Available online at www.sciencedirect.com

ScienceDirect

Nuclear Physics A 956 (2016) 717–720

NUCLEAR PHYSICS A

www.elsevier.com/locate/nuclphysa

Spectral functions from anisotropic lattice QCD

G. Aarts[a], C. Allton[a], A. Amato[b], W. Evans[c], P. Giudice[d], T. Harris[e], A. Kelly[f], S.Y. Kim[g], M.P. Lombardo[h], K. Praki[a], S.M. Ryan[e], J.-I. Skullerud[f]

[a]*Department of Physics, Swansea University, Swansea SA2 8PP, Wales, UK*
[b]*Helsinki Institute of Physics and University of Helsinki, Helsinki, Finland*
[c]*Albert Einstein Center for Fundamental Physics, Institute for Theoretical Physics Universitat Bern, Sidlerstrasse 5, CH-3012 Bern, Switzerland*
[d]*Institut für Theoretische Physik, Universität Münster, D–48149 Münster, Germany*
[e]*School of Mathematics, Trinity College, Dublin 2, Ireland*
[f]*Department of Mathematical Physics, Maynooth University, Maynooth, Co Kildare, Ireland*
[g]*Department of Physics, Sejong University, Seoul 143-747, Korea*
[h]*INFN–Laboratori Nazionali di Frascati, I–00044 Frascati (RM), Italy*

Abstract

The FASTSUM collaboration has been carrying out lattice simulations of QCD for temperatures ranging from one third to twice the crossover temperature, investigating the transition region, as well as the properties of the Quark Gluon Plasma. In this contribution we concentrate on quarkonium correlators and spectral functions. We work in a fixed scale scheme and use anisotropic lattices which help achieving the desirable fine resolution in the temporal direction, thus facilitating the (ill posed) integral transform from imaginary time to frequency space. We contrast and compare results for the correlators obtained with different methods, and different temporal spacings. We observe robust features of the results, confirming the sequential dissociation scenario, but also quantitative differences indicating that the methods' systematic errors are not yet under full control. We briefly outline future steps towards accurate results for the spectral functions and their associated statistical and systematic errors.

1. Computing Spectral Functions: hard to be right, hard to be wrong

Spectral functions play an important role in our understanding of the composite states in the plasma[1]. Narrow, delta function-like shapes indicating stable states are deformed into broad peaks at higher temperature signalling the appearance of a thermal width of a still discernable bound state. At even larger temperatures quarks unbind and the peak structure is eventually lost, with the higher states melting first. For the light mesons and baryons this process is entangled with the pattern of chiral symmetry restoration. For charmonium and bottomonium things are subtler: the low-lying states are bound by the Coulombic part of the potential which is less affected by the thermal transition while the breaking of the string affects higher states leading to their dissolution. This is a sketchy picture of what is known as sequential melting of quarkonia.

In recent years great progress has been made in this field, and so far all realistic model or lattice calculations fully confirm this qualitative picture. For instance there is a satisfactory qualitative agreement for the S-waves. This means that the scenario of sequential melting is robust: it is very hard to be wrong in this

http://dx.doi.org/10.1016/j.nuclphysa.2016.05.005

respect. However, it turns out that it is very hard to be right too: the results of all these calculations differ quantitatively, producing different shapes of the spectral functions for the same temperatures. This comes as no surprise, as we have learnt that even for simpler bulk observables accuracy is achieved at the price of a great effort[2].

Our work is motivated by the desire to reach accurate results for spectral functions and their associated statistical and systematic errors.

2. Anisotropic lattices

Reaching the continuum limit is a major issue for any lattice study. Once the real time axis is rotated to an imaginary one, QCD is formulated on a four dimensional Euclidean space. It is in principle important and interesting to study the continuum limit by disentangling space and time discretization effects. This is achieved by using anisotropic lattices, characterized by different spacings a_s and a_τ. When this approach is used for thermodynamics, a finer a_τ renders practical the so-called fixed scale approach: since the temperature is given by $T = 1/(N_\tau a_\tau)$, one can realize different temperatures simply by changing N_τ while a_τ is kept fixed, thus bypassing the complications associated with an independent scale setting for each temperature.

The analysis of spectral functions poses specific problems, and anisotropic lattices are especially apt to address them[3]. For instance it is well known that spectral functions may have spurious peaks and structures on finite lattices, simply because the propagators always admit a spectral decomposition into exponentials and there is a frequency cutoff: it is only when $N_s, N_\tau \to \infty$ that we recover the spectral function of the continuum theory. Such evolution towards the continuum results can be monitored by changing N_τ while keeping a constant temperature T, which requires a suitable tuning of $a_\tau = 1/TN_\tau$. The larger N_τ which can be easily reached on anisotropic lattices is also beneficial when computing the integral transform of the Euclidean propagator $G(\tau)$ to obtain its associated spectral function $\rho(\omega)$:

$$G(\tau) = \int_0^\infty \frac{d\omega}{2\pi} K(\tau, \omega)\rho(\omega), \qquad 0 \leq \tau < \frac{1}{T}, \tag{1}$$

where, for mesons, the kernel $K(\tau, \omega)$ is $K(\tau, \omega) = \frac{\left(e^{-\omega\tau} + e^{-\omega(1/T-\tau)}\right)}{1-e^{-\omega/T}}$. It is well known that the inverse transform Eq. (1) is ill posed since the transform itself is a smoothing operator: we have to reconstruct peaks, or structures, in $\rho(\omega)$ from a smooth function $G(\tau)$. Clearly many points in the direct space — the Euclidean time — are needed to attempt this inversion, whichever method one wishes to use. In particular in our studies we rely primarily on Maximum Entropy Methods (MEM).

A third point which makes anisotropic lattices especially helpful concerns NRQCD, which is appropriate for the bottom quarks. The NRQCD evolution for the quark propagator is solved as an initial value problem: $G(\vec{n}, n_\tau) = H_\tau G(\vec{n}, n_\tau - a_\tau \vec{e}_\tau)$, and this discretized evolution becomes increasingly accurate with a smaller a_τ. In the Quark Gluon Plasma the significant range of frequencies for bottom quarks $\omega \sim 2M \sim 8$ GeV still exceeds $T < 0.5$ GeV and NRQCD remains applicable. Moreover $K(\tau, \omega) \simeq \left(e^{-\omega\tau} + e^{-\omega(1/T-\tau)}\right)$, hence backwards and forwards propagations are decoupled and the spectral relation reduces to an inverse Laplace transform: $G(\tau) = \int_{\omega_0}^\infty \frac{d\omega}{2\pi} \exp(-\omega\tau)\rho(\omega)$.

In this note we present results for temperatures ranging from $0.4T_c$ to $2T_c$, with a pion mass of 390 MeV, still larger than its physical value, and an almost physical strange quark. The lattice spacing is $a_s = 123$ am (attometers), and we consider two values of a_τ, $a_\tau = (35, 18)$ am, corresponding to anisotropy factors $a_s/a_\tau \simeq (3.5, 7)$. The first set of calculations – which are sometimes referred to as Generation 2 – is completed and published[5], while the results with the finer a_τ — referred to as Generation 3 — are still preliminary[4].

3. Charmonia and bottomonia on the 2nd Generation lattices: comparing and contrasting methods

Charmonium (relativistic) and bottomonium (non-relativistic) propagators have been computed on the 2nd Generation lattices. As a part of our program to understand and control the systematics of spectral

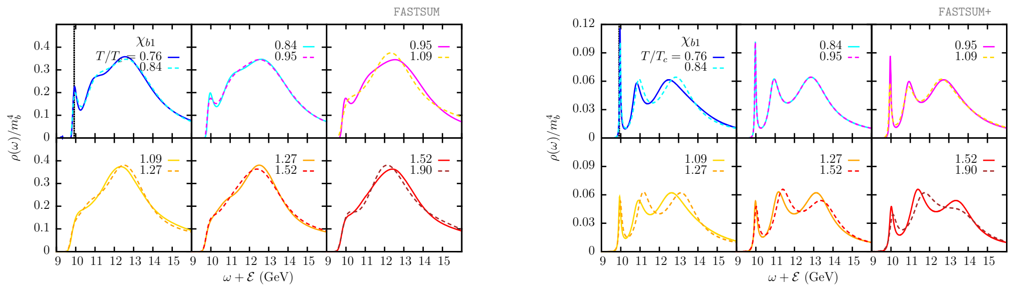

Fig. 1. Results for the χ_b spectral functions obtained by using the same set of correlators and different inversion methods: MEM on the left and the novel Bayesian method of [7] on the right.

function calculations we have analyzed them with different methods[3]. Let us summarize here the main findings: the behaviour of the S-wave bottomonia spectral functions for temperatures ranging from $T/T_c = 0.76$ to $T/T_c \simeq 2$ shows that the fundamental state — the Upsilon — broadens with temperature, however its peak still remains discernable, while the excited states melt above T_c. We have studied the stability of the MEM results with respect to the choice of the Euclidean time interval: the general features are robust, which is good as the observed pattern is clearly reminiscent of experimental observations. However the shape of the peak is affected by the details of the analysis, and we conclude that on these lattices we can only estimate an upper bound for the width[6]. For the P-wave states, no peak survives above T_c suggesting the melting of the χ_b.

The same set of correlators has been analyzed by use of the Burnier-Rothkopf(BR) method[7] and we observe a qualitative agreement for the S-wave states, while results for the P-wave states clearly still need to settle down. In Figure 1, from Ref. [3] we show the results for the latter case: in the BR approach the χ_b not only survives in the plasma, but the peak associated with the first excited state remains fairly constant with temperature, even as the ground state peak becomesweaker.

Similar analyses are in progress for charmonia: so far the MEM results indicate that the J/Ψ melts at about $1.5T_c$, while all P-wave states, including χ_c, melt in the QGP. In view of the discussion above more analysis and checks are of course needed before drawing conclusions, and this work is in progress.

As a generic statement, the BR method seems to have a stronger affinity to identify peaks, which might however include spurious ones. For instance, for P-waves the BR method also finds peaks in the free case, where no peaks are expected, which are attributed to ‘ringing’. On the other hand, MEM has no problem in reconstructing the free spectral function.

To make further progress we might, for instance: a) Try to disentangle spurious peaks by comparing interacting results with free lattice ones, where one has a clear picture of the meaning of different structures[8] b) Experiment with different methods[9, 10] c) Take the continuum limit: spurious peaks should disappear and the two methods might have a better chance to converge to the same results d) Have a closer look at the Euclidean propagators where different parametrizations might have a more transparent physical intepretation.

Our 3rd Generation data have been produced with the last two possibilities in mind: recall here that the anisotropy increases from 3.5 to 7. Before discussing some of our preliminary results, let us recall some aspects of the direct analysis of the 2nd Generation propagators. Consider first the infinite temperature limit, i.e. the limiting case of free heavy quarks: in continuum NRQCD the correlation functions behave as $G_{\rm free}(\tau) \propto \frac{e^{-\omega_0 \tau}}{\tau^{\alpha+1}}$, with $\alpha = (1/2, 3/2)$ for (S, P)-wave states. Defining an effective exponent[11] $\gamma_{\rm eff}(\tau) \equiv -\frac{\tau}{G(\tau)} \frac{dG(\tau)}{d\tau}$ we have indeed found some indication that $\gamma_{\rm eff}(\tau)$ approaches its free field values at high temperatures, and large time τ. It is then natural to consider the ansatz

$$G(\tau) = \frac{e^{-\omega_0(T)\tau}}{\tau^{\alpha(T)+1}} \tag{2}$$

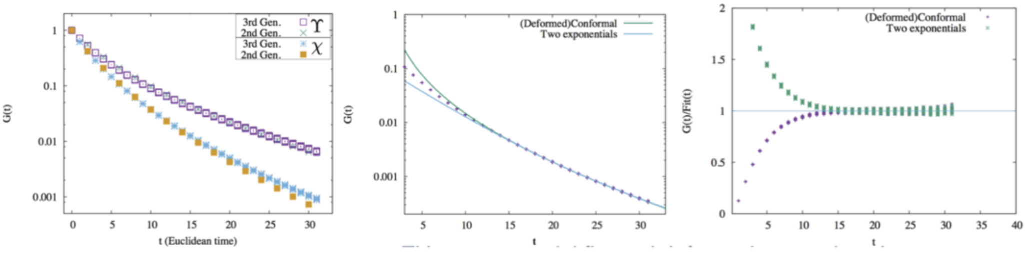

Fig. 2. 3rd Generation propagators contrasted with 2nd Generation propagators with rescaled Euclidean time (left). 3rd Generation χ_b propagator with fits superimposed (center). Ratio of the 3rd Generation results for the χ_b to the fitted ones. See text for details.

for the detailed analysis of correlators which are possible on lattices with many temporal points – the 3rd Generation lattices.

4. Understanding systematic errors on spectral functions: first results on the 3rd Generation lattices

In Figure 2, left, we present an overview of the results for $T = 2T_c$. We observe that $G_{3rdGen}(n_\tau) = G_{2ndGen}(2n_\tau)$ within the (small) errors, as it should be. The results of fits to the χ_b correlator are shown in Figure 2, center and right. In addition to the ansatz Eq. (2), we consider the standard spectral decomposition $G(\tau) = \sum_n a_n e^{-m_n \tau}$. Both fits apparently describe the data nicely, showing explicitly that very similar correlators might well be associated to vastily different spectral functions — a concrete demostration of the ill-posedness of the Laplace transform. Let us look again at the plots of Figure 1, for the χ_b at the highest temperature $T = 2T_c$: the peaks of the BR reconstruction might be reminiscent of the exponential fit, while the broader structure observed in the MEM reconstructed spectral function is indeed expected of a deformed power-law decay in Euclidean time Eq. (2).

By increasing statistics hopefully we should be able to identify with confidence the functional form which best fits the correlator. Correspondingly, and hopefully, both inversion methods should converge to the same — correct — spectral function. This analysis is in progress.

Finally, we note that Eq. (2) has the form expected from a mass deformed conformal theory with anomalous dimension. This is not surprising since the Quark Gluon Plasma is continously connected with the cold, conformal strongly interacting phase of QCD: our ansatz Eq. (2) might well be a natural baseline for the analysis of correlators, capturing the evolution from exponential to the free field power law decay.

5. Acknowledgements

We acknowledge the Science Foundation Ireland, grant 11-RFP.1-PHY-3201 (SMR), the grant NRF-2015R1A2A2A01005916 (SK), the STFC grants ST/K000411/1, ST/H008845/1, ST/K005804/1, ST/K005790/1 and ST/L000369/1, the STFC DiRAC HPC Facility (www.dirac.ac.uk), PRACE grants 2011040469 and Pra05_1129q, CINECA grant INF14_FTeCP (CINECA-INFN agreement).

[1] A. Andronic *et al.*, Eur. Phys. J. C **76** (2016) no.3, 107
[2] C. Ratti, this Volume
[3] J. I. Skullerud *et al.*, AIP Conf. Proc. **1701** (2016) 060018 and references therein
[4] For the tuning of parameters we rely on the analysis of the Hadron Spectrum Collaboration
[5] G. Aarts, C. Allton, T. Harris, S. Kim, M. P. Lombardo, S. M. Ryan and J. I. Skullerud, JHEP **1407** (2014) 097
[6] T.Harris, PhD Thesis, Trinity College, Dublin
[7] Y. Burnier and A. Rothkopf, Phys. Rev. Lett. **111**, 182003 (2013)
[8] A. Rothkopf, this Volume
[9] H. Ohno, this Volume
[10] H. T. Shu, H. T. Ding, O. Kaczmarek, S. Mukherjee and H. Ohno, arXiv:1510.02901 [hep-lat]
[11] G. Aarts, S. Kim, M. P. Lombardo, M. B. Oktay, S. M. Ryan, D. K. Sinclair and J.-I. Skullerud, Phys. Rev. Lett. **106** (2011) 061602.