The Classical Approximation for Real-Time Scalar Field Theory at Finite Temperature*

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The use of classical thermal field to approximate real-time quantum thermal field theory is discussed. For a $\lambda\phi^4$ theory, it is shown that the classical Rayleigh-Jeans divergence can be canceled with the appropriate counterterms, and a comparison is made between the classical and quantum perturbative expansion. It is explained why Hard Thermal Loops prevent the same method to work for gauge theories.

I. INTRODUCTION

Real-time dynamics of quantum fields plays an important role in the early universe (baryogenesis, inflation) and in heavy ion collisions. Some quantities cannot be determined within perturbation theory, even after possible resummations. An example is the rate of sphaleron transitions at high temperature in (extensions of) the Standard Model, relevant for baryogenesis. A non-perturbative real-time calculation on a lattice, using Monte Carlo methods, is complicated, because of the complex Boltzmann weight.

A relatively simple way to do the dynamics is to use the classical equations of motion. Depending on the choice of initial conditions, this describes an equilibrium or a non-equilibrium situation. In this talk I discuss classical $\lambda\phi^4$ theory, in thermal equilibrium. This talk is based on work done in collaboration with Jan Smit [1] (see also [2]). More recent work is presented at this Workshop at the poster session [3].

II. EFFECTIVE FIELD THEORIES AND COARSE-GRAINING

It often happens that a Quantum Field Theory (QFT) is too complicated to be solved completely. If there are several scales present in the model, a useful approach is to construct an effective field theory for the degrees of freedom that are important at the scale under consideration. The other 'less important' degrees are integrated out, one way or the other. This very general idea has been applied successfully in many different physical situations, under names as coarse-graining, Wilson renormalization group, dimensional reduction, and so on.

These ideas have also been applied when considering the real-time dynamics in a QFT at finite temperature. This typically leads to a transport or kinetic theory desciption. As an explicit example, consider real-time $\lambda\phi^4$ theory at finite temperature [4]. The low-momentum or soft modes are considered to be the degrees of freedom that are of interest, and the high-momentum or hard modes as those that can be integrated out. In general, the resulting semi-classical dynamics for the soft modes is very complicated. They are subject to noise and dissipation, due to the coarse-grained interaction with hard modes, and the effective equations of motion are non-local in space and time. Especially this non-locality makes it difficult to use the resulting equations directly for a numerical treatment.

Therefore I discuss here another possibility to approximate the dynamics, originally proposed in [5], which is related to dimensional reduction (DR) and 3-d effective field theories for static quantities: classical thermal field theory.

III. DYNAMICS AND CLASSICAL THERMAL FIELD THEORY

As already mentioned in the Introduction, a relatively simple way to deal with the dynamics, is to use the classical equations of motions. One simply solves the equations, with given initial conditions, and then averages over the initial conditions with the Boltzmann weight $\exp -\beta H$ as distribution function. If the hamiltonian in the Boltzmann weight is the same as the one that determines the equations of motion, this leads to an equilibrium description. For e.g. the classical 2-point function, this procedure means the following ($x = (\mathrm{x}, t)$)

$$S(x-x') = \langle\phi(x)\phi(x')\rangle_{\rm cl} = Z_{\rm cl}^{-1}\int \mathcal{D}\pi\mathcal{D}\phi\, e^{-\beta H(\pi,\phi)}\,\phi(x)\phi(x'), \tag{1}$$

with the classical partition function

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$$Z_{\rm cl} = \int \mathcal{D}\pi\mathcal{D}\phi\, e^{-\beta H(\pi,\phi)},$$

$\beta = 1/T$, and the hamiltonian and potential

$$H = \int d^3x\, \frac{1}{2}\pi^2 + V(\phi),$$
$$V = \int d^3x \left(\frac{1}{2}(\nabla\phi)^2 + \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4\right).$$

In (1), $\phi(x)$ is the solution of the classical equations of motion $\dot\phi(x) = \{\phi(x), H\}$, $\dot\pi(x) = \{\pi(x), H\}$, with the initial conditions $\phi({\rm x}, t_0) = \phi({\rm x})$, $\pi({\rm x}, t_0) = \pi({\rm x})$. The integration over phase space is over the initial conditions at $t = t_0$, weighted with the Boltzmann weight.

It is necessary to make two remarks at this point. First of all, classical thermal field theory contains the well-known Rayleigh-Jeans divergence. Hence, everything is formulated with a (lattice) cutoff, to regularize this divergence. We will show that this cutoff can be removed in the end, if the parameters in the classical theory are chosen in the correct way. And secondly, when restricting to time-independent correlation functions, the canonical momenta can be integrated out, and the resulting partition function has precisely the form of that of a 3-d, superrenormalizable field theory,

$$Z_{\text{3-d}} = \int \mathcal{D}\phi\, e^{-\beta V(\phi)},$$

which has been studied in great detail in DR [6]. I will come back to DR results at the appropriate places.

In the remainder of this talk, I will discuss what role the cutoff and the Rayleigh-Jeans divergence play and what the relation with the dynamics in the quantum theory is. This will be done in perturbation theory.

IV. PERTURBATION THEORY IN QUANTUM AND CLASSICAL THERMAL FIELD THEORY

Perturbation theory in the classical theory is obtained by writing the field as $\phi(x) = \phi_0(x) + \lambda\phi_1(x) + \ldots$. Solving the equations of motion, order by order in λ, gives

$$\phi_0(x) = \int \frac{d^3k}{(2\pi)^3}\, e^{i{\rm k}\cdot{\rm x}}\, [\phi({\rm k})\cos\omega_{\rm k}(t - t_0) + \frac{\pi({\rm k})}{\omega_{\rm k}}\sin\omega_{\rm k}(t - t_0)],$$
$$\phi_1(x) = -\lambda\int d^4x'\, G_0^R(x - x')\frac{1}{3!}\phi_0^3(x'), \quad \text{etc},$$

with $\omega_{\rm k}^2 = {\rm k}^2 + m^2$. Here we introduced the retarded Green function

$$G_{\rm cl}^R(x - x') = -\theta(t - t')\langle\{\phi(x), \phi(x')\}\rangle_{\rm cl}. \qquad (2)$$

In the unperturbed case, it reads (after spatial Fourier transformation)

$$G_0^R({\rm k}, t - t') = \theta(t - t')\frac{\sin\omega_{\rm k}(t - t')}{\omega_{\rm k}}. \qquad (3)$$

If one now calculates (1), products of ϕ_0 have to be averaged with the Boltmann weight, and for the unperturbed case this gives

$$S_0({\rm k}, t) = \int d^3x\, e^{-i{\rm k}\cdot{\rm x}}\langle\phi_0(x)\phi_0(0)\rangle_{\rm cl} = T\frac{\cos\omega_{\rm k}t}{\omega_{\rm k}^2}. \qquad (4)$$

As is well-known, there is a zoo of methods available to formulate perturbation theory in real-time quantum thermal field theory [7]. When using the Schwinger-Keldysh contour, the quantum field ϕ is denoted with ϕ^+, ϕ^-, if it lives on resp. the upper and the lower branch. A version that is particulary convenient here, is the 'center-of-mass/relative' coordinates version, where the basic field variables are taken as

$$\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix} = \begin{pmatrix}(\phi_+ + \phi_-)/2\\ \phi_+ - \phi_-\end{pmatrix},$$

in terms of ϕ^+, ϕ^-. The reason is that the matrix propagator in this basis compares directly with the 2-point functions that we found in classical perturbation theory. Namely, the 2×2 propagator is given by

$$\mathbf{G}(x - x') = \begin{pmatrix} iF(x - x') & G_{\rm qm}^R(x - x')\\ G_{\rm qm}^A(x - x') & 0\end{pmatrix},$$

with in terms of the original field

$$F(x - x') = \frac{1}{2}\langle\phi(x)\phi(x') + \phi(x')\phi(x)\rangle,$$
$$G_{\rm qm}^R(x - x') = G_{\rm qm}^A(x' - x) = i\theta(t - t')\langle[\phi(x), \phi(x')]\rangle.$$

Indeed, in the naive classical limit, i.e. fields commute and commutators go to Poisson brackets, these become (1) and (2).

In the unperturbed case, the relation becomes very explicit. The free G_0^R is given by the classical expression (3), and the free F_0 is given by

$$F_0({\rm k}, t) = [n(\omega_{\rm k}) + \frac{1}{2}]\frac{\cos\omega_{\rm k}t}{\omega_{\rm k}} = T\sum_n \frac{\cos\omega_{\rm k}t}{\omega_n^2 + \omega_{\rm k}^2},$$

where $n(\omega) = (e^{\beta\omega} - 1)^{-1}$ is the Bose distribution. We have written the real-time 2-point function that contains all the temperature dependence as a sum over Matsubara frequencies $\omega_n = 2\pi nT$, familiar from the imaginary-time formalism. Comparing this result with (4) indicates that the classical theory is an effective theory for the $n = 0$ term, just as in DR for time-independent quantities.

An important ingredient in the setup of perturbation theory in the quantum case is the KMS condition [8]. Using the KMS condition to determine the 2-point function in temporal momentum space, ensures that the so-called vertical part of the contour is taken into account, or in more physical terms, that the system is in thermal equilibrium. It turns out that it is possible to derive a KMS condition also in the classical theory [9], it reads

$$\frac{d}{dt}\langle\phi(x)\phi(x')\rangle_{\rm cl} = T\langle\{\phi(x),\phi(x')\}\rangle_{\rm cl}.$$

It can be used to express S in terms of G^R, G^A. Explicitly, we find in resp. the quantum and the classical case the following relation in temporal momentum space ($k=(k^0,\mathrm{k})$)

$$\begin{aligned} F_0(k) &= -i(n(k^0)+\frac{1}{2})\left(G_0^R(k)-G_0^A(k)\right), \\ S_0(k) &= -i\frac{T}{k^0}\left(G_0^R(k)-G_0^A(k)\right). \end{aligned}$$

Using this, we can work in temporal momentum space throughout, which is convenient from a technical point of view.

All the found relations between the quantum and the classical expressions are nice, but rather academic until we start to calculate perturbative corrections. Hence we calculated the 2-point function to two loops and the 4-point function to one loop. Both in the quantum as in the classical theory, it is straightforward to recognize diagrams and identify 1PI parts. It turns out that there is a direct correspondence with the diagrams in the quantum theory if one uses the ϕ_1,ϕ_2 basis. I will show this for the one loop and two loop setting sun retarded self-energy $\Sigma_R(p)$.

One loop self-energy

In the quantum theory, the one loop retarded self-energy is given by (see Fig. 1)

$$\begin{aligned} \Sigma_R^{(1)} &= \frac{1}{2}\lambda\int\frac{d^4k}{(2\pi)^4}\,F_0(k) \\ &= \frac{\lambda T^2}{24\hbar}-\frac{\lambda mT}{8\pi}+\mathcal{O}(\hbar\lambda\log(T/\hbar)), \end{aligned} \tag{5}$$

where I indicated explicitly the $\hbar$ dependence. The leading term is the so-called thermal mass, $m_{\rm th}^2$, which is the only Hard Thermal Loop (HTL) contribution in scalar field theory. It needs to be resummed in order to have a true perturbative expansion (in $\lambda^{1/2}$).

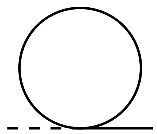

FIG. 1. Retarded self energy: one loop diagram. The full line represents F_0 resp. S_0 and the dashed-full line G_0^R.

In the classical theory, we find (with a momentum cutoff Λ)

$$\Sigma_{R,\rm cl}^{(1)} = \frac{1}{2}\lambda\int\frac{d^4k}{(2\pi)^4}\,S_0(k) = \frac{\lambda\Lambda T}{4\pi^2}-\frac{\lambda mT}{8\pi}. \tag{6}$$

Compared with the quantum expression, F_0 is replaced by S_0. The $\hbar^0$ term in (5) is correctly reproduced by the classical theory, and the higher order terms in $\hbar$ are absent, as expected. The $\hbar^{-1}$ term turns up in the classical theory as a linear divergence, and not as the thermal mass. However, if we use our knowledge from DR, we can simply cancel the divergence and put in the correct thermal mass by the appropriate choice of classical mass parameter, which is written as $m^2 = m_{\rm th}^2-\delta m^2$. In DR, this is called matching. Note that since this one loop diagram is momentum independent, the analysis is not very complicated.

Two loop setting sun diagrams

The two loop setting sun contribution to the self-energy is momentum-dependent and gives in the quantum theory rise to e.g. Landau damping, because of an imaginary part. Hence it is worthwhile to analyse this diagram also in the classical theory. In the quantum theory we have two diagrams (see Fig. 2)

$$\begin{aligned} \Sigma_R^{\rm sun}(p) = -\frac{1}{2}\lambda^2\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\Big[&F_0(k_1)F_0(k_2)G_0^R(k_3) \\ &-\frac{1}{12}G_0^R(k_1)G_0^R(k_2)G_0^R(k_3)\Big], \end{aligned}$$

with $k_3 = p-k_1-k_2$. Note that the second diagram does not contain (explicit) temperature dependence. A closer look shows that it is indeed subdominant at high T and small λ.

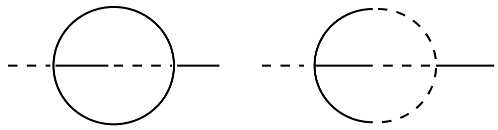

FIG. 2. Retarded self energy: two loop setting sun diagrams. The second diagram is absent in the classical theory.

It turns out that in the classical theory we only find the dominant contribution, i.e. the classical counterpart ($F_0\to S_0$) of the first diagram

$$\begin{aligned} \Sigma_{R,\rm cl}^{\rm sun}(p) &= -\frac{1}{2}\lambda^2\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}S_0(k_1)S_0(k_2)G_0^R(k_3) \\ &= -\frac{\lambda^2T^2}{6}\int\frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}\frac{1}{\omega_{\mathrm{k}_1}^2\omega_{\mathrm{k}_2}^2\omega_{\mathrm{k}_3}^2} \qquad (7)\\ &\quad +\frac{p^0}{T}\int\frac{d\Omega}{2\pi}\frac{w(\mathrm{p},\Omega)}{p^0+i\epsilon+\Omega}, \qquad (8) \end{aligned}$$

where we introduced the classical scattering integral

$$
w(\mathrm{p},\Omega) = \frac{\lambda^2}{6}\sum_{\{\pm\}}\int\prod_{j=1}^{3}\left[\frac{d^3k_j}{(2\pi)^3 2\omega_{\mathrm{k}_j}}\frac{T}{\omega_{\mathrm{k}_j}}\right] \times(2\pi)^4\delta(\Omega\pm\omega_{\mathrm{k}_1}\pm\omega_{\mathrm{k}_2}\pm\omega_{\mathrm{k}_3}).
$$

The sum is over all +'s and −'s. The first term (7) is independent of p^0, real, and logarithmically divergent. It is actually the result obtained in DR. Hence the divergence is canceled again by the appropriate choice of classical mass parameter, just as in the one loop diagram. The divergences we have encountered by now are the only ones in DR, because of superrenormalizability. A closer look at other diagrams in the classical theory makes it plausible that these divergences are also the only ones in the general time-dependent classical case. Hence the classical divergences are completely under control. The second term (8) is finite. It contains all the p^0 i.e. time dependence, and it has an imaginary part.

It is instructive to compare the imaginary parts in the quantum and classical case. Using the notation [10] (for $p^0 > 0$)

$$
\mathrm{Im}\,\Sigma_R^{\mathrm{sun}}(p) = -g_1(p) - g_2(p),
$$

for both the quantum and classical case, we find, in the quantum theory

$$
\begin{aligned}
g_1(p) &= \frac{\lambda^2}{96}(e^{p^0/T}-1)\int\frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}\frac{n_{\mathrm{k}_1}}{\omega_{\mathrm{k}_1}} \\
&\quad\times\frac{n_{\mathrm{k}_2}}{\omega_{\mathrm{k}_2}}\frac{n_{\mathrm{k}_3}}{\omega_{\mathrm{k}_3}}2\pi\delta(p^0-\omega_{\mathrm{k}_1}-\omega_{\mathrm{k}_2}-\omega_{\mathrm{k}_3}), \\
g_2(p) &= \frac{\lambda^2}{32}(e^{p^0/T}-1)\int\frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}\frac{(1+n_{\mathrm{k}_1})}{\omega_{\mathrm{k}_1}} \\
&\quad\times\frac{n_{\mathrm{k}_2}}{\omega_{\mathrm{k}_2}}\frac{n_{\mathrm{k}_3}}{\omega_{\mathrm{k}_3}}2\pi\delta(p^0+\omega_{\mathrm{k}_1}-\omega_{\mathrm{k}_2}-\omega_{\mathrm{k}_3}),
\end{aligned}
$$

and in the classical theory

$$
\begin{aligned}
g_{1,\mathrm{cl}}(p) &= \frac{\lambda^2}{96}\frac{p^0}{T}\int\frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}\frac{T^3}{\omega_{\mathrm{k}_1}^2\omega_{\mathrm{k}_2}^2\omega_{\mathrm{k}_3}^2} \\
&\quad\times 2\pi\delta(p^0-\omega_{\mathrm{k}_1}-\omega_{\mathrm{k}_2}-\omega_{\mathrm{k}_3}), \\
g_{2,\mathrm{cl}}(p) &= \frac{\lambda^2}{32}\frac{p^0}{T}\int\frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}\frac{T^3}{\omega_{\mathrm{k}_1}^2\omega_{\mathrm{k}_2}^2\omega_{\mathrm{k}_3}^2} \\
&\quad\times 2\pi\delta(p^0+\omega_{\mathrm{k}_1}-\omega_{\mathrm{k}_2}-\omega_{\mathrm{k}_3}),
\end{aligned}
$$

$g_{1,(\mathrm{cl})}(p)$ represents three body decay, and $g_{2,(\mathrm{cl})}(p)$ Landau damping. The classical expressions are indeed the leading order expressions from the quantum theory. The reason is that, after replacement of the Bose distributions by the classical distributions, the resulting integrals are still finite.

To demonstrate this very explicitly, consider the on-shell plasmon damping rate [11,10]. It is given by

$$
\gamma = \frac{-\mathrm{Im}\,\Sigma_R^{\mathrm{sun}}(\mathbf{0},m)}{2m} = \frac{g_2(\mathbf{0},m)}{2m}.
$$

It turns out that, to leading order in λ and T, $g_2(\mathbf{0},m)$ equals $g_{2,\mathrm{cl}}(\mathbf{0},m)$, and we find (to leading order)

$$
\gamma_{\mathrm{cl}} = \gamma = \frac{\lambda^2T^2}{1536\pi m}.
$$

Using that $m \approx m_{\mathrm{th}}$, because of resummation in the quantum theory and because of matching in the classical theory, gives for the plasmon damping rate (again to leading order)

$$
\gamma_{\mathrm{cl}} = \gamma = \frac{\lambda\sqrt{\lambda\hbar}T}{128\sqrt{6}\pi}.
$$

This example shows in a very explicit way that the classical theory, with the appropriate choice of parameters, indeed approximates the quantum theory.

Let me end the discussion on scalar field theory with the remark that also for the 4-point function the leading order expressions are reproduced by the classical theory.

V. CLASSICAL THERMAL GAUGE THEORY AND HARD THERMAL LOOPS

Of course, one of the reasons the scalar theory was studied, is because of possible applications to the dynamics of hot gauge theories. We know that the DR approach has been succesfully applied to study e.g. the electroweak phase transition, and the presence of (non-)abelian gauge fields did not pose any fundamental problems.

It turns out that, concerning the dynamics, there are such problems [12,13]. The reason for this are the Hard Thermal Loops in gauge theories [14]. In contrast to the scalar theory, HTL's in gauge theories are momentum dependent, in a complicated non-analytical way. As a typical example, consider the longitudinal part of the gauge boson self-energy in e.g. scalar electrodynamics. In the HTL approximation, it is given by [7]

$$
F(p^0,\mathrm{p}) = 2m^2\left(1-\frac{p^{0\,2}}{\mathrm{p}^2}\right)\left(1-\frac{p^0}{2|\mathrm{p}|}\log\frac{p^0+|\mathrm{p}|}{p^0-|\mathrm{p}|}\right),
$$

with the prefactor

$$
m^2 = \frac{e^2}{\pi^2}\int_0^\infty dk\,k\,n(k) = \frac{e^2T^2}{6\hbar},
$$

where I indicated again the $\hbar$ dependence. This term gives rise to e.g. the Debye screening mass, $m_D^2 = F(0,\mathrm{p}\to 0) = 2m^2 = e^2T^2/3\hbar$.

In the classical theory, the Bose distribution $n(\omega)$ is replaced by the classical one T/ω, which leads to a linearly divergent prefactor

$$
m_{\mathrm{cl}}^2 = \frac{e^2}{\pi^2}\int_0^\Lambda dk\,k\frac{T}{k} = \frac{e^2T\Lambda}{\pi^2},
$$

where we used a momentum cutoff for simplicity. This is similar to what happens in the scalar theory (compare

(5) and (6)). However, now it is not possible to cancel the divergence with a local mass counterterm, because of the complicated momentum dependence. In the DR approach for static quantities, these problems do not arise, because for $p^0 = 0$, $F(0, \mathrm{p}) = 2m^2$, i.e. momentum independent. Hence the linear divergent Debye mass in the effective 3-d theory can, in DR, simply be canceled by a mass counterterm.

We can conclude that HTL's in gauge theories make a straightforward use of classical thermal gauge theory to approximate the dynamics questionable. In the several proposals that exist to incorporate HTL effects in a classical-like theory, new local degrees of freedom are added to make up for them in one way or the other [12,15–17]. The idea to add classical particles (instead of fields) has been succesfully implemented numerically [16].

VI. CONCLUSIONS

To summarize, we have shown how classical thermal field theory can be used to approximate real-time quantum field at finite temperature, for the $\lambda\phi^4$ case. Instead of explicitly integrating out hard modes to construct an effective theory for the soft modes, we showed that it is possible to take all the modes into account. The resulting Rayleigh-Jeans divergence can be dealt with in a straightforward manner, namely by using counterterms that are dictated by dimensional reduction. Furthermore, by using real-time perturbation theory, we have shown that the classical theory approximates the quantum one if also the finite part of the classical parameters are chosen according to the DR matching rules.

Essential is that the HTL effects in the quantum theory can be easily incorporated in the classical theory: only the thermal mass has to be put in. This is also the reason why the same prescription does not work for gauge theories. Here HTL's are momentum dependent and in the classical theory, they give rise to divergences that cannot be canceled with local counterterms (i.e. in $3+1$ dimensions; the classical approximation does work for gauge theories in $1+1$ dimensions, because here the classical contribution is dominant and finite, and the HTL contribution is subdominant [18]). Adding new degrees of freedom to represent the HTL contributions seems to be a possible way out.

ACKNOWLEDGMENTS

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