a high temperature approximation to the quantum theory. Then the parameters in the classical theory are effective parameters which are to be chosen such that the classical correlation functions match those in the quantum theory as well as possible. For the static theory the approximation is equivalent to a form of dimensional reduction and this makes it possible to determine the parameters in the potential energy part of the hamiltonian in terms of the parameters in the quantum theory [4]. General matching rules are obtained in Ref. [7]. For example, matching the cosmological constant, mass and coupling parameters to one loop order we find

$$\epsilon = \bar{\epsilon} - \frac{\pi^2 T^4}{90} + \frac{\bar{m}^2 T^2}{24} - \frac{\bar{m}^4}{64\pi^2} \ln\frac{\bar{\nu}^2}{\nu_T^2} - \frac{\Lambda^3 T}{6\pi^2}\left(\ln\frac{\Lambda}{T} - \frac{1}{3}\right) - \frac{\Lambda T m^2}{4\pi^2},$$

$$\mu^2 = \bar{m}^2 + \bar{\lambda}\left(\frac{T^2}{24} - \frac{\bar{m}^2}{32\pi^2}\ln\frac{\bar{\nu}^2}{\nu_T^2} - \frac{\Lambda T}{4\pi^2}\right), \quad \lambda = \bar{\lambda}\left(1 - \frac{3\bar{\lambda}}{32\pi^2}\ln\frac{\bar{\nu}^2}{\nu_T^2}\right),$$

where $\nu_T \equiv 4\pi \exp(-\gamma_E) T$ and less dominant terms as $T \to \infty$ are suppressed. The quantities on the right hand side are dimensionally renormalized parameters in the MS-bar scheme and $\bar{\nu}$ is the corresponding mass scale. Notice how the Rayleigh-Jeans divergence is canceled by the Λ^3 counterterm, leaving the familiar quantum expression for the pressure as just a finite renormalization in the classical theory.

The above relation for μ^2 now implies with (1), choosing the MS-bar scale $\bar{\nu} = \nu_T$,

$$m^2 = \bar{m}^2 + \lambda\left(\frac{T^2}{24} - \frac{\bar{m}T}{8\pi}\right).$$

Inserting this relation in the result (3) for the damping rate gives for large T

$$\gamma = \frac{\lambda^{3/2} T}{128\sqrt{6}\,\pi},$$

which is indeed the plasmon rate obtained in the quantum theory [10].

7. Discussion

Among the time-independent correlation functions only the two-point function is superficially divergent, because of the superrenormalizability. Inspection of the time-dependent correlation functions leads to the conclusion that, after performing the intermediary time integrals and replacing the remaining sines and cosines by 1, the overall degree of divergence of the $(n > 2)$-point functions is less than zero. Only the $n = 2$ case is then non-trivial and we have shown to second order in λ that the time-dependent classical two-point function is finite, after taking into account the counterterms of the static theory. We conclude therefore (but have of course not given a proof) that with these counterterms all finite temperature time dependent correlation functions in ϕ^4 theory are cutoff independent.

The coupling λ was taken to be cutoff independent, consistent with the superrenormalizability of the static theory. We would like to remark that this static three-dimensional theory is quite non-generic from the statistical physics point of view. The coupling λ_3 of a three-dimensional theory has the dimension of [mass] and the dimensionless coupling at scale ν is $\lambda(\nu) = \lambda_3(\nu)/\nu$. We may call $\lambda_B = \lambda(\Lambda)$ the bare coupling and $\lambda_R = \lambda(m)$ the renormalized coupling. Since the system gets critical as the ratio Λ/m approaches infinity, one might expect that the renormalized coupling approaches the nontrivial infrared stable fixed point λ^* of Ising-like systems (the Wilson-Fischer point). This is not the case here, because $\lambda_B = \lambda T/\Lambda$ is non-generic: it is is automatically tuned to the gaussian fixed point as $\Lambda/m \to \infty$. The renormalized coupling $\lambda_R \approx \lambda T/m$, which can be arbitrary in the interval $(0, \lambda^*)$.

For a clearer understanding of the classical theory as an approximation to the quantum theory, one would like to go beyond the usual dimensional reduction involving only time independent quantities. Matching time dependent quantities may introduce at two loop order also a finite renormalization of the coefficient of π^2 in the hamiltonian. We have not yet carried out such time dependent matching. For a different approach to time dependent quantities, based on an expansion in Planck's constant $\hbar$, see Ref. [11].

The fact that the analytic expression for the classical plasmon damping rate combined with parameter matching according to dimensional reduction leads to the correct quantum mechanical result is quite encouraging for the physically more interesting case of the SU(2)-Higgs model (cf. Ref. [12] where Eq. (4.3) gives the impression that the classical gluon damping rate is uv-divergent; quantum mechanically the gluon damping rate is of course uv-finite but subtle in the infrared, see, e.g., [13]). Numerical simulations in the classical SU(2)-Higgs model clearly show the damping phenomenon and appear to be compatible with lattice spacing independence of the rates [14].

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